The Radius of a Dust-Filled Static Universe Equals the Distance Light Travels While a Disturbance Grows by e

— Einstein's static universe balances at $\Lambda c^2 = 4\pi G\rho$, disturbances grow at the rate $\sqrt{(4\pi G\rho)}$, and the curvature radius is $c/\sqrt{(4\pi G\rho)}$ — the separator is pressure, and the ratio of the radius to the e-folding time becomes $\sqrt{(1+3w)}$ — [Paper 1001]

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September 2026

— From thinking to syncing. —

Abstract

The static universe Einstein set up in 1917 stops by letting a cosmological term Λ cancel the pull of gravity, and, as Eddington showed in 1930, it cannot stay put once slightly displaced.This paper counts, both with a Newtonian shell and with the acceleration equation of general relativity, the condition for standing still, the rate at which a displacement grows, and the curvature radius of space, and compares them while varying w in the equation of state $p = w\rho c^2$.No new theorem or law is claimed. First, the balance is $\Lambda c^2 = 4\pi G\rho(1+3w)$, and shells of every radius balance at once——for dust (w = 0) both the shell equation and the relativistic equation return $\Lambda c^2 =$ 12.566370614 ($\rho = 1$), and shells of radius 0.3, 1 and 7 stop at the same Λ. Filled with light, Λ becomes 2.000000 times that of dust (Section 2). **Second, a displacement grows at the rate $\sqrt{(4\pi G\rho)}$ whether space expands or contracts——integrating from a displaced rest state, the dust shell gives 3.544907 for expansion and 3.544909 for contraction, matching 3.544908 from the formula. For general w the square of the rate is $4\pi G\rho(1+3w)(1+w)$, and a light-filled universe collapses 1.632993 times faster than dust** (Section 3). **Third, and this is the core. The curvature radius of a dust-filled static universe equals the distance cτ that light travels in the time τ a displacement takes to grow by e——the ratio of the radius from Friedmann's first equation to cτ is 1.000000 for dust, 1.414213 for light ($\sqrt{2} = 1.414214$), 1.140175 for w = 0.1 and 1.999999 for w = 1, all matching $\sqrt{(1+3w)}$** (Section 4). **Fourth, reading the shell's energy as curvature gives the same radius——the energy per unit mass of a static shell of radius 1 is −6.283185 ($-2\pi G\rho$); reading it as $-c^2r^2/(2R^2)$ gives R = 0.282095, the radius from the first equation** (Section 4). **Fifth, for dust, with M the product of the three-sphere's volume and the density, $2GM/c^2$ equals the distance to the antipodal point——writing M for the volume $2\pi^2a^3$ times the density, $2GM/(c^2a)$ is $\pi/(1+w)$: 3.141593 for dust and 2.356194 for light** (Section 5). **Sixth, the separator is pressure——w enters Λ as 1+3w, the squared collapse rate as (1+3w)(1+w) and the inverse squared radius as 1+w, and only $\sqrt{(1+3w)}$ survives in the ratio. With the representative $\rho = 1.0\times10^{-26}$ kg/m³, τ for dust is 10.941787 billion years and the radius is 10.941787 billion light years** (Section 6). **Keywords**: Einstein static universe, cosmological constant, Friedmann equations, Eddington instability, equation of state, three-sphere.

Scope of this paper (scope note): **No new theorem or law is claimed——Einstein's static universe (1917) with its balance $\Lambda = 4\pi G\rho/c^2$, the instability shown by Eddington (1930), Friedmann's two equations, and the Newtonian reading that puts the cosmological term into a shell as the force $\Lambda c^2 r/3$ are all known.No claim is made about the actual universe——the model is counted with $G = c = 1$ and $\rho = 1$, and the SI value $\rho = 1.0\times10^{-26}$ kg/m³ is a representative value for seeing orders of magnitude. No comparison with the observed Λ is made.Only uniform expansion and contraction are considered——only displacements in which the whole universe stretches or shrinks uniformly are counted; local fluctuations (the Jeans instability) are not treated.The total mass of a closed universe is not defined——M is only written as the product of the three-sphere's volume and the density, and no meaning as the mass of a closed universe is claimed.w is restricted to constants——w is a constant greater than −1/3, and the numbers were checked at the four points $w = 0, 0.1, 1/3$ and 1. A w that changes with time is not treated.Relation to earlier papers**: **the scope note of Paper 907 said that it does not treat the cosmological constant, leaving aside dark energy, the cosmological constant itself and the geometry of spatial curvature; this paper sits in the seat of the cosmological constant** (no earlier occupant).**Paper 907 showed that the expansion equation for dust follows exactly from a Newtonian shell, while for light the deceleration is off by a factor of two——here it is counted that the same term $\rho + 3p/c^2$ multiplies the Λ needed for standing still by $1+3w$ and makes the ratio of the radius to the collapse time $\sqrt{(1+3w)}$.Paper 72 read Λ as an integration constant rather than a vacuum energy——here the origin of Λ's value is not questioned; only the fact that standing still fixes one ratio of Λ to ρ is used.Paper 13 showed that the equations of gravity and electromagnetism share the solid angle 4π——here it is counted that the same 4π enters both the balance $\Lambda c^2 = 4\pi G\rho$ and the collapse rate $\sqrt{(4\pi G\rho)}$.Paper 08 set the volume $2\pi^2$ of the three-sphere as the four-dimensional unit patch——here the space of the static universe is itself a three-sphere of radius a, and its volume $2\pi^2a^3$ is counted.**What is added is **finding the static Λ by bisection in the shell equation and the relativistic equation and matching it to $4\pi G\rho(1+3w)$ for four values of w, measuring the collapse rate in both directions by integrating from a displaced state, showing that the ratio of the radius to the e-folding time is $\sqrt{(1+3w)}$ and exactly 1 for dust, rederiving the same radius from the shell's energy**, and **counting $2GM/(c^2a) = \pi/(1+w)$ from the three-sphere's volume**.

1. Notation and the Question

Setting: **the Newtonian shell is a shell of radius r at the edge of a sphere of uniform density ρ, pulled by the inner mass $M = (4\pi/3)\rho r^3$ and pushed by the cosmological force $\Lambda c^2 r/3$.**

$$\ddot{r} = -\frac{GM}{r^2} + \frac{\Lambda c^2}{3} r.$$

In general relativity the acceleration of the scale factor a and Friedmann's first equation (spatial curvature k) are used. The pressure is $p = w\rho c^2$ and the density changes as $\rho \propto a^{-3(1+w)}$.

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}, \qquad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}.$$

The model is counted with $G = c = 1$ and $\rho = 1$; only Section 6 uses representative SI values.

Question: at what Λ does a static universe stand still, at what rate does it collapse, what radius does it have, and what relation remains among the three?

2. The Balance

Computation 2.1: the Λ at which the acceleration vanishes was found by bisection.

equation	w	$\Lambda c^2/(4\pi G\rho)$
shell (no pressure)	none	**1.000000**
general relativity	**0**	**1.000000**
general relativity	**0.1**	**1.300000**
general relativity	**1/3**	**2.000000**
general relativity	**1**	**4.000000**

For dust both the shell and general relativity return $\Lambda c^2 = 12.566370614$, and for the four values of w the result agrees with $4\pi G\rho(1+3w)$ within 10^{-9}.

> **Observation 2.2**
>
> **shells of every radius balance at once.Putting $\Lambda c^2 = 4\pi G\rho$ into shells of radius 0.3, 1 and 7 gives accelerations of 2.220×10^{-16}, 0 and 3.553×10^{-15}——the shell equation has no length scale, and the condition for standing still is fixed by the density alone. For dust, the shell equation with fixed mass and the relativistic equation with density varying as a^{-3}, written as separate functions, returned the same acceleration (difference 0).**

> **Observation 2.3**
>
> **counting a light-filled universe with a pressureless shell underestimates the needed Λ by half.The shell gives $\Lambda c^2 = 12.566371$ and general relativity 25.132741, a ratio of 2.000000——the pressure term $3p/c^2$ raises the push needed to stop.**

3. The Collapse Rate

Computation 3.1: starting from rest displaced by 10^{-8}, the equation was integrated and the rate s was obtained by assuming the displacement grows as $\cosh(st)$.For the dust shell, by $t = 2$ the displacement grew 599.842 times outward and 599.845 times inward, giving s = 3.544907 and 3.544909 ($\sqrt{4\pi G\rho} = 3.544908$).

w	s by integration	square root of acceleration slope	$\sqrt{4\pi G\rho(1+3w)(1+w)}$
0	3.544907	3.544908	3.544908
0.1	4.239092	4.239093	4.239093
1/3	5.788808	5.788810	5.788810
1	10.026506	10.026513	10.026513

> **Observation 3.2**
>
> a static universe collapses both outward and inward.The growth of the displacement is the same in both directions, and the square of the rate is $4\pi G\rho(1+3w)(1+w)$——a light-filled universe collapses 1.632993 times faster than dust. The integration time was chosen for each w so that the displacement stays in the linear range (up to about 600 times).

4. The Radius and the Collapse Time

Computation 4.1: the curvature radius a was taken from the first equation at rest ($\dot{a} = 0$) with $k = +1$ and compared with the time $\tau = 1/s$ for a displacement to grow by e.

w	a	τ	$a/(c\tau)$	$\sqrt{1+3w}$	Λa^2
0	0.282095	0.282095	1.000000	1.000000	1.000000
0.1	0.268967	0.235900	1.140175	1.140175	1.181818
1/3	0.244301	0.172747	1.414213	1.414214	1.500000
1	0.199471	0.099736	1.999999	2.000000	2.000000

Observation 4.2

this is the core. The radius of a dust-filled static universe equals the distance light travels while a displacement grows by e.For dust $a/(c\tau) = 1.000000$, and $\Lambda a^2 = 1.000000$ is Einstein's relation $\Lambda = 1/a^2$ itself——the inverse squared radius is $4\pi G\rho(1+w)/c^2$ and the squared collapse rate is $4\pi G\rho(1+3w)(1+w)$, so only $1+3w$ remains in the squared ratio. For light the ratio becomes $\sqrt{2}$.

Observation 4.3

reading the shell's energy as curvature gives the same radius.The energy per unit mass of a static shell of radius 1, $E = -GM/r - \Lambda c^2 r^2/6$, is -6.283185 ($-2\pi G\rho$); reading it as $E = -c^2 r^2/(2R^2)$ gives $R = 0.282095$, which agrees with the radius 0.282095 from the first equation within 10^{-12}——the shell equation itself has no length scale, but reading its energy as the curvature term fixes a single radius.

5. The Volume of the Three-Sphere

Computation 5.1: the volume of the unit three-sphere, integrated as $\int_0^\pi 4\pi \sin^2\chi\, d\chi$ by Simpson's rule with 20000 steps, is 19.739208802, agreeing with $2\pi^2$. The volume $2\pi^2 a^3$ times the density was written as M.

w	$2GM/(c^2 a)$	$\pi/(1+w)$	distance to the antipode πa	$2GM/c^2$
0	3.141593	3.141593	0.886227	0.886227
0.1	2.855993	2.855993	0.844984	0.768167
1/3	2.356194	2.356194	0.767495	0.575621
1	1.570796	1.570796	0.626657	0.313329

Observation 5.2

for dust, 2GM/c² equals the distance πa to the antipodal point on the three-sphere.For dust both $2GM/c^2$ and πa are 0.886227, and with pressure their ratio shrinks as $1/(1+w)$——this is an equality about the product of volume and density, not a definition of the total mass of a closed universe.

6. The Separator and Representative Values

> **Observation 6.1**
>
> **the separator is pressure.w enters the Λ needed for standing still as $1+3w$, the squared collapse rate as $(1+3w)(1+w)$, and the inverse squared radius as $1+w$——of these, only $\sqrt{1+3w}$ survives in the ratio of the radius to the e-folding time. For dust the pressure term vanishes and the ratio is exactly 1.**

Computation 6.2: **in SI, $G = 6.67430 \times 10^{-11}$, $c = 299792458$ m/s, $\rho = 1.0 \times 10^{-26}$ kg/m^3, and a year of 365.25 days were used.**

quantity	dust	light (same ρ)
Λ (m^{-2})	9.331988×10^{-53}	1.866398×10^{-52}
τ (billion years)	10.941787	6.700448
a (billion light years)	10.941787	9.475865

For dust $\tau = 3.452965 \times 10^{17}$ s and $a = 1.035173 \times 10^{26}$ m; counting the radius in light years and the collapse time in years gives the same number.

> **Observation 6.3**
>
> **what is not said.No comparison is made with the density of the actual universe or with the observed Λ. Local fluctuations (the Jeans instability) and a time-varying w are not treated——M was only written as the product of volume and density, and no claim is made that gives a closed universe a total mass.**

7. Census and Closing

1. $\Lambda c^2 = 4\pi G\rho(1+3w)$ **(12.566370614 for dust)** (Computation 2.1): **shells balance at every radius.**
2. **counting light without pressure halves Λ (ratio 2.000000)** (Observation 2.3): **the pressure term raises the push.**
3. **collapse rate 3.544907 and 3.544909 (formula 3.544908)** (Computation 3.1): **outward and inward.**
4. **light collapses 1.632993 times faster than dust** (Observation 3.2): **the squared rate is $4\pi G\rho(1+3w)(1+w)$.**
5. $a/(c\tau) = 1.000000$ **(dust)** (Observation 4.2): **the core.**

6. **1.414213 for light, 1.999999 for $w = 1$** (Computation 4.1): **the ratio is $\sqrt{1 + 3w}$.**
7. **shell energy -6.283185 gives $R = 0.282095$** (Observation 4.3): **the same radius by the reading of Paper 907.**
8. **$2GM/(c^2a) = 3.141593$ (dust)** (Observation 5.2): **$2GM/c^2$ is the distance to the antipode.**
9. **$\tau = 10.941787$ billion years, $a = 10.941787$ billion light years** (Computation 6.2): **orders of magnitude for a representative value.**
10. **pressure leaves only $\sqrt{1 + 3w}$ in the ratio** (Observation 6.1): **the separator; Papers 907, 72, 13, 08.**

Closing: **The radius of a dust-filled static universe equals the distance light travels while a disturbance grows by e.Einstein's static universe balances at Λc² = 4πGρ(1+3w) in both the shell equation and the relativistic equation, and a displacement grows outward and inward at the rate √(4πGρ(1+3w)(1+w))——for the dust shell, integration gave 3.544907 and 3.544909, matching 3.544908 from the formula.The ratio of the radius from Friedmann's first equation to the e-folding time is 1.000000 for dust and 1.414213 for light, all matching √(1+3w)——reading the shell energy −2πGρr² as curvature gives the same radius 0.282095.The separator is pressure——w enters the balance, the collapse rate and the radius in different forms, and only √(1+3w) remains in the ratio.Placed among the earlier papers——Paper 907 showed that the expansion of dust follows exactly from a shell, Paper 72 read Λ as an integration constant, Paper 13 showed the shared 4π, and Paper 08 set the volume 2π² of the three-sphere. Here it was counted that in a universe standing still the balance, the collapse rate and the radius all come from the same 4πGρ, and that for dust the radius is the distance light covers in the collapse time.To be honest——the model covers only uniform expansion and contraction and is compared with neither the actual universe nor the observed Λ. That 2GM/c² equals the distance to the antipode is an equality about the product of volume and density, not a definition of the mass of a closed universe.**

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