

Only Dimensionless Numbers Can Be Fine-Tuned

— 3rd Edition: What $n = 2$ Selects and What It Does Not — [Paper 10]

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— *From thinking to syncing.* —

ABSTRACT

This paper called the divergence of $dm/dr = 4\pi r^{2-n}$ for $n \neq 2$ a “system crash theorem” and approached the fine-tuning problem from it. The second edition **withdrew the claim that the anthropic principle had been “debugged”** and separated the metaphors from the theorem. **The 3rd edition settles the four points that remained. First, only dimensionless numbers can be fine-tuned. c , $\hbar$ and G are dimensionful — that is, choices of units — so “if it had been a few percent different” cannot be distinguished from a redefinition of the units (Duff 2002). The six numbers of Rees's *Just Six Numbers*, cited by this paper, are all dimensionless ($N \sim 10^{36}$, $\epsilon = 0.00723$, Ω , λ , $Q \sim 10^{-5}$, $D = 3$) — the professionals had listed only dimensionless quantities from the start. That §3 of the second edition ranged c , $\hbar$, G , α , Λ in a single list was a confusion of categories. Second, the real boundary of “crash” is $n = 3$, not $n = 2$. The enclosed mass $m(R) = 4\pi\rho_0\int_0^R r^{2-n}dr$ is finite for every $n < 3$ and diverges logarithmically at $n = 3$. Nothing breaks for $2 < n < 3$. Third, real density profiles have $n \neq 2$ all the time. Uniform density is $n = 0$ — our own universe on large scales (FLRW) — where $dm/dr \rightarrow \infty$ and yet nothing has broken. An NFW halo has $n = 1$ inside and $n = 3$ outside. So “ $n < 2$ is a macroscopic crash” is incompatible with observation. Where the second edition reserved judgement, saying that “the distance between divergence and the absence of life has not been closed”, the 3rd edition shows that the distance is not merely open but runs the other way. Fourth, what then is $n = 2$? It is the unique exponent for which dm/dr does not depend on r — a different root from the finiteness of the enclosed mass ($n < 3$). And it is merely the case $d = 3$ of $n = d - 1$, which Paper 114 classified on the side that does not select a dimension. Scope: a structural account within established physics (Layer 2). This paper claims neither the refutation nor the endorsement of the anthropic principle or the multiverse hypothesis.**

Positioning of this paper (discipline): This paper offers a **structural interpretation** of established physical laws. It claims neither the discovery of new physical laws nor the proof of open problems. The relations shown are transformations of known expressions; the aim is to propose a structural reading of them. No claim contradicting established theory is made. **The appearance of the same number in several contexts is not treated as evidence of a common origin** (Papers 48, 86, 100, 115). $m = 4\pi$ and the like are conventional choices within a system of units, not new facts about nature (Papers 35, 38, 95).

1. Introduction: the theorem this paper showed, and the question that remained

The algebraic content of this paper is one thing. For a spherically symmetric distribution with $\rho \propto r^{-n}$ in three dimensions,

$$\frac{dm}{dr} = 4\pi r^{2-n},$$

which diverges as $r \rightarrow \infty$ if $2 - n > 0$, and as $r \rightarrow 0$ if $2 - n < 0$. This elementary fact is correct.

The second edition made explicit that the labels “memory overflow”, “freeze” and “crash” attached to this theorem are metaphors and not part of its content, and reserved judgement: “it does not follow immediately from a divergence that the universe cannot exist — a separate dynamical argument is required”. It also withdrew the claim that the anthropic principle had been “debugged”.

The 3rd edition closes that reservation.

The separate argument is in fact available — and it runs opposite to the direction the reservation suggested.

2. Only dimensionless numbers can be fine-tuned

Established Fact 2.1 (A dimensionful constant cannot be “tuned”)

c , $\hbar$ and G are **dimensionful**. Their numerical values depend entirely on the system of units, and **physics does not decide which value is correct**. Hence the question “what if c had been a few percent different” cannot be distinguished from a redefinition of the units (Duff 2002; the 3rd edition of Paper 34 turned the same criticism on itself).

Only dimensionless quantities can be tuned — the fine structure constant $\alpha = 1/137.035999$, the cosmological constant in Planck units $\Lambda \ell_p^2 \sim 10^{-122}$, and so on.

Observation 2.2 (The professionals had listed only dimensionless quantities from the start)

The six numbers of Rees's *Just Six Numbers* (1999), cited in §1 of this paper, are **all dimensionless**:

quantity	value	meaning
N	$\sim 10^{36}$	ratio of electromagnetic to gravitational force
ε	0.00723	nuclear binding efficiency $H \rightarrow He$ (computed here)
Ω	~ 1	density parameter of the universe
λ	~ 0.7	contribution of the cosmological constant
Q	$\sim 10^{-5}$	amplitude of density fluctuations
D	3	spatial dimension (an integer)

The fine-tuning debate has been framed from the beginning as a discussion of dimensionless numbers.

Corollary 2.3 (The list in §3 of the second edition confused categories)

Section 3 of the second edition **ranged $c, \hbar, G, \alpha, \Lambda$ in a single list**, attaching to each a label such as “bandwidth”, “resolution” or “compression ratio”.

But the first three are choices of units and the latter two are (once made dimensionless) facts. Things that must not be ranged together were ranged together — the same type of error as the 3rd edition of Paper 15 found in grouping $m = 4\pi$ (a choice of units) and $\varepsilon_0 = r$ (a function of the configuration) under the one word “Yamagishi patch”. **The format of a list concealed the difference of category.**

3. The real boundary of “crash” is $n = 3$

Established Fact 3.1 (The enclosed mass is finite for every $n < 3$)

For $\rho = \rho_0 r^{-n}$, the mass within a sphere of radius R is

$$m(R) = 4\pi\rho_0 \int_0^R r^{2-n} dr,$$

which converges at the origin iff $2 - n > -1$, i.e. $n < 3$ (verified here):

n	$m(R)$	consequence
$n < 3$	$4\pi\rho_0 R^{3-n}/(3-n)$	finite
$n = 3$	$\propto \ln R$	logarithmically divergent (the real boundary)
$n > 3$	divergent at the origin	the enclosed mass is infinite

For $2 < n < 3$ the enclosed mass remains finite. Nothing breaks.

Corollary 3.2 (The divergence of dm/dr and the divergence of m are different things)

What the theorem of the second edition stated was **the divergence of a rate, dm/dr , not the divergence of the mass itself**. A rate may diverge while its integral stays finite — which is exactly the case for $2 < n < 3$.

The word “crash” was too strong for the divergence of a rate.

4. Real density profiles have $n \neq 2$ all the time

Observation 4.1 ($n = 0$ is our own universe)

profile	n	note
uniform density (large-scale FLRW; interior of a uniform sphere)	0	$dm/dr = 4\pi\rho_0r^2 \rightarrow \infty$
NFW halo, inner region	1	the cusp
isothermal sphere	2	this paper's $n = 2$
NFW halo, outer region	3	mass diverges logarithmically; a cutoff is used

Uniform density is $n = 0$, and there dm/dr diverges as $r \rightarrow \infty$. And that is the standard picture cosmology adopts on large scales. Nothing has broken.

Corollary 4.2 (“ $n < 2$ is a macroscopic crash” is incompatible with observation)

The second edition reserved judgement, writing that “what a divergent quantity means physically requires a separate dynamical argument, which this paper does not supply”. The 3rd edition supplies part of that argument — and it runs the other way.

On the $n < 2$ side, the most basic case of all, $n = 0$ (uniform), actually holds, and the divergence of dm/dr there signifies no breakdown whatever. The 3rd edition withdraws the label “ $n < 2$ is a macroscopic memory overflow”.

The label “microscopic singularity freeze” on the $n > 2$ side is withdrawn likewise — for $2 < n < 3$ the enclosed mass is finite, and the inner cusp ($n = 1$) and outer slope ($n = 3$) of NFW haloes are structures actually observed.

5. What, then, is $n = 2$? — the two special values are distinct roots

Observation 5.1 (What $n = 2$ satisfies is that dm/dr does not depend on r)

At $n = 2$, $dm/dr = 4\pi\rho_0$ is a constant independent of r , giving the linear extension $m(R) = 4\pi\rho_0 R$. This is correct and peculiar to $n = 2$.

But this is linearity, not stability. Which of the two words is used changes the strength of the claim completely.

Corollary 5.2 (The two special values of n must not be mixed)

value	what is special	root
$n = 2$	dm/dr independent of r (linear extension)	geometry: balance against r^{d-1}
$n = 3$	the enclosed mass ceases to be finite	analysis: convergence of an integral

These are distinct roots. The first and second editions spoke of the uniqueness of the former as though it carried the “boundary of breakdown” that belongs to the latter. In the vocabulary of Papers 86 and 100, two roots had been folded into one claim.

And in the vocabulary of Paper 113, the module boundary is the one at $n = 3$ — there the description changes (a cutoff becomes necessary). At $n = 2$ nothing changes.

6. $n = 2$ is merely the case $d = 3$ of $n = d - 1$

Established Fact 6.1 (The same condition in general dimension)

In $\mathbb{R}^d$ we have $dm/dr = \text{Vol}(S^{d-1}) r^{d-1-n}$, so the condition of independence from r is $n = d - 1$. Verified here: $d = 2, 3, 4, 5$ give $n = 1, 2, 3, 4$.

Corollary 6.2 (This formula selects no dimension)

Paper 114 took a census of **which structures select a dimension and which do not**, and **classified $n = d - 1$ on the side that does not**. The 3rd edition of Paper 06 repeated the same judgement.

The uniqueness of $n = 2$ is therefore uniqueness given $d = 3$. Section 4(3) of the second edition correctly wrote that **“why $d = 3$ lies outside this paper”**, but the 3rd edition writes it back beside the theorem — **an assumption belongs next to the theorem, not in the scope section**.

7. The “universe OS system API” is Layer 3

Observation 7.1 (A definition environment made a metaphor look like mathematics)

Section 3 of the second edition placed a **“universe OS system API specification”** inside a definition environment, describing c as “bandwidth”, $\hbar$ as “resolution” and G as “compression ratio”. The second edition noted at the head of the section that **“this section is an interpretation, not a theorem”**, but **the typesetting remained that of a definition**.

Paper 82 explicitly quarantined this kind of language (OS, rendering, bandwidth) in Layer 3, the non-scientific layer. The 3rd edition removes the section from the body and moves it to the Layer-3 box.

Noting something in a remark and showing it in the typesetting are different acts — the reader reads the name of the environment. For the same reason the 3rd edition of Paper 17 wrote that **“naming is not deriving”**, a **definition environment is used only for definitions**.

Personal reflection of the author (Layer 3; labelled non-scientific / no claim of mechanism)

In the first edition I wrote that I had “completed the debugging of the anthropic principle”. The second edition withdrew that and reserved judgement: **“the distance between divergence and the absence of life has not been closed”**. Reserving judgement, I thought, had made me honest.

Going back over it for the 3rd edition, I found that **reserving judgement had not been enough. The distance is not open. It runs the other way.** Uniform density — $n = 0$ — lies on the side I had called a “macroscopic crash”. **And that is the standard picture of our own universe.**

Writing “I have not shown this yet” looks safe, and is not. Whether it is unshown, or whether the opposite can be shown, **cannot be known without looking.** I had reserved without looking.

And something about $n = 2$ does remain. **It is the unique exponent for which dm/dr does not depend on r** — that is correct. **But it is linearity, not stability, and it is uniqueness given $d = 3$.** Say a true thing at its true size. That was the work this time.

8. Scope / Conclusion

What is shown: that only dimensionless numbers can be fine-tuned, $c, \hbar, G$ being choices of units, and that Rees's six numbers are all dimensionless (§2) / that the real boundary at which the enclosed mass ceases to be finite is $n = 3$, nothing breaking for $2 < n < 3$ (§3) / that uniform density $n = 0$ is our own universe, so “ $n < 2$ is a macroscopic crash” is incompatible with observation (§4, the metaphorical labels withdrawn) / that what is special about $n = 2$ is the linearity of dm/dr , a distinct root from $n = 3$ (convergence of an integral) (§5) / that $n = 2$ is merely the case $d = 3$ of $n = d - 1$ (§6, Paper 114) / that the “universe OS system API” is removed from the body (§7, Paper 82).

What is not shown (restraint): no refutation of the anthropic principle or the multiverse hypothesis — the first edition's “debugging completed” was withdrawn in the second and lies outside the scope of this paper / **nor any endorsement of them** / **not that observers fail to arise in a universe with $n \neq 2$** (§4 gives evidence in the opposite direction) / **nothing about the fine-tuning of α, Λ and the rest** / **why $d = 3$** (§6, outside this paper) / no new physical law and no prediction (Layer 2).

What the 3rd edition adds is the closing of a reservation. Not stopping at “I have not shown this yet”, but looking — and this time what the looking found was that the metaphor collided with observation.

The theorem itself stands unchanged from the first edition.

$n = 2$ is the unique exponent for which dm/dr does not depend on r , giving the linear extension

$$m(R) = 4\pi\rho_0 R.$$

But that is linearity, not stability, and it is uniqueness given $d = 3$.

Revision Record

3rd edition (August 2026): the title was changed to name **what the paper shows**.

Added in the 3rd edition: a statement that **only dimensionless numbers can be fine-tuned**, $c, \hbar, G$ being dimensionful and hence choices of units (Duff 2002), with a table showing that **the six numbers of Rees's *Just Six Numbers***, cited by this paper, are all dimensionless (§2; $\varepsilon = 0.00723$ computed here). A demonstration that **the enclosed mass $m(R)$ is finite for every $n < 3$, the real boundary being $n = 3$ (logarithmic divergence) (§3)**. A table of real density profiles — uniform ($n = 0$, the standard large-scale picture), NFW inner ($n = 1$) and outer ($n = 3$) — showing that **$n \neq 2$ occurs constantly in nature (§4)**. An explicit statement that **$n = 2$ (linearity, geometry) and $n = 3$ (convergence, analysis) are distinct roots (§5)**. A note that **$n = 2$ is merely the case $d = 3$ of $n = d - 1$, classified by Paper 114 on the side that selects no dimension (§6)**. All numbers were computed here (no external tables consulted).

Withdrawn in the 3rd edition:

1. The metaphorical labels “macroscopic memory overflow”, “microscopic singularity freeze” and “system crash”. The second edition retained them while noting that they were “**metaphors, not part of the theorem**”; but **$n = 0$ (uniform density) is the standard picture of our own universe, and for $2 < n < 3$ the enclosed mass is finite. The labels are withdrawn because they collide with observation (§§3–4)**.
2. Section 3, the “universe OS system API specification”, is removed from the body. The second edition noted that it was “**an interpretation, not a theorem**”, but the typesetting remained a **definition environment**. It is moved to **Layer 3 under the discipline of Paper 82 (§7)**. It is also recorded that **ranging $c, \hbar, G, \alpha, \Lambda$ in one list was a confusion of categories (§2)**.
3. The second edition's reservation that “**the distance between divergence and the absence of life requires a separate argument**”. The 3rd edition supplies part of that argument and shows that **the distance is not merely open but runs the other way (§4)**.

Withdrawn in the 2nd edition (July 2026): the first edition stated in its title, abstract and conclusion that it “**proves**”, that “**the debugging of the anthropic principle is complete**”, and that “**the need to rely on unverifiable multiverse hypotheses has entirely vanished**”. All were withdrawn. What the paper did was not a proof but a **characterisation of one parameter, together with a limitation of its scope**. The second edition also **separated the metaphors from the theorem**,

making explicit that **what is algebraically established is the fact of divergence alone**.

First edition: Zenodo 10.5281/zenodo.21404556.

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