The Three-Body Moment of Inertia Is a Parabola Only in Four Dimensions, Where the Spinning Triangle Has Zero Energy

— in D-dimensional gravity Ï = 4E + 2(D−4)V holds, and only in four dimensions does the potential term drop out, so three bodies with negative energy stop before the root of the parabola — the separator is whether the force exponent n = D−1 equals 3 — [Paper 1002]

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September 2026

— From thinking to syncing. —

Abstract

The second time derivative Ï of the N-body moment of inertia $I = \Sigma m|x|^2$ can be written in terms of the kinetic and potential energies (the Lagrange-Jacobi identity).This paper counts that identity and the motion of three bodies under D-dimensional gravity, where the force falls off as the (D−1)-th power of r because the field spreads over $4\pi r^2$ in three dimensions and $2\pi^2 r^3$ in four.No new theorem or law is claimed. First, $Ï = 4E + 2(D−4)V$, and only in four dimensions does the potential term drop out——Ï built from the force function and $4E + 2(D−4)V$ built from the potential function agreed within a relative difference of 8.882×10^{-16} for 20 random states each at D = 3, 4 and 5 (Section 2). **Second, the energy of the spinning triangle changes sign in four dimensions——spinning three equal masses on an equilateral triangle of side 1 gives the energy $3(D−4)/(2(D−2))$: -1.500000 in three dimensions, 0 in four, 0.500000 in five and 1.000000 in eight** (Section 3). **Third, and this is the core. The moment of inertia of three bodies in four dimensions follows a parabola in time exactly——slowing the spinning triangle to 0.99 times its speed gives the energy -0.029850, and the time at which I falls to 1 percent of its initial value, 4.072212491, agrees with the parabola's prediction in every digit. The integration stops at 4.092370, just before the root 4.092728, where I also agrees with the parabola at 1.748861×10^{-4}** (Section 4). **Fourth, three bodies with negative energy stop by collision before the root of the parabola——five random three-body systems in four dimensions kept I within a relative difference of 1.362×10^{-12} of the parabola while two bodies closed to a distance of 10^{-3} at 0.162020 to 0.853067 times the root** (Section 5). **Fifth, the triangle in three dimensions keeps spinning at negative energy——the three-dimensional triangle has energy -1.500000, and by the four-dimensional rule I would reach 0 at 0.577350, yet I stayed constant within 3.215×10^{-12} over two turns of period 3.627599** (Section 6). **Sixth, the separator is whether the force exponent $n = D−1$ equals 3——the coefficient of the potential energy is $2(n−3)$ and the energy of the spinning triangle is proportional to $n−3$, both vanishing at $n = 3$** (Section 7). **Keywords**: Lagrange-Jacobi identity, three-body problem, moment of inertia, Lagrange equilateral solution, dimension of space, inverse-cube force.

Scope of this paper (scope note): **No new theorem or law is claimed——the Lagrange-Jacobi identity (Jacobi), the virial theorem, Lagrange's equilateral solution (1772), the fact that gravity in D dimensions falls off as the (D−1)-th power of r, and the marginality of circular orbits under an inverse-cube force are all known.No claim is made about an actual four-dimensional world——D is only varied to count equations and motions; no claim is made that a world with four-dimensional gravity exists.What happens after a collision is not treated——integration stops when two bodies close to 10^{-3}, when I falls to 1 percent of its initial value, or when the integrator halts at its step limit; regularization of collisions and motion after them are not treated.Kinds of singularity are not distinguished——total collapse and earlier binary collisions are counted only by the single fact that motion stops before the root of the parabola. Whether non-collision singularities exist is not treated.The conditions for the triangle to keep its shape are not discussed——the spinning triangle was turned for two periods only, and the conditions under which its shape is kept are not treated. Why the change of I in five dimensions is larger than in three was not examined either.The masses are limited——the triangle uses equal masses only, and only five random three-body cases were counted.Relation to earlier papers**: **the scope note of Paper 490 said that it builds no celestial mechanics and looks only inside the circular restricted three-body problem; this paper sits in the seat outside it, a three-body problem in which all three masses act** (no earlier occupant).**Paper 01 showed n = 2 in three dimensions and Paper 08 showed n = 3 in four dimensions as the unique exponents at which the mass of a spherical shell grows uniformly——here it is counted that the same n = D−1 enters the three-body moment of inertia as the exponent of the gravitational force, and that the potential term drops out at n = 3.Paper 116 showed that the 3 in c^3 of the Planck length is a weight of time, and that the dimension of space is found in the denominator 1/n (n = d−1)——here it is counted that the same n fixes the coefficient 2(n−3) of the potential energy.Paper 56 stated that d = 3 is the only dimension where flux conservation agrees with orbital stability——here it is counted that one dimension higher, in four, the triangle spinning with its shape kept has zero energy.**What is added is **building the identity separately from the force function and the potential function and checking it in three dimensions of space**, **showing in six dimensions that the triangle's energy is 3(D−4)/(2(D−2)) with the sign change at four**, **confirming by integration that in four dimensions I follows the parabola and collapses at the predicted time**, **counting in five cases that random three-body systems stop by collision before the root**, and **showing that the four-dimensional rule fails for the triangle in three and five dimensions**.

1. Notation and the Question

Setting: **three bodies of mass m_i are placed in D-dimensional space. The field spreads over the unit sphere of area S_{D-1} (4π in three dimensions, $2\pi^2$ in four), so the force between two bodies has magnitude Gm_im_j/r^{D-1} and the potential energy is $-Gm_im_j/((D-2)r^{D-2})$.The centre of mass is at the origin and the moment of inertia is $I=\sum_i m_i|x_i|^2$.**

$$\ddot{I}=2\sum_i m_i\left(|v_i|^2+x_i\cdot a_i\right)=4T+2\sum_i x_i\cdot F_i.$$

The potential energy V is homogeneous of degree $-(D-2)$ in r, so $\sum_i x_i \cdot F_i = (D-2)V$, and with $E = T + V$ one obtains:

$$\ddot{I} = 4E + 2(D-4)V.$$

Question: in the dimension where the potential energy drops out of this identity, what happens to the motion of three bodies, and how does it differ from other dimensions?

2. The Identity

Computation 2.1: 20 sets of masses between 0.5 and 2 and random positions and velocities were made for each D, and $\ddot{I}$ built numerically from the force function was compared with $4E + 2(D-4)V$ built from the potential function.

D	S_{D-1}	force	coefficient of V, $2(D-4)$	largest relative difference
3	12.566371	$1/r^2$	-2	8.882×10^{-16}
4	19.739209	$1/r^3$	0	2.555×10^{-16}
5	26.318945	$1/r^4$	$+2$	3.305×10^{-16}

> **Observation 2.2**
>
> only in four dimensions does the potential term drop out, leaving $\ddot{I} = 4E$.In three dimensions $\ddot{I} = 4E - 2V$, and $V < 0$ pushes $\ddot{I}$ up; in five dimensions $\ddot{I} = 4E + 2V$, which pushes it down——in four dimensions $I(t) = I_0 + \dot{I}_0 t + 2Et^2$ holds exactly until a collision. If $E < 0$ the parabola eventually goes below 0, so the motion must stop before that.

3. The Spinning Triangle

Computation 3.1: three bodies of equal mass 1 were placed on an equilateral triangle of side 1 and spun at the speed where the inward force $\sqrt{3}Gm^2/a^{D-1}$ balances the centrifugal force (the angular speed is 1.732051 in every D).

D	energy E	$3(D-4)/(2(D-2))$
3	-1.500000	-1.500000
4	0.000000	0.000000
5	0.500000	0.500000
6	0.750000	0.750000
7	0.900000	0.900000
8	1.000000	1.000000

Observation 3.2

the energy of the spinning triangle changes sign in four dimensions.The kinetic energy is $(3/2)Gm^2/a^{D-2}$ and the potential energy $-3Gm^2/((D-2)a^{D-2})$, and their sum is proportional to $D-4$——the triangle is bound in three dimensions, has exactly $E=0$ in four, and spins with positive energy in five or more.

4. Changing the Speed in Four Dimensions

Computation 4.1: **the four-dimensional triangle was integrated at 1, 0.99 and 1.01 times its speed (DOP853, relative tolerance 10^{-12}).**

speed	E	what was seen	difference from I_0+2Et^2
1 times	2.220446×10^{-16}	$I=1.000000$ at $t=10$	7.170×10^{-11}
0.99 times	-0.029850	time 4.072212491 for I to fall to 1 percent of I_0	4.074×10^{-12}
1.01 times	0.030150	$I=7.030000$ at $t=10$	1.251×10^{-10}

Observation 4.2

this is the core. The moment of inertia of three bodies in four dimensions follows a parabola in time exactly.At 0.99 times the speed, the time for I to fall to 1 percent of I_0 is 4.072212491, agreeing in every digit with the parabola's prediction 4.072212491——run without stopping, the integrator halts 0.000358 before the root 4.092728, at $t = 4.092370$, when its step becomes smaller than the spacing of double-precision numbers, and the $I = 1.748861 \times 10^{-4}$ there also agrees with the parabola's value 1.748861×10^{-4}. The triangle collapses toward its centre when it is only slightly too slow, and flies apart when slightly too fast.

5. Random Three Bodies in Four Dimensions

Computation 5.1: three bodies with random masses, positions and velocities were placed in four dimensions, five cases with $E < 0$ were chosen, and integration was stopped when two bodies closed to a distance of 10^{-3}.

case	E	I_0	root of parabola	stopping time	largest relative difference of I
1	-0.334297	5.060610	2.442948	1.562183	5.393×10^{-13}
2	-0.098366	14.202612	3.192558	2.723466	4.593×10^{-13}
3	-0.056191	7.508706	4.857038	4.113223	1.362×10^{-12}
4	-0.195875	18.811864	10.372401	1.680540	5.949×10^{-14}
5	-0.142467	5.522696	9.767905	7.139431	8.450×10^{-13}

Observation 5.2

three bodies with negative energy stop by collision before the root of the parabola.The ratios of the stopping time to the root were 0.639466, 0.853067, 0.846858, 0.162020 and 0.730907, all less than 1——since I cannot be negative, the root of the parabola gives an upper bound on when the motion stops. In all five cases, the stopping point was where two bodies closed to a distance of 10^{-3}.

6. The Triangle in Three and Five Dimensions

Computation 6.1: a triangle of side 1 was spun for two periods in three and in five dimensions (the period is 3.627599 in both).

D	E	largest change of I over two periods	I after two periods by the four-dimensional rule
3	-1.500000	3.215×10^{-12}	-156.913670
5	0.500000	6.842×10^{-8}	53.637890

Observation 6.2

the triangle in three dimensions keeps spinning at negative energy.By the four-dimensional rule I would reach 0 at $t = 0.577350$, yet the three-dimensional triangle kept I constant over two turns of period 3.627599——in three dimensions $-2V$ pushes $\ddot{I}$ up and cancels $4E$; in five dimensions $+2V$ pushes it down and cancels a positive $4E$. Both give $\ddot{I} = 0$, not a parabola. The change 6.842×10^{-8} of I in five dimensions is larger than the 3.215×10^{-12} in three dimensions, but stayed within 10^{-6} over two periods.

7. The Separator and What Is Not Said

Observation 7.1

the separator is whether the force exponent n = D−1 equals 3.Writing the force exponent as $n = D - 1$, the coefficient of the potential energy in the identity is $2(n - 3)$ and the energy of the spinning triangle is $3(n - 3)/(2(n - 1))$, both vanishing at $n = 3$——at $n = 2$ in three dimensions the potential term remains, so a triangle with negative energy can keep I constant. At $n = 3$ in four dimensions the bending $\ddot{I}$ of I is set by the energy alone.

Observation 7.2

what is not said.No claim is made that a world with four-dimensional gravity exists. Motion after collisions, regularization of collisions, and whether non-collision singularities exist are not treated——the conditions under which the spinning triangle keeps its shape are not treated either. The relation to $m = 4\pi$ goes only as far as the area over which the field spreads fixing the force exponent; no further numerical coincidence is claimed.

8. Census and Closing

1. $\ddot{I} = 4E + 2(D-4)V$ **within a relative difference of** 8.882×10^{-16} (Computation 2.1): **force and potential built separately agree.**
2. **triangle energies** -1.500000, 0, 0.500000 (Computation 3.1): **the sign changes at four dimensions.**
3. **energy** -0.029850 **at** 0.99 **times the speed** (Computation 4.1): **the four-dimensional triangle collapses when slightly slow.**
4. **collapse time** 4.072212491 **agreeing with the prediction in every digit** (Observation 4.2): **the core.**
5. $I = 1.748861 \times 10^{-4}$ **at** 4.092370**, before the root** 4.092728 (Observation 4.2): **a parabola until just before stopping.**
6. **five random cases stopping at** 0.162020 **to** 0.853067 **times the root** (Observation 5.2): **the root bounds the stopping time.**
7. **the three-dimensional triangle turning twice past** $t = 0.577350$ (Observation 6.2): **the four-dimensional rule fails.**
8. **period** 3.627599 (Computation 6.1): $\ddot{I} = 0$ **in three and in five dimensions.**
9. $I = 7.030000$ **at** $t = 10$ **at** 1.01 **times the speed** (Computation 4.1): **with positive energy it follows the parabola outward.**
10. **whether the force exponent** $n = D - 1$ **equals 3** (Observation 7.1): **the separator; Papers 490, 01, 08, 116, 56.**

Closing: **The three-body moment of inertia is a parabola only in four dimensions, where the spinning triangle has zero energy.In D-dimensional gravity Ï = 4E + 2(D−4)V holds, and building it separately from the force function and the potential function gave agreement within a relative difference of 8.882×10^{-16}——the energy of the spinning triangle is 3(D−4)/(2(D−2)): negative in three dimensions, 0 in four and positive in five or more.Slowing the four-dimensional triangle to 0.99 times its speed, the time 4.072212491 at which I falls to 1 percent of its initial value agreed with the parabola in every digit, and five random three-body systems reached collision at 0.162020 to 0.853067 times the root——the triangle in three dimensions keeps I constant and turns on at negative energy.The**

separator is whether the force exponent n = D−1 equals 3.Placed among the earlier papers——Paper 01 and Paper 08 showed n = 2 in three dimensions and n = 3 in four as the exponents at which a shell's mass grows uniformly, Paper 116 showed that the dimension of space is found in the denominator 1/n (n = d−1), and Paper 56 showed that d = 3 is the only dimension where flux conservation agrees with orbital stability. Here it was counted that the same n decides the dimension in which the potential drops out of the three-body moment of inertia, and that in four dimensions the bending of I is set by the energy alone.To be honest——no claim is made that a world with four-dimensional gravity exists, and motion after collisions, non-collision singularities and the conditions under which the triangle keeps its shape are not treated. The three-body systems checked numerically are only the equal-mass triangle and five random cases.

References

1. Ehrenfest, P. (1917). In what way does it become manifest in the fundamental laws of physics that space has three dimensions? *Proceedings of the Amsterdam Academy*.
2. Goldstein, H., Poole, C., & Safko, J. (2002). *Classical Mechanics* (3rd ed.). Addison-Wesley.
3. Jacobi, C. G. J. (1866). *Vorlesungen über Dynamik*. Reimer.
4. Lagrange, J.-L. (1772). Essai sur le problème des trois corps. *Prix de l'Académie Royale des Sciences de Paris*.
5. Yamagishi, Y. (2026). *The Uniqueness Theorem of $n = 2$ for Spherically Symmetric Density Distributions* [Paper 01]. Zenodo. 10.5281/zenodo.21403186
6. Yamagishi, Y. (2026). *A Structural Interpretation of the 4D Poincaré Conjecture and Exotic Structures via the n=3 Uniqueness Theorem and Hopf Fibration* [Paper 08]. Zenodo. 10.5281/zenodo.21404419
7. Yamagishi, Y. (2026). *A Structural Study of the 4π Patch* [Paper 56]. Zenodo. 10.5281/zenodo.21597500
8. Yamagishi, Y. (2026). *What Does the Exponent of c Count?* [Paper 116]. Zenodo. 10.5281/zenodo.22003794
9. Yamagishi, Y. (2026). *Something Is Resting on the Summit of a Hill* [Paper 490]. Zenodo. 10.5281/zenodo.22760033

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