Without Stimulated Emission, Radiation Follows Wien's Law, and Only the 8π of A over B Carries the Polarization Count

— removing stimulated emission from Einstein's ledger makes the ratio of the balancing radiation to Planck's law $1 - \exp(-h\nu/kT)$ — the separator is whether the number of polarizations enters the ratio of A to B or the ratio of stimulated to spontaneous emission — [Paper 1005]

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— From thinking to syncing. —

Abstract

Einstein (1917) wrote the rates of absorption, stimulated emission and spontaneous emission between two levels as B12 u N1, B21 u N2 and A N2, and showed that if they balance in thermal equilibrium with Boltzmann level populations, the radiation u is Planck's law and the ratio of A to B21 is $8\pi h\nu^3/c^3$.This paper counts what happens when stimulated emission is removed from this ledger, and where in the coefficient 8π the number of polarizations enters.No new theorem or law is claimed. First, and this is the core. Without stimulated emission, the balancing radiation is Wien's law——its ratio to Planck's law is $1 - \exp(-x)$ with $x = h\nu/kT$: 0.095163 at x = 0.1, 0.632121 at x = 1 and 0.999955 at x = 10. It is within 1 percent only for x above 4.605170, which at 300 K means wavelengths shorter than 10.414 μm. The missing fraction $\exp(-x)$ equals the share of stimulated emission in all emission within Planck radiation (Section 4). **Second, 8π is the product of the polarization count 2 and the area 4π of the unit sphere in k-space——counting integer points in a periodic box, the number in a shell is proportional to the area of the unit sphere (ratio 0.994578 for d = 3), and the number of directions perpendicular to the wave is $d - 1$. The product of 2 polarizations and 4π is 25.132741, equal to 8π** (Section 2). **Third, the ledger does settle to the Boltzmann distribution——integrating the rate equations in time inside Planck radiation, for degeneracy ratios 1 and 3 and x from 0.5 to 10, all 8 cases settled to level ratios within a relative difference of 7.077×10^{-14} of Boltzmann** (Section 3). **Fourth, the ratio of stimulated to spontaneous emission is the occupation $1/(\exp(x) - 1)$, and it moves by 45.453 orders of magnitude across four settings——it was 0.006884 at 500 nm on the solar surface (5772 K), 2.203651×10^{-42} at 500 nm at room temperature and 6.250486×10^{3} at 1 GHz at room temperature. The ratio is 1 at a wavelength of 69.1905 μm at 300 K** (Section 5). **Fifth, stimulated emission beats absorption only at negative temperature——at 632.8 nm, a state with twice as many atoms in the upper level as in the lower is T = −32802.090 K; at positive temperature the lower level holds more and absorption wins** (Section 6). **Sixth, the separator is whether the number of polarizations enters the ratio of A to B or the ratio of stimulated to spontaneous emission——the coefficient of A over B in d dimensions is $d - 1$ times the area of the unit sphere (2π, 8π, $6\pi^2$, 105.275780) and carries the polarization count. The ratio of stimulated to spontaneous emission in radiation built from lattice-counted modes was the same with $d - 1$ polarizations as with 1** (Section 7). **Keywords**: stimulated emission, spontaneous emission, Einstein coefficients, Wien's law, number of polarizations, negative temperature.

Scope of this paper (scope note): **No new theorem or law is claimed——Einstein's relations between the A and B coefficients (1917), Planck's radiation law, Wien's radiation law, that the occupation of radiation is 1/(exp(x) − 1), and that a population inversion can be expressed as a negative temperature are all known.No causal reason is given for why stimulated emission is needed——what is said is only the fact that removing stimulated emission from the ledger makes the balancing radiation Wien's law rather than Planck's.No claim is made about a world whose dimension is not 3——d − 1 times the area of the unit sphere is a value of the formula, and no such world is claimed to exist.Lasers are not treated——devices, linewidths, pumping and the degeneracies of real atoms are not included; 632.8 nm is used only as an example wavelength.The numerics of the rate equations are limited——A was set to 10^6 per second, and only degeneracy ratios 1 and 3 and four values of x were integrated.Relation to earlier papers**: **the scope note of Paper 902 said that it removed the number of polarizations from the coefficient, leaving it out of the integrals that fit exponents and putting in 2 only when checking the 3-dimensional coefficient; this paper sits in that seat, puts the polarization count back into the coefficient of A over B, and counts that it does not enter the ratio of stimulated to spontaneous emission** (no earlier occupant).**Paper 902 showed that the 4π in the coefficient of black-body energy density is not a solid angle but the area of the unit sphere in k-space——here it is counted that multiplying that 4π by the 2 polarizations gives the 8π of A over B.Paper 265 showed that the 3 in the T^3 of low-temperature heat capacity comes from the dimension of the density of states——here it is counted that the same density of states, together with the polarization count, enters the ratio of A to B.Paper 403 showed that negative temperatures are hotter than all positive ones——here it is counted that the population inversion in which stimulated emission beats absorption is a negative temperature.**What is added is **counting the ratio of Wien to Planck when stimulated emission is removed and finding the range within 1 percent**, **counting shell points and polarizations with integer points in a periodic box and splitting 8π into 2 and 4π**, **confirming by integrating the rate equations that populations settle to Boltzmann**, **counting the stimulated-to-spontaneous ratio in four settings**, and **confirming with lattice modes that the polarization count enters the ratio of A to B but not the stimulated-to-spontaneous ratio**.

1. Notation and the Question

Setting: **consider a transition of frequency ν between a lower level 1 (degeneracy g_1, population N_1) and an upper level 2 (degeneracy g_2, population N_2). With u the radiation energy density per unit frequency, the transitions per unit time are set as absorption $B_{12}uN_1$, stimulated emission $B_{21}uN_2$ and spontaneous emission AN_2.If $B_{12}uN_1 = B_{21}uN_2 + AN_2$ holds in thermal equilibrium with $N_2/N_1 = (g_2/g_1)e^{-x}$ ($x = h\nu/kT$), the following are required.**

$$u = \frac{A/B_{21}}{(g_1B_{12}/g_2B_{21})\,e^{x} - 1}, \qquad g_1B_{12} = g_2B_{21}, \qquad \frac{A}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$$

Question: **what happens to the balancing radiation when stimulated emission is removed from this ledger, and where in the coefficient 8π does the number of polarizations enter, and where does it not?**

2. Counting Modes and 8π

Computation 2.1: integer vectors n in a periodic box were counted with $|n| \le R$ and compared with the ball volume $V_d R^d$, and the shell $R-1 < |n| \le R$ with the unit-sphere area $S_{d-1}(R-1/2)^{d-1}$.

d	R	points in ball	ratio to $V_d R^d$	shell points over $S_{d-1}(R-1/2)^{d-1}$
2	400	502625	0.999941	1.002338
3	120	7236577	0.999772	0.994578
4	50	30849449	1.000225	1.002939

Computation 2.2: the dimension of the space perpendicular to a random wave vector k (the rank of the projector $I - kk^{\mathsf{T}}/|k|^2$) was 1, 2, 3, 4 for $d = 2, 3, 4, 5$. The product of 2 polarizations and $S_2 = 4\pi$ was 25.132741, equal to 8π.

> **Observation 2.3**
>
> 8π is the product of the polarization count 2 and the area 4π of the unit sphere in k-space.The number of points in a shell is proportional to the area of the unit sphere, and each wave vector carries $d-1$ transverse polarizations——the number of modes per unit frequency is $(d-1)S_{d-1}\nu^{d-1}/c^d$, which is $8\pi\nu^2/c^3$ for $d = 3$.

3. Integrating the Rate Equations

Computation 3.1: with $\nu = c/500\text{nm}$, $A = 10^6$ per second, $A/B_{21} = 8\pi h\nu^3/c^3$ and $g_1 B_{12} = g_2 B_{21}$, the equations were integrated in time inside Planck radiation from $N_1 = 1, N_2 = 0$.

g_2/g_1	x	integrated N_2/N_1	Boltzmann	relative difference
1	0.5	0.606530660	0.606530660	1.318×10^{-14}
1	1.0	0.367879441	0.367879441	7.077×10^{-14}
1	10.0	0.000045400	0.000045400	5.403×10^{-14}
3	0.5	1.819591979	1.819591979	5.735×10^{-14}
3	10.0	0.000136200	0.000136200	3.483×10^{-14}

Observation 3.2

the ledger does settle to the Boltzmann distribution.The largest relative difference over the 8 cases (part of the table shown) was 7.077×10^{-14}——with stimulated emission included and $g_1 B_{12} = g_2 B_{21}$ and $A/B_{21} = 8\pi h\nu^3/c^3$ imposed, Planck radiation and the level populations do not contradict each other.

4. Removing Stimulated Emission

Computation 4.1: **setting $B_{21} = 0$ and putting $N_2/N_1 = e^{-x}$ into the balance $BuN_1 = AN_2$ gives $u = (A/B)e^{-x}$ (Wien's radiation law). Its ratio to Planck's law was computed.**

x	Wien/Planck	$1 - e^{-x}$
0.1	0.095163	0.095163
1.0	0.632121	0.632121
3.0	0.950213	0.950213
10.0	0.999955	0.999955

Observation 4.2

this is the core. Without stimulated emission, the balancing radiation is Wien's law.The ratio of Wien to Planck is $1 - e^{-x}$, only $\mathbf{0.095163}$ at $x = 0.1$, and within 1 percent only for $x > \mathbf{4.605170}$ (at 300 K, wavelengths shorter than 10.414 µm)——the missing fraction e^{-x} equals the share of stimulated emission in all emission within Planck radiation: $\mathbf{0.904837}$ at $x = 0.1$ and $\mathbf{0.367879}$ at $x = 1$. At $x = 4.6$ the ratio is more than 1 percent off, and at $x = 4.7$ within.

5. The Ratio of Stimulated to Spontaneous Emission

Computation 5.1: **the ratio of stimulated to spontaneous emission $B_{21}u/A = 1/(e^{x} - 1)$ was computed in four settings.**

setting	x	stimulated/spontaneous
solar surface 5772 K, 500 nm	4.985367	6.884339×10^{-3}
room temperature 300 K, 500 nm	95.918459	2.203651×10^{-42}
room temperature 300 K, 10 µm	4.795923	8.332221×10^{-3}
room temperature 300 K, 1 GHz	0.000160	6.250486×10^{3}

Observation 5.2

the ratio of stimulated to spontaneous emission is the occupation, and it moves by 45.453 orders of magnitude across the four settings.The ratio is 1 at $x = \ln 2 = 0.693147$, at a wavelength of $\mathbf{69.1905}$ µm at 300 K and $\mathbf{3.5962}$ µm at 5772 K——for visible light at room temperature the ratio is 2.203651×10^{-42} and spontaneous emission wins, while at 1 GHz at room temperature it is 6.250486×10^{3} and stimulated emission wins.

6. Population Inversion and Negative Temperature

Computation 6.1: **at 632.8 nm, $h\nu/k = 22736.676$ K. With $g_2 = g_1$, the temperature with $N_2/N_1 = 2$ is $T = -h\nu/(k \ln 2) = -32802.090$ K, which gives 2 when put back into $e^{-h\nu/kT}$.The net of stimulated emission minus absorption is proportional to $N_2 - N_1$: $-0.5N_1$, 0, $+1.0N_1$ for $N_2/N_1 = 0.5, 1, 2$.**

Observation 6.2

stimulated emission beats absorption only at negative temperature.At positive temperature (for example 10000 K), $N_2 < N_1$ and absorption wins; the inversion $N_2 > N_1$ corresponds to the negative temperature -32802.090 K——in thermal equilibrium at positive temperature, there is no amplification.

7. d Dimensions and the Separator

Computation 7.1: **the coefficient $(d-1)S_{d-1}$ of A/B (multiplying $h\nu^d/c^d$) was listed, radiation at $x = 2$ was built from lattice-counted modes, and the stimulated-to-spontaneous ratio was compared with $d-1$ polarizations and with 1.**

d	polarizations	S_{d-1}	coefficient of A/B	lattice ratio ($d-1$, 1)
2	1	6.283185	6.283185	0.156883505, 0.156883505
3	2	12.566371	25.132741	0.155669064, 0.155669064
4	3	19.739209	59.217626	0.156977574, 0.156977574
5	4	26.318945	105.275780	(lattice not counted)

Observation 7.2

the separator is whether the number of polarizations enters the ratio of A to B or the ratio of stimulated to spontaneous emission.The coefficient of A over B is proportional to the polarization count $d-1$. In radiation built from the lattice, the stimulated-to-spontaneous ratio was the same with $d-1$ polarizations as with 1, and agreed with the occupation $1/(e^2-1) = 0.156517643$ within the shell-counting error (ratios 1.002338, 0.994578, 1.002939)——the polarization count multiplies both the radiation u and A/B equally, so it does not remain in their ratio, stimulated over spontaneous. What carries the polarization count is the 8π of A over B.

8. Census and Closing

1. **Wien/Planck $= 1 - e^{-x}$ (0.095163, 0.632121, 0.999955)** (Computation 4.1): **the core.**
2. **within 1 percent only for $x > 4.605170$** (Observation 4.2): **at 300 K, only wavelengths shorter than 10.414 μm.**

3. $8\pi = 2 \times 4\pi = \mathbf{25.132741}$ (Computation 2.2): **polarization count and the area of the unit sphere in k-space.**
4. **shell-point ratio $\mathbf{0.994578}$ ($d = 3$)** (Computation 2.1): **modes are proportional to the unit-sphere area.**
5. **integrated N_2/N_1 within 7.077×10^{-14}** (Observation 3.2): **the ledger settles to Boltzmann.**
6. **stimulated/spontaneous 6.884339×10^{-3} for the Sun at 500 nm, 6.250486×10^{3} at 1 GHz at room temperature** (Computation 5.1): **the occupation.**
7. **wavelength of ratio 1, $\mathbf{69.1905}$ µm (300 K)** (Observation 5.2): $x = \ln 2$**.**
8. **inversion temperature $-\mathbf{32802.090}$ K** (Computation 6.1): **amplification at negative temperature.**
9. **the coefficient $(d-1)S_{d-1}$ of A/B and a lattice ratio independent of polarization** (Computation 7.1): **the separator; Papers 902, 265, 403.**

Closing: **Without stimulated emission, radiation follows Wien's law, and only the 8π of A over B carries the polarization count.Removing stimulated emission from Einstein's ledger makes the ratio of the balancing radiation to Planck's law 1 − exp(−x), which is 0.095163 at x = 0.1 and within 1 percent only for x above 4.605170——the ledger with stimulated emission settled to Boltzmann in all 8 cases of integrating the rate equations, within a relative difference of 7.077×10^{-14}.8π is the product of the polarization count 2 and the area 4π of the unit sphere in k-space, and in d dimensions it becomes d − 1 times the area of the unit sphere——the ratio of stimulated to spontaneous emission is the occupation, 0.006884 at 500 nm on the Sun and 2.203651×10^{-42} at 500 nm at room temperature, and in lattice-built radiation it did not change between d − 1 polarizations and 1. A population inversion is a negative temperature: at 632.8 nm, twice as many upper as lower gives −32802.090 K.The separator is whether the number of polarizations enters the ratio of A to B or the ratio of stimulated to spontaneous emission.Placed among the earlier papers——Paper 902 read the 4π of the coefficient as the area of the unit sphere in k-space and left the polarization count out of the coefficient. Paper 265 showed the dimension of the density of states, and Paper 403 that negative temperatures are hotter than positive ones. Here the polarization count that was left out was put back into the ratio of A to B, and it was counted that it does not remain in the stimulated-to-spontaneous ratio.To be honest——no causal reason is given for why stimulated emission is needed, and neither worlds whose dimension is not 3 nor laser devices are treated. The rate equations were integrated only for limited A, degeneracies and x.**

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