The Disk That Cancels Intersections Avoids Itself from Dimension Five Up and Meets Itself in Points Only in Dimension Four

— the Whitney disk meets itself in general position in dimension 2 + 2 − n, and the three intersections of the disk with itself and with the two spheres all become points only for the split 2 + 2 in dimension 4 — the separator is whether the disk dimension 2 reaches the middle dimension n/2 — [Paper 1004]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
September 2026

— From thinking to syncing. —

Abstract

The Whitney trick works when a p-dimensional and a q-dimensional submanifold (p + q = n) of an n-manifold meet in two points of opposite sign: the circle joining the two points is filled by a 2-dimensional disk, and pushing along the disk removes both points.The h-cobordism theorem (Smale 1962) uses this trick when the ends have dimension at least 5; for 4-dimensional ends, Freedman (1982) showed that it holds in the topological category and Donaldson (1987) gave examples where it fails in the smooth category.This paper counts the dimension a + b − n in which an a-dimensional and a b-dimensional piece in general position meet, using a table and random triangles and tetrahedra. No new theorem is claimed. First, the disk meets itself in dimension 4 − n, which is negative from dimension 5 up and zero only in dimension 4——together with the disk and P (2 + p − n) and the disk and Q (2 + q − n), the three are all zero only for (n, p, q) = (4, 2, 2) among the 36 splits with n = 2 to 12. The three are all negative for the 16 splits with p and q at least 3, the smallest n being 6 (Section 2). **Second, and this is the core. Two random triangles meet in points in R^4 and do not meet in R^5——with 400000 pairs each, the fraction that meet was 0.279247 in R^3, 0.055670 in R^4 and 0 in R^5 and R^6. All 20000 intersections planted in each of R^5 and R^6 were found, so the 0 is not a miss** (Section 3). **Third, 2 and 3 dimensions meet exactly in points in R^5——random triangles and tetrahedra met with fraction 0.024450 in R^5 and 0 in R^6. For the split (2, 3) in dimension 5, the disk does not meet itself but meets the 3-dimensional Q in points** (Section 3). **Fourth, a bent disk meets itself in lines in R^3, points in R^4 and nowhere in R^5——placing the same disk of 1152 triangles, the number of meeting pairs among the 656443 pairs sharing no vertex was 253 in R^3, 3 in R^4 and 0 in R^5** (Section 4). **Fifth, the signs of intersections in R^4 are nearly half positive and half negative——of the 22268 intersections of random triangle pairs, the positive fraction was 0.504221; the trick removes pairs of points of opposite sign** (Section 3). **Sixth, the separator is whether the disk dimension 2 reaches the middle dimension n/2——the sum of the two dimensions being intersected sets where they meet in points: 3 for a circle and a disk, 4 for two disks, 5 for a disk and a 3-dimensional piece** (Section 5). **Keywords**: Whitney trick, general position, h-cobordism theorem, self-intersection, intersection dimension, random triangles.

Scope of this paper (scope note): **No new theorem is claimed——that a-dimensional and b-dimensional pieces in general position meet in dimension a + b − n, the trick of Whitney (1944), the h-cobordism theorem (Smale 1962, ends of dimension at least 5, simply connected), that the topological h-cobordism theorem holds for 4-dimensional ends (Freedman 1982), and that there are smooth examples where it fails (Donaldson 1987) are all known.The failure of the theorem in dimension 4 is not derived from this count——what is counted is only in which dimension the disk used in the proof meets itself and the spheres.The details of the trick are not entered——the framing of the disk, conditions on fundamental groups, the constructions used in place of the disk in dimension 4, and the arguments that get past the points of the disk and Q for the split (2, 3) in dimension 5 are not treated.Meeting pairs are not intersection points——what was counted for the bent disk is the number of pairs of meeting triangles, and one intersection point may be counted by several pairs.The random shapes are limited——vertices of triangles and tetrahedra are uniform in the unit cube, and the disk is a superposition of three sine waves only.Relation to earlier papers**: **the scope note of Paper 1003 said only that dimension 4 lies outside the range of the h-cobordism theorem, as the reason for not using the value of Θ in dimension 4; this paper sits in that seat and counts in which dimension the disk used in the proof meets itself at the edge of that range** (no earlier occupant).**Paper 254 counted the dimension 2 + 1 − n in which a circle and a disk meet in general position, and showed that knots come undone in dimension 4——here the 2 + 2 − n of two disks is counted, confirming that the dimension of point intersections rises from 3 to 4.Paper 279 counted the dimension 2 + 2 − d in which two random walk paths meet, and showed that the cost of self-avoidance disappears above dimension 4——here the same 2 + 2 is counted as the self-intersection of the disk that cancels intersections.Paper 158 listed the dimension 2n of smooth embeddings of n-manifolds (Whitney) and wrote 4 for n = 2——here it is counted with random pieces that in that 4 a 2-dimensional disk meets itself in points, and in 5 it does not.Paper 08 cited the topological 4-dimensional Poincaré conjecture as Freedman's theorem and wrote that exotic $\mathbb{R}^4$ follows from the results of Freedman and Donaldson——here it is counted that at 4-dimensional ends, where the results of those two diverge, the intersections of the disk become points.**What is added is **listing the three intersection dimensions of the disk with itself and with the two spheres for every split with n = 2 to 12, and showing that they are all points only for (4, 2, 2)**, **counting the fractions of meeting random triangles and tetrahedra in R^3 to R^6 and confirming with planted intersections that zeros are not misses**, **placing the same bent disk in R^3, R^4 and R^5 and counting its self-intersecting pairs**, and **setting the count beside those of 254 and 279 to show that the sum of the intersected dimensions sets the boundary**.

1. Notation and the Question

Setting: **in an n-manifold M, let a p-dimensional P and a q-dimensional Q ($p + q = n$) meet in two points x, y of opposite sign. The circle made of an arc joining x and y in P and one in Q is filled by a 2-dimensional disk D.If D does not meet itself and its interior meets neither P nor Q, pushing Q along D removes x and y together (the Whitney trick).In general position, pieces of dimensions a and b meet in dimension $a + b − n$, and do not meet if this is negative.**

$$\dim(D \cap D) = 4 - n, \qquad \dim(D \cap P) = 2 + p - n, \qquad \dim(D \cap Q) = 2 + q - n$$

Question: for which dimension and split do the three intersections all vanish, and where do they all become points? Do random pieces meet as the count says?

2. A Table of Intersection Dimensions

Computation 2.1: the 36 splits with $p \le q$ for $n = 2$ to 12 were listed (part shown).

n	(p, q)	$D \cap D$	$D \cap P$	$D \cap Q$
3	$(1, 2)$	1	0	1
4	$(2, 2)$	0	0	0
5	$(2, 3)$	-1	-1	0
6	$(3, 3)$	-2	-1	-1
7	$(3, 4)$	-3	-2	-1
8	$(4, 4)$	-4	-2	-2

> **Observation 2.2**
>
> the dimension $4 - n$ in which the disk meets itself is negative for every dimension from 5 up and zero only in dimension 4.The three are all zero only for $(n, p, q) = (4, 2, 2)$ among the 36 splits, and all negative for the 16 splits with $p \ge 3$ and $q \ge 3$, the smallest n being 6——for $(2, 3)$ in dimension 5, the disk meets neither itself nor P, but meets Q in points. For $(2, 2)$ in dimension 4, the disk dimension 2 equals the middle dimension $n/2 = 2$, and the three intersections all become points.

3. Counting with Random Pieces

Computation 3.1: 400000 pairs of triangles with vertices uniform in the unit cube were placed in each space, and $a_0 + Us = b_0 + Vt$ was solved; the pair was counted as meeting when the solution lies in both triangles (in R^3, by whether an edge pierces the other triangle).

space	$2+2-n$	meeting pairs	fraction
R^3	1	111699	0.279247
R^4	0	22268	0.055670
R^5	-1	0	0.000000
R^6	-2	0	0.000000

Computation 3.2: in R^5 and R^6, 20000 pairs of triangles built to meet were planted and put through the same test. Both found 20000, with largest residuals 7.114×10^{-14} and 1.273×10^{-14}.The signs of the 22268 intersections in R^4 (the sign of the determinant of the four edges) were 11228 positive and 11040 negative.

Computation 3.3: triangles and tetrahedra ($2+3-n$) were placed in the same way, 400000 pairs each. Meeting pairs were 9780 in R^5 (fraction 0.024450) and 0 in R^6.

> **Observation 3.4**
>
> this is the core. Two random triangles meet in points in R⁴ and do not meet in R⁵.The fraction that meet was **0.279247** in R^3, **0.055670** in R^4 and 0 in R^5 and R^6, and all planted intersections were found. The positive fraction of intersections in R^4 was **0.504221**, and triangles and tetrahedra met in R^5 with fraction **0.024450**——in the dimensions where $2+2-n$ and $2+3-n$ are zero, intersections appeared with positive fraction, and in the dimensions where they are negative they vanished. The signs are nearly half and half, and the trick removes two points of opposite sign at once.

4. Self-Intersections of a Bent Disk

Computation 4.1: a disk of 1152 triangles, a square divided 24×24, was placed in R^3, R^4 and R^5 by a superposition of three sine waves (with the same random coefficients), and the meeting pairs among the 656443 pairs sharing no vertex were counted.

space	self-intersection dimension $4-n$	meeting triangle pairs
R^3	1	253
R^4	0	3
R^5	-1	0

Observation 4.2

a bent disk meets itself in lines in R³, points in R⁴ and nowhere in R⁵.Meeting pairs fell from 253 in R^3 to 3 in R^4 and to 0 in R^5——by the dimensions of Computation 2.1, the self-intersection is a line in R^3, a point in R^4 and empty in R^5. What was counted is pairs, not intersection points.

5. Comparing Counts and the Separator

Computation 5.1: **for each pair of intersected dimensions a, b, the n with $a+b-n=0$ was listed.**

the two	n meeting in points	earlier paper
circle and disk ($1+2$)	3	Paper 254 (knots come undone in dimension 4)
disk and disk ($2+2$)	4	this paper; Paper 279 (two random walk paths)
disk and 3-dimensional piece ($2+3$)	5	this paper (the split $(2,3)$ in dimension 5)

Observation 5.2

the separator is whether the disk dimension 2 reaches the middle dimension n/2.Among even $n=2$ to 12, the disk dimension 2 equals the middle dimension $n/2$ only for $n=4$, where the disk and the two spheres have the same dimension and the three intersections all become points——the circle and disk of 254 meet in points in dimension 3 and vanish in 4. The two disks here meet in points in dimension 4 and vanish in 5. What sets the boundary is the sum of the two intersected dimensions, and the same count appears in knots (3), in random walks and the disk that cancels intersections (4), and in a disk and a 3-dimensional piece (5).

6. What Is Not Said

Observation 6.1

what is not said.It is not claimed that the failure of the smooth h-cobordism theorem in dimension 4 is derived from this count. What was counted is only in which dimension the disk used in the proof meets things——the framing of the disk, conditions on fundamental groups, the constructions used in place of the disk in dimension 4, and the arguments that get past the points with Q for $(2, 3)$ in dimension 5 are not entered. Meeting pairs of the bent disk are not called intersection points.

7. Census and Closing

1. **the disk's self-intersection $4 - n$** (Computation 2.1): **negative from 5 up, zero only in 4.**
2. **all three zero only for $(4, 2, 2)$ among 36 splits** (Observation 2.2): **all negative for 16 splits, smallest $n = 6$.**
3. **meeting fraction of two triangles 0.055670 (R^4) and 0 (R^5)** (Computation 3.1): **the core.**
4. **all 20000 planted pairs found** (Computation 3.2): **the 0 is not a miss.**
5. **positive fraction of intersections in R^4, 0.504221** (Computation 3.2): **signs nearly half and half.**
6. **triangles and tetrahedra 0.024450 (R^5) and 0 (R^6)** (Computation 3.3): **the boundary of $2 + 3 - n$.**
7. **meeting pairs of the bent disk $253, 3, 0$** (Computation 4.1): **line, point, nothing.**
8. **point-meeting n: 3 for $1 + 2$, 4 for $2 + 2$, 5 for $2 + 3$** (Computation 5.1): **Papers 254 and 279.**
9. **whether the disk dimension 2 reaches the middle dimension** (Observation 5.2): **the separator; Papers 1003, 158, 08.**

Closing: **The disk that cancels intersections avoids itself from dimension five up and meets itself in points only in dimension four.The Whitney disk meets itself in dimension 4 − n, and the three intersections of the disk with itself and with the two spheres all become points only for (2, 2) in dimension 4 among the 36 splits with n = 2 to 12——two random triangles met in points with fraction 0.055670 in R^4, and 0 in R^5 and R^6 under a test that finds every planted intersection. The self-intersecting pairs of the same bent disk were 253 in R^3, 3 in R^4 and 0 in R^5.The separator is whether the disk dimension 2 reaches the middle dimension n/2.Placed among the earlier papers——Paper 254 counted dimension 3 for a circle and a disk, Paper 279 dimension 4 for two random walk paths, and Paper 158 the 2n of Whitney's embedding. Here it was counted that in dimension 4, which Paper 1003 only said lies outside the range of the h-cobordism theorem, the intersections of the disk used in the proof all become points, giving the dimension count at the place where the results of Freedman and Donaldson cited by Paper 08 diverge.To be honest——the failure of the theorem in dimension 4 is not derived from this count, and neither the details of the trick nor**

the constructions used in place of the disk in dimension 4 are entered. The random shapes are only unit-cube vertices and a disk of three sine waves.

References

1. Donaldson, S. K. (1987). Irrationality and the h-cobordism conjecture. *Journal of Differential Geometry*.
2. Freedman, M. H. (1982). The topology of four-dimensional manifolds. *Journal of Differential Geometry*.
3. Smale, S. (1962). On the structure of manifolds. *American Journal of Mathematics*.
4. Whitney, H. (1944). The self-intersections of a smooth n-manifold in 2n-space. *Annals of Mathematics*.
5. Yamagishi, Y. (2026). *A Structural Interpretation of the 4D Poincaré Conjecture and Exotic Structures via the n=3 Uniqueness Theorem and Hopf Fibration* [Paper 08]. Zenodo. 10.5281/zenodo.21404419
6. Yamagishi, Y. (2026). *Whatever Can Be Measured from Inside Has a Home Somewhere* [Paper 158]. Zenodo. 10.5281/zenodo.22092143
7. Yamagishi, Y. (2026). *That Knots Come Undone in Four Dimensions Is Not One Phenomenon* [Paper 254]. Zenodo. 10.5281/zenodo.22162025
8. Yamagishi, Y. (2026). *What Changed the Exponent of a Chain Was Self-Avoidance Alone, and above Four Dimensions That Cost Disappears* [Paper 279]. Zenodo. 10.5281/zenodo.22170222
9. Yamagishi, Y. (2026). *The Poincaré Conjecture Is a Different Question in Each Category, and Only Dimension Four Stays Open in Both PL and Smooth* [Paper 1003]. Zenodo. 10.5281/zenodo.22823587

— *From thinking to syncing.* —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)