The Einstein–Rosen Bridge Opens to a Throat of Circumference $4\pi GM/c^2$ and Closes within $2\pi GM/c^3$, and Not Even Light Crosses

— on a slice of constant time the throat joins two exteriors, but because the light-direction coordinate V never decreases, a path leaving one exterior enters the black hole before reaching the other — the separator is whether things connect on a spatial slice or along causal paths that include time — [Paper 1006]

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— From thinking to syncing. —

Abstract

Extending the Schwarzschild solution as far as it goes joins two exteriors through a throat (Einstein and Rosen 1935, Kruskal 1960).This paper counts the shape and lifetime of that throat, and whether it can be crossed, numerically in units G = c = M = 1.No new theorem or law is claimed. First, on a slice of constant time, the throat joins the two exteriors symmetrically——replacing the isotropic radius ρ by $M^2/(4\rho)$ leaves r unchanged (relative difference 7.077×10^{-16} over 1000 random points), and the minimum of r is 2M at ρ = M/2. The proper distances from the throat to the same r on both sides are equal, 7.252585093 at ρ = 5 (Section 2). **Second, Flamm's paraboloid is a picture of the throat, and its curvature comes out the same from the metric alone——the proper distance from the throat to r = 10M agreed at 11.831542860 by integral, closed form and arc length on the paraboloid. The curvature $-M/r^3$ is the same from the embedded shape and from the metric alone, −0.125000 at the throat** (Section 3). **Third, and this is the core. The throat opens and closes, and its lifetime is $2\pi GM/c^3$——following the worldline that stays at the middle of the throat in Kruskal coordinates, the throat radius shrinks from 2M to 0 in a proper time of 3.141592654 (πM). It agreed within 8.882×10^{-16} at four points with the cycloid-shaped parametrization r = M(1 + cos η), τ = M(η + sin η), and from 0 through opening to closing at 0 it is 2πM** (Section 4). **Fourth, not even light crosses to the other side——the light-direction coordinate V = T + X does not decrease along future-directed causal paths. Of 20000 random paths, with speeds at most that of light, 0 reached the other exterior even when made to move only leftward, and all entered the black hole. Allowing faster-than-light paths let 11657 arrive, so the 0 is not a miss of the test** (Section 5). **Fifth, for one solar mass the throat circumference is 18555.817 m and the lifetime 3.094777×10^{-5} s——GM/c^2 is 1476.625 m; the 4π of the circumference is the 2π of a circle times the 2 of the radius $2GM/c^2$, and the 2π of the lifetime is twice the π of opening or closing** (Section 6). **Sixth, the separator is whether things connect on a spatial slice or along causal paths that include time——on a slice of constant time the throat joins the two exteriors, but once time is included, V never decreases, so a path leaving one exterior enters the black hole before reaching the other** (Section 7). **Keywords**: Einstein-Rosen bridge, Kruskal coordinates, Flamm paraboloid, causal structure, Schwarzschild solution, cycloid.

Scope of this paper (scope note): **No new theorem or law is claimed——the Schwarzschild solution, isotropic coordinates, Flamm's (1916) paraboloid, the Einstein and Rosen (1935) bridge, Kruskal's (1960) extension, the fact that the bridge cannot be crossed (Fuller and Wheeler 1962), and that the proper time of free fall can be written in cycloid form are all known.Traversable wormholes are not treated——constructions that hold a throat open with matter of negative energy are not included.Rotating or charged holes are not treated——the bridges of the Kerr and Reissner–Nordström solutions, whose interiors differ, are not counted.Quantum theory is not treated——arguments linking the bridge to quantum entanglement are not entered.The random paths are limited——only radial motion is considered, and paths leaving the exterior at time 0 were advanced 4000 steps of 0.002.Relation to earlier papers**: **the scope note of Paper 485 said that it does not discuss the event horizon, and that causal structure, the distinction between coordinate and true singularities, and the interior solution are not its subject; this paper sits in that seat and counts, within the interior and the causal structure, that the throat opens and closes and cannot be crossed** (no earlier occupant).**Paper 485 showed that Michell's radius for a dark star, $2GM/c^2$, was right but not for the reason given——here it is counted that the sphere of that same radius is the narrowest throat on a slice of constant time.Paper 91 showed that curvature is an intrinsic quantity measurable from within a surface——here the curvature of Flamm's paraboloid is counted in two ways, from the embedded shape and from the metric alone, and the two agree.Paper 488 showed that the requirement that the fall time not depend on height fixes the cycloid uniquely——here it is counted that the throat radius and proper time are written by a cycloid-shaped parametrization; only the shape is shared, and no claim is made that it comes from the isochrony requirement.**What is added is **confirming numerically the symmetry of isotropic coordinates and the proper distances from the throat to both sides**, **matching the arc length and curvature of the paraboloid by embedding and by metric**, **integrating the lifetime of the throat along a Kruskal worldline and checking it against the cycloid parametrization**, **counting, with a faster-than-light control, that no random causal path reaches the other side**, and **giving the throat circumference and lifetime for one solar mass**.

1. Notation and the Question

Setting: **the Schwarzschild solution of mass M is written in units $G = c = M = 1$. In isotropic coordinates $r = \rho(1 + M/2\rho)^2$, and in Kruskal coordinates T, X with $(1 - r/2M)e^{r/2M} = T^2 - X^2$,**

$$ds^2 = \frac{32M^3}{r}\, e^{-r/2M}\left(-dT^2 + dX^2\right) + r^2 d\Omega^2.$$

With $U = T - X$ and $V = T + X$, exterior I is $U < 0 < V$, the other exterior III is $V < 0 < U$, the black hole II is $U > 0$ and $V > 0$, and the singularity is $T^2 - X^2 = 1$.

Question: **what shape is the throat joining the two exteriors, how long does it exist, and can one pass through it to the other side?**

2. The Symmetry of Isotropic Coordinates and the Throat

Computation 2.1: 1000 random values of ρ from 0.01 to 50 were taken and $r(\rho)$ compared with $r(M^2/4\rho)$; the largest relative difference was 7.077×10^{-16}. The minimum of r was $r = 2.000000000$ at $\rho = M/2$.The proper distance from the throat $\rho = M/2$ to $\rho = 5$ ($r = 6.050000$) and to the mirror point $\rho = 0.050$ were both 7.252585093.

> **Observation 2.2**
>
> on a slice of constant time, the throat joins the two exteriors symmetrically.$\rho \to M^2/4\rho$ exchanges the exterior $\rho > M/2$ with the other exterior $\rho < M/2$ and changes neither r nor proper distance——the sphere of minimum $r = 2M$ is the throat at the border of the two exteriors.

3. Flamm's Paraboloid and Curvature

Computation 3.1: for $z = \sqrt{8M(r - 2M)}$, $1 + (dz/dr)^2$ agreed with $1/(1 - 2M/r)$ within a relative difference of 5.806×10^{-16} over 1000 random points. The proper distance from the throat to $r = 10M$ was obtained in three ways.

method	proper distance
integral $\int dr/\sqrt{1 - 2M/r}$	11.831542860
closed form	11.831542860
polyline arc length on the paraboloid	11.831542860

Computation 3.2: the curvature was obtained in two ways: from the embedded shape, $z'z''/(r(1 + z'^2)^2)$, and from the metric alone, $A'/(2rA^2)$ with $A = 1/(1 - 2M/r)$.

r	from embedding	from metric alone	$-M/r^3$
2.5	-0.064000000	-0.064000000	-0.064000000
3.0	-0.037037037	-0.037037037	-0.037037037
6.0	-0.004629630	-0.004629630	-0.004629630
20.0	-0.000125000	-0.000125000	-0.000125000

Observation 3.3

Flamm's paraboloid is a picture of the throat, and its curvature comes out the same from the metric alone.The arc length on the paraboloid matched the proper distance $\mathbf{11.831542860}$, and the curvature was $-M/r^3$ both ways, -0.125000 at the throat——the paraboloid is a picture placed in an outside three-dimensional space, but the value of the curvature does not depend on the picture.

4. The Lifetime of the Throat

Computation 4.1: **the worldline staying at $X = 0$ in Kruskal coordinates was followed. At $T = 0$, $r = 2M$; as $T \to 1$, $r \to 0$; and the proper time is $\int_0^{2M} \sqrt{r/(2M - r)}\,dr = 3.141592654$ ($\pi M = 3.141592654$).**

η	cycloid r	cycloid τ	Kruskal T	proper time up to T
0.5	1.877582562	0.979425539	0.395605738	0.979425539
1.0	1.540302306	1.841470985	0.704623889	1.841470985
2.0	0.583853163	2.909297427	0.973711271	2.909297427
2.8	0.057777659	3.134988150	0.999787274	3.134988150

Observation 4.2

this is the core. The throat opens and closes, and its lifetime is 2πGM/c³.The throat radius is $r = M(1 + \cos\eta)$ and the proper time $\tau = M(\eta + \sin\eta)$, matching the Kruskal worldline within 8.882×10^{-16} at four points. From $r = 2M$ to 0 takes πM, and from 0 through opening to closing at 0 takes 6.283185307 ($2\pi M$)——the throat on a slice of constant time is not a passage that stays there, but a shape that opens and closes.

5. No Crossing

Computation 5.1: **radial future-directed causal paths satisfy $dT \geq |dX|$, hence $dU \geq 0$ and $dV \geq 0$. From the exterior at time 0 (X random in 0.01 to 3), 20000 paths each were advanced with the speed dX/dT chosen at random at every step.**

speed range	reached III	entered II	minimum of V
-1 to 1	0	20000	0.010316
-1 to 0 (leftward only)	0	20000	0.010237
-3 to -1 (faster than light)	11657		

Computation 5.2: **a path moving leftward at the speed of light from $X_0 = 0.5$ reaches the singularity at $T = 1.250000$, $X = -0.750000$, with $T - |X| = 0.500000$, inside the black hole.**

Observation 5.3

not even light crosses to the other side.Paths no faster than light reached the other exterior 0 times even when moving only leftward, and all entered the black hole with V staying positive. Faster-than-light paths arrived 11657 times——$V > 0$ in exterior I and $V < 0$ in exterior III, and V does not decrease along causal paths, so I cannot reach III. The first run used a zero-mean control with speeds from -3 to 3, which did not drift left and gave 0 arrivals; the control was changed to drift left, confirming that the test does not simply return 0.

6. Values for One Solar Mass

Computation 6.1: values were obtained with $GM_\odot = 1.3271244 \times 10^{20}$ m³/s² and the defined value of c.

quantity	value
GM/c^2	**1476.625** m
throat radius $2GM/c^2$	**2953.250** m
throat circumference $4\pi GM/c^2$	**18555.817** m
throat area $16\pi G^2M^2/c^4$	1.095999×10^{8} m²
GM/c^3	4.925491×10^{-6} s
throat lifetime $2\pi GM/c^3$	3.094777×10^{-5} s
half of it, $\pi GM/c^3$	1.547389×10^{-5} s

Observation 6.2

for one solar mass the throat circumference is 18555.817 m and the lifetime 3.094777×10^{-5} s.The 4π of the circumference is the 2π of a circle times the 2 of the radius $2GM/c^2$, not a solid angle——the 2π of the lifetime is the π from $2M$ to 0 counted twice, once opening and once closing.

7. The Separator and What Is Not Said

Observation 7.1

the separator is whether things connect on a spatial slice or along causal paths that include time.On a slice of constant time, the throat joins the two exteriors symmetrically, with finite proper distance and curvature ——once time is included, the throat opens and closes within $2\pi GM/c^3$, and since V does not decrease along causal paths, a path leaving one exterior enters the black hole before reaching the other.

Observation 7.2

what is not said.Traversable wormholes held open by negative energy, the interiors of rotating or charged black holes, and quantum arguments are not treated——the 4π of the circumference is not called a solid angle, and the cycloid shape is not claimed to share a root with the isochronous curve. The random paths are radial only.

8. Census and Closing

1. $r(\rho) = r(M^2/4\rho)$**, minimum** $2M$ (Computation 2.1): **the throat on a slice of constant time.**
2. **proper distances to both sides** $\mathbf{7.252585093}$ (Computation 2.1): **symmetric.**
3. **paraboloid arc length** $\mathbf{11.831542860}$ **and curvature** $-M/r^3$ (Computations 3.1, 3.2): **curvature the same from the metric alone.**
4. **half-lifetime of the throat** $\mathbf{3.141592654}$ **(**πM**)** (Computation 4.1): **the core.**
5. **cycloid match within** 8.882×10^{-16} **at four points** (Observation 4.2): **lifetime** $\mathbf{6.283185307}$ **(**$2\pi M$**).**
6. $\mathbf{0}$ **of 20000 subluminal paths reach III,** $\mathbf{11657}$ **faster-than-light paths do** (Computation 5.1): **no crossing.**
7. $T - |X| = 0.500000$ **even moving left at light speed** (Computation 5.2): **the singularity is reached inside the black hole.**
8. **for one solar mass, throat circumference** $\mathbf{18555.817}$ **m and lifetime** 3.094777×10^{-5} **s** (Computation 6.1): $GM/c^2 = \mathbf{1476.625}$ **m.**
9. **spatial slice or causal path** (Observation 7.1): **the separator; Papers 485, 91, 488.**

Closing: **The Einstein–Rosen bridge opens to a throat of circumference 4πGM/c² and closes within 2πGM/c³, and not even light crosses.On a slice of constant time, the symmetry of isotropic coordinates joins two exteriors through the throat, the proper distances from the throat to both sides are equal, and the arc length and curvature of Flamm's paraboloid come out the same from the metric alone——following the middle of the throat in Kruskal coordinates, its radius shrinks from 2M to 0 in 3.141592654 (πM), matching the cycloid-shaped parametrization, and its lifetime is 2πM.The light-direction coordinate V does not decrease along causal paths, and of 20000 random paths none slower than light reached the other side. For one solar mass the throat circumference is 18555.817 m and the lifetime 3.094777×10⁻⁵ s——the 4π of the circumference is the 2π of a circle times the 2 of the radius, not a solid angle.The separator is whether things connect on a spatial slice or along causal paths that include time.Placed among the earlier papers——Paper 485 showed that Michell's radius 2GM/c² was right for a different reason and left the event horizon and interior aside, Paper 91 showed that curvature is intrinsic, and Paper 488 that the isochrony requirement fixes the cycloid. Here, in the interior and causal structure that 485 did not treat, it was counted that**

the throat opens and closes and cannot be crossed.To be honest——traversable wormholes, the interiors of rotating or charged black holes and quantum theory are not entered, and the random paths are radial only.

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