The Moon’s Path around the Sun Is Always Convex, and Convexity Fails Near the Point Where the Sun and Earth Pull Equally

— integrating the Sun, Earth and Moon, the lunar radius at which convexity fails is 259214.846 km, 434.074 km beyond the equal-gravity point, and the Moon now sits at 1.482940 times that distance — the separator is whether the lunar radius lies outside the convexity boundary, between the loop radius 454.841 km and that boundary, or inside the loop radius — [Paper 1010]

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— From thinking to syncing. —

Abstract

The Moon circles the Earth, but seen from the Sun it travels around the Sun with the Earth, and its path can look wavy.This paper counts the sign of the curvature of the Moon's path as seen from the Sun, using a superposition of two circles and a planar three-body integration of the Sun, Earth and Moon, and finds the lunar radius at which convexity fails by changing only that radius.No new theorem or law is claimed. First, the Sun's pull on the Moon exceeds twice the Earth's pull everywhere——it was 2.209655 times at new moon, 2.187060 times at full moon and 2.198314 times on average (Section 2). **Second, seen from the Sun, the Moon's path always bends toward the Sun——in the three-body integration the curvature stayed positive for 60 days, with radii of curvature between 0.6350 au and 1.9559 au. In the two-circle superposition too, the ratio of the Moon's centripetal acceleration to the Earth's is 0.459240, and the sign of the curvature did not change over a year** (Section 3). **Third, and this is the core. The radius at which convexity fails nearly coincides with the point where the Earth and Sun pull equally——the convexity boundary found by bisection over the lunar radius alone is 259214.846 km, 434.074 km beyond the equal-gravity point 258780.772 km of Paper 505, a relative difference of 0.001677. The Moon now sits at 1.482940 times that distance** (Section 4). **Fourth, the two-circle boundary lies 1636.120 km beyond the three-body boundary——the radius at which the minimum curvature of the two-circle path is 0 is the au times the square root of the Earth–Moon mass sum over the total mass, 260850.966 km, matching the numerical bisection 260850.980 km** (Section 4). **Fifth, the Moon's path has three shapes——in the two-circle path the sign of the minimum curvature changes at two radii, 454.841 km and 260850.98 km. Outside the boundary the path is convex; between them the bending reverses; inside 454.841 km the Moon moves faster than the orbital speed and draws loops, and the curvature has one sign again** (Section 5). **Sixth, the separator is whether the lunar radius lies outside the convexity boundary, between the loop radius and that boundary, or inside the loop radius——the Moon now lies outside the boundary, and the loop radius is inside the Earth's radius** (Section 6). **Keywords**: lunar orbit, convexity, curvature, three-body problem, sphere of influence, loop.

Scope of this paper (scope note): **No new theorem or law is claimed——that the Moon's path is convex as seen from the Sun, that the Sun's pull on the Moon exceeds the Earth's, the curvature condition for superposed circles, and the radii of spheres of influence are all known.It is not claimed that the path is convex because the Moon lies beyond the equal-pull point——the convexity boundary and the equal-gravity point differ by 434.074 km and are not the same thing; what can be said is that they are close, a relative difference of 0.001677.The lunar orbit is simplified——the Earth and Moon start on circles, and only planar motion over 60 days was integrated; eccentricity, orbital inclination and tidal recession of the Moon are not included.Other satellites are not treated——the path shapes of other planets' satellites are not counted.Relation to earlier papers**: **the scope note of Paper 505 said that it counts nothing beyond the Hill radius, the Laplace sphere of influence and the equal-gravity point, and does not claim that three is the end; this paper sits in that seat and counts, as a fourth radius, the radius at which the Moon's path loses convexity (the earlier occupant is Paper 784, which cited the same scope note and counted the Roche limit).Paper 505 showed that the Earth's equal-gravity point is at 258780.772 km and that the Moon lies beyond it——here it is counted that this point differs from the radius at which the Moon's path loses convexity by only 434.074 km.Paper 833 showed that a satellite's orbit tells the sum of the planet's and satellite's masses——here the Earth–Moon mass sum is used in the formula for the two-circle boundary as well.**What is added is **giving the ratio of pulls on the Moon at three positions**, **counting the curvature of the Moon's path as seen from the Sun by three-body integration**, **finding by bisection over the lunar radius alone the radius at which convexity fails, and comparing it with 505's point and the two-circle boundary**, and **counting the two radii at which the curvature sign changes and separating the three path shapes**.

1. Notation and the Question

Setting: **the planar three-body problem of the Sun, Earth and Moon is integrated with** $GM_S = 1.32712440018 \times 10^{20}$, $GM_E = 3.986004418 \times 10^{14}$, $GM_M = 4.9028 \times 10^{12}$ **(m³/s²), 1 au** $= 1.495978707 \times 10^{11}$ **m and mean lunar distance** 3.844×10^{8} **m. The Earth–Moon barycentre starts at circular speed about the Sun, and the Moon starts at new moon with circular speed about the Earth.With** v **and** a **the velocity and acceleration of the Moon as seen from the Sun, the curvature**

$$\kappa = \frac{v \times a}{|v|^3}$$

is measured, positive in the direction of revolution.

Question: **does the Moon's path as seen from the Sun ever reverse its bending? How close to the Earth would the Moon have to be for it to reverse?**

2. The Pulls on the Moon

Computation 2.1: **the Sun's pull on the Moon was compared with the Earth's pull (2.697559×10^{-3} m/s²) at three positions.**

position	Sun's pull (m/s²)	Sun / Earth
new moon ($a - r$ from the Sun)	5.960677×10^{-3}	2.209655
full moon ($a + r$)	5.899725×10^{-3}	2.187060
mean (a)	5.930084×10^{-3}	2.198314

> **Observation 2.2**
>
> **the Sun's pull on the Moon exceeds twice the Earth's pull everywhere.It is 2.209655 times at new moon and 2.187060 times at full moon——for the Moon, the Sun pulls harder than the Earth.**

3. The Curvature of the Moon's Path Seen from the Sun

Computation 3.1: **in the two-circle superposition (Earth with the sidereal year of 365.256363 days, Moon with the sidereal month of 27.321661 days), $r\omega_M^2/(a\omega_E^2) = 0.459240$, and the curvature $v \times a$ (m²/s³) over a year had minimum 9.223158×10^{1} and maximum 2.665942×10^{2}, staying positive.The three-body integration at the present radius over 60 days gave the following.**

quantity	curvature (/m)	radius of curvature (au)
minimum	3.417636×10^{-12}	1.9559
maximum	1.052700×10^{-11}	0.6350
minimum / maximum	0.324654	

Observation 3.2

seen from the Sun, the Moon's path always bends toward the Sun.The curvature stayed positive for 60 days, and even at its gentlest the radius of curvature was 1.9559 au——the Moon's path may look wavy, but its bending never reverses. The ratio of the gentlest to the sharpest bending is 0.324654.

4. The Radius at Which Convexity Fails

Computation 4.1: changing only the lunar radius (with the Moon at circular speed about the Earth), the radius at which the 60-day minimum curvature is 0 was found by bisection. The end values were -1.611×10^{-12} at 230640.0 km and 1.191×10^{-12} at 288300.0 km.

radius	value (km)
three-body radius at which convexity fails	259214.846
equal-gravity point of Paper 505 ($q = 3.002735 \times 10^{-6}$)	258780.772
the same point from these constants' GM_E/GM_S	258813.232
two-circle boundary $a\sqrt{(M_E + M_M)/(M_S + M_E + M_M)}$	260850.966
numerical bisection of the two-circle boundary	260850.980

Observation 4.2

this is the core. The radius at which convexity fails nearly coincides with the point where the Earth and Sun pull equally.The three-body boundary lies 434.074 km beyond 505's point (relative 0.001677) and 1636.120 km inside the two-circle boundary (relative -0.006272). The Moon now sits at 1.482940 times the boundary——505's point is a value rounded to the fourth decimal of 10^4 km, and the check reproduced it within that rounding. The first run set the tolerance at 0.1 km, narrower than the rounding, and failed (difference 0.228 km).

5. Three Shapes of the Moon's Path

Computation 5.1: the radii at which the sign of the minimum curvature of the two-circle path changes were sought from 50 m to 400,000 km.

lunar radius	minimum two-circle curvature	Moon speed / orbital speed
384400.000 km (now)	9.201×10^{1}	(Moon 1.025 km/s, orbit 29.785 km/s)
129607.423 km (half the boundary)	-5.069×10^{2}	
909.682 km	-4.254×10^{6}	0.707107
227.420 km	9.625×10^{7}	1.414213

Observation 5.2

the Moon's path has three shapes.The sign changes at the two radii 454.841 km and 260850.98 km, and the smaller is exactly the radius $a(M_E + M_M)/(M_S + M_E + M_M)$ at which the Moon's speed equals the orbital speed——outside the boundary the path is convex; between them it is a wave whose bending reverses; inside 454.841 km the Moon circles faster than the orbit and draws loops, and the curvature has one sign again. The first run saw the minimum curvature turn positive again on the loop side, and the range was widened and recounted.

6. The Separator and What Is Not Said

Observation 6.1

the separator is whether the lunar radius lies outside the convexity boundary, between the loop radius and that boundary, or inside the loop radius.The Moon now lies outside, at 1.482940 times the boundary, and the loop radius 454.841 km is inside the Earth's radius of 6371 km——a satellite of the Earth would have to orbit inside the Earth to draw loops.

Observation 6.2

what is not said.It is not claimed that the path is convex because the Moon lies beyond the equal-gravity point; the two radii differ by 434.074 km——eccentricity, inclination and tidal recession of the lunar orbit are not included, and no other satellites are counted.

7. Census and Closing

1. **Sun / Earth pulls 2.209655, 2.187060, 2.198314** (Computation 2.1): **above 2 everywhere.**
2. **radii of curvature 0.6350 to 1.9559 au, curvature positive** (Computation 3.1): **always convex.**
3. **convexity boundary 259214.846 km and 505's point 258780.772 km** (Computation 4.1): **the core; relative 0.001677.**
4. **the Moon at 1.482940 times the boundary** (Observation 4.2): **outside.**
5. **two-circle boundary 260850.966 km** (Computation 4.1): **1636.120 km beyond the three-body boundary.**
6. **sign-change radii 454.841 km and the boundary** (Computation 5.1): **three shapes.**
7. **outside, between or loops** (Observation 6.1): **the separator; Papers 505, 784, 833.**

Closing: **The Moon's path around the Sun is always convex, and convexity fails near the point where the Sun and Earth pull equally.The Sun's pull on the Moon is 2.187060 to 2.209655 times the Earth's, and in the integration of the Sun, Earth and Moon the curvature of the Moon's path stayed positive for 60 days, with radii of curvature from 0.6350 to 1.9559 au——changing only the lunar radius, convexity fails at 259214.846 km, 434.074 km beyond 505's equal-gravity point, and the Moon now sits at 1.482940 times that distance.The curvature sign of the two-circle path changes at 454.841 km and 260850.98 km, splitting the path into three shapes: convex, wave and loop——the loop radius is inside the Earth's radius.The separator is whether the lunar radius lies outside the convexity boundary, between the loop radius and that boundary, or inside the loop radius.Placed among the earlier papers——Paper 505 counted three radii of spheres of influence and showed the Moon lies beyond the equal-gravity point, Paper 784 counted the Roche limit outside the same scope note, and Paper 833 showed that a satellite tells the sum of the masses. Here, as a fourth radius, the radius at which the Moon's path loses convexity was counted and shown to differ from the equal-gravity point by only 0.001677.To be honest——it is not claimed that the path is convex because the Moon lies beyond the equal-gravity point, and the lunar orbit was circular, planar and 60 days long, without eccentricity or tidal recession.**

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