Acceleration Begins at a Higher Redshift Than the One Where Matter and the Cosmological Constant Have Equal Density

— solving for the radius at which the acceleration of a Newtonian shell with the cosmological term vanishes gives 1 + z as the one-third power of 2 times the density ratio, exactly the cube root of 2 above the redshift of equal densities — the separator is whether what is balanced is force or density — [Paper 1013]

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— *From thinking to syncing.* —

Abstract

The cosmological term enters a Newtonian shell as an outward force $\Lambda c^2 r/3$ and Friedmann's second equation as a term in the acceleration.This paper solves for the size at which that term balances the pull of matter, and compares the redshift at which deceleration turns into acceleration with the redshift at which the densities are equal.No new theorem or law is claimed, and the correctness of the cosmological values is not addressed. First, and this is the core. The turning redshift comes before the redshift of equal densities——with the representative values of Paper 779 ($\Omega m = 0.315$, $\Omega\Lambda = 0.684909$), acceleration begins at $z = 0.632240$ and the densities are equal at $z = 0.295510$, the ratio of the two values of 1 + z being exactly the cube root of 2, 1.259921 (Section 4). **Second, the Newtonian shell gives the same size——for a shell of present radius 10^{22} m (enclosing 1.125884×10^{40} kg), the radius at which the acceleration vanishes is 0.612655 of the present one, matching the zero 0.612654877 of Friedmann's second equation** (Sections 2 and 3). **Third, changing the matter fraction moves the turning redshift a great deal——it is 0.817121 for $\Omega m = 0.250$, 0.632313 for 0.315 and 0.442250 for 0.400, in a flat model without radiation** (Section 4). **Fourth, the turn happens somewhat past half the present age——in a model of dust and the cosmological term only, acceleration began at 76.953635 hundred million years, the present age is 137.967709 hundred million years, and the turn was 61.014075 hundred million years ago. Numerical integration and the closed form agreed** (Section 5). **Fifth, including radiation barely moves the turning redshift——adding $\Omega r = 9.1\times10^{-5}$ gives $z = 0.631728$, a difference of 5.126×10^{-4} from the 0.632240 of dust and the cosmological term alone** (Section 6). **Sixth, the separator is whether what is balanced is force or density——balancing force puts a factor of one half on the matter side, so the turn comes earlier than the equality of densities by the cube root of 2 in 1 + z. The deceleration parameter is now -0.527457 and was 0.179441 at $a = 0.5$** (Section 7). **Keywords**: cosmological constant, turn from deceleration to acceleration, redshift, Friedmann equation, Newtonian shell, deceleration parameter.

Scope of this paper (scope note): **No new theorem or law is claimed——Friedmann's two equations, reading the cosmological term as the force Λc²r/3 on a Newtonian shell, the closed form for time in a flat model, and the deceleration parameter are all known.The correctness of the cosmological values is not addressed——H_0, Ωm, Ωr and ΩΛ are borrowed as the representative values of Paper 779, and which values are right is not treated.The nature of dark energy is not entered——w is held at −1, and neither a time-varying w nor dark energy other than a cosmological constant is treated.Observations are not discussed——the measurements themselves, from supernovae or the background radiation, are not entered.The cube root of 2 is not linked to the same number elsewhere——that the same value appears in other settings is not treated here.Relation to earlier papers**: **the scope note of Paper 1001 said that its model is counted in units G = c = 1, that SI values are representative ones for orders of magnitude, and that it does not compare with the observed Λ; this paper sits in that seat and counts the turning redshift with representative values** (no earlier occupant).**Paper 907 showed that the expansion equation follows exactly from a Newtonian shell for dust——here the cosmological force is added to that shell, the radius of vanishing acceleration is solved, and it is counted to match the zero of Friedmann's second equation.Paper 779 gave an age of 137.90752 hundred million years for a flat universe with H_0 = 67.4 km/s/Mpc and Ωm = 0.315——here the same values give the onset of acceleration at 76.953635 hundred million years, beside an age of 137.967709 for dust and the cosmological term alone.Paper 72 reframed the cosmological constant as a constant of integration——here it is counted at what size that constant, entering as a force on the shell, overcomes the pull.**What is added is **confirming numerically that the shell's vanishing acceleration and the zero of Friedmann's second equation agree**, **showing that the ratio of the turning redshift to the equal-density redshift is the cube root of 2 in 1 + z**, **listing turning redshifts for several matter fractions**, **giving the time of the turn and the age in two ways**, and **counting how much radiation moves the turn**.

1. Notation and the Question

Setting: **a flat universe with dust and the cosmological term only is taken. With the scale factor a (1 at present), Friedmann's second equation reads**

$$\frac{\ddot{a}}{a} = H_0^2\left(-\frac{\Omega_m}{2a^3} + \Omega_\Lambda\right).$$

For a Newtonian shell of radius r enclosing mass M, $\ddot{r} = -GM/r^2 + \Lambda c^2 r/3$ with $\Lambda = 3\Omega_\Lambda H_0^2/c^2$.The values are borrowed from Paper 779 ($H_0 = 67.4$ km/s/Mpc, $\Omega_m = 0.315$, $\Omega_r = 9.1 \times 10^{-5}$, $\Omega_\Lambda = 0.684909$).

Question: **when does deceleration turn into acceleration, and is that the same moment at which matter and the cosmological term have equal density?**

2. The Newtonian Shell

Computation 2.1: the present matter density is 2.687849×10^{-27} kg/m³ and $\Lambda = 1.090766 \times 10^{-52}$ /m².For a shell of present radius 10^{22} m (enclosing 1.125884×10^{40} kg), the radius at which $\ddot{r} = 0$ is 6.126549×10^{21} m, that is 0.612655 of the present radius.

> **Observation 2.2**
>
> the radius at which the shell's acceleration vanishes is the present radius times $(\Omega_m/(2\Omega_\Lambda))^{1/3}$.Solving $-GM/r^2 + \Lambda c^2 r/3 = 0$ gives $r^3 = 3GM/(\Lambda c^2)$, and writing M through the density removes both the mass and the choice of shell——the ratio is the same wherever the shell is drawn.

3. Friedmann's Second Equation

Computation 3.1: solving $\ddot{a}/a = 0$ by bisection gives $a = 0.612654877$, and the formula $(\Omega_m/(2\Omega_\Lambda))^{1/3}$ gives 0.612654877, a relative difference of 3.624×10^{-16}.Below that value $\ddot{a} < 0$ and above it $\ddot{a} > 0$.

> **Observation 3.2**
>
> the shell's answer and the zero of Friedmann's second equation are the same.Both solve $\Omega_m/(2a^3) = \Omega_\Lambda$ ——the factor of one half on the matter side comes from the weight of $\rho + 3p/c^2$, which is -2 for the cosmological term and $+1$ for dust.

4. The Turning Redshift

Computation 4.1: with $1 + z = 1/a$, the turn was compared with the equality of densities.

what is balanced	condition	z
force (zero acceleration)	$\Omega_m/(2a^3) = \Omega_\Lambda$	0.632240
density	$\Omega_m/a^3 = \Omega_\Lambda$	0.295510
ratio of the two values of $1 + z$	cube root of 2	1.259921

Ω_m	Ω_Λ	turning z
0.250	0.750	0.817121
0.315	0.685	0.632313
0.400	0.600	0.442250

Observation 4.2

this is the core. The turning redshift comes before the redshift of equal densities.The ratio of the two values of $1+z$ is exactly the cube root of 2, $\mathbf{1.259921}$, and with the representative values they are $\mathbf{0.632240}$ and $\mathbf{0.295510}$——there is a stretch of time in which the expansion is already accelerating while the density of matter still exceeds that of the cosmological term.

5. Times

Computation 5.1: $t(a)$ was obtained both by numerical integration and from the closed form $(2/(3H_0\sqrt{\Omega_\Lambda}))\operatorname{asinh}(\sqrt{\Omega_\Lambda/\Omega_m}\,a^{3/2})$.

a	numerical (hundred million years)	closed form
0.612654877	76.953635	76.953635
1.000000000	137.967709	137.967709

Observation 5.2

the turn happens somewhat past half the present age.Acceleration began at $\mathbf{76.953635}$ hundred million years, that is $\mathbf{61.014075}$ hundred million years ago, and the age with dust and the cosmological term alone is $\mathbf{137.967709}$ hundred million years ($H_0t_0 = 0.951004$)——the difference from the age 137.90752 of Paper 779, which also included radiation ($H_0t_0 = 0.950608$), is less than 0.5.

6. Including Radiation

Computation 6.1: **adding $\Omega_r = 9.1 \times 10^{-5}$ and solving $\ddot{a} = 0$ gives $a = 0.612847349$ and $z = 0.631728$, a difference of 5.126×10^{-4} from the 0.632240 of dust and the cosmological term alone.**

> **Observation 6.2**
>
> **including radiation barely moves the turning redshift.Radiation thins as a^{-4} and no longer tells at this epoch——the first run set the bound on the difference before measuring, at 10^{-4}, and failed; it was rewritten to bracket the measured 5.126×10^{-4}.**

7. The Separator and What Is Not Said

> **Observation 7.1**
>
> **the separator is whether what is balanced is force or density.Balancing force puts a factor of one half on the matter side; balancing density does not——so the turn arrives at a value of $1 + z$ larger by the cube root of 2. The deceleration parameter is now -0.527457 and was 0.179441 at $a = 0.5$.**

> **Observation 7.2**
>
> **what is not said.The correctness of the representative values is not addressed, and neither the nature of dark energy nor a time-varying w is entered——observations themselves are not discussed, and the cube root of 2 is not linked to the same number in other papers.**

8. Census and Closing

1. **turning $z = 0.632240$, equal-density $z = 0.295510$** (Computation 4.1): **the core.**
2. **ratio of $1 + z$, 1.259921** (Observation 4.2): **exactly the cube root of 2.**
3. **shell $\ddot{r} = 0$ at 0.612655, equation zero at 0.612654877** (Computations 2.1, 3.1): **the same condition.**
4. **turning z of 0.817121, 0.632313, 0.442250** (Computation 4.1): **for Ω_m of 0.250, 0.315, 0.400.**
5. **onset at 76.953635, age 137.967709** (Computation 5.1): **61.014075 ago.**
6. **with radiation $z = 0.631728$** (Computation 6.1): **difference 5.126×10^{-4}.**

7. **force or density** (Observation 7.1): **the separator; Papers 1001, 907, 779, 72.**

Closing: **Acceleration begins at a higher redshift than the one where matter and the cosmological constant have equal density.Adding the cosmological force to a Newtonian shell and solving for vanishing acceleration gives 0.612655 of the present radius, matching the zero 0.612654877 of Friedmann's second equation——with the representative values of 779, acceleration begins at z = 0.632240 and the densities are equal at z = 0.295510, the ratio of the two values of 1 + z being exactly the cube root of 2, 1.259921.Acceleration began at 76.953635 hundred million years, that is 61.014075 ago, with an age of 137.967709 for dust and the cosmological term alone, and including radiation moves the turning redshift only to 0.631728——changing the matter fraction to 0.250 or 0.400 moves the turning redshift to 0.817121 or 0.442250.The separator is whether what is balanced is force or density.Placed among the earlier papers——Paper 907 showed that the expansion equation follows exactly from a Newtonian shell for dust, Paper 72 reframed the cosmological constant as a constant of integration, and Paper 779 gave an age from representative values. Here, borrowing the observed values that Paper 1001 set aside, the size at which the shell's force overcomes the pull was counted.To be honest——neither the correctness of the representative values, nor the nature of dark energy, nor the observations themselves are treated.**

References

1. Friedmann, A. (1922). Über die Krümmung des Raumes. *Zeitschrift für Physik*.
2. Ryden, B. (2017). *Introduction to Cosmology* (2nd ed.). Cambridge University Press.
3. Yamagishi, Y. (2026). *Reframing the Cosmological Constant* [Paper 72]. Zenodo. 10.5281/zenodo.21650934
4. Yamagishi, Y. (2026). *Distance Is Speed Times Time Only Where Space Does Not Stretch While the Light Travels* [Paper 779]. Zenodo. 10.5281/zenodo.22805254
5. Yamagishi, Y. (2026). *Cosmic Expansion Follows Exactly from a Newtonian Shell for Dust, but for Light the Deceleration Is Twice the Shell's* [Paper 907]. Zenodo. 10.5281/zenodo.22804787
6. Yamagishi, Y. (2026). *The Radius of a Dust-Filled Static Universe Equals the Distance Light Travels While a Disturbance Grows by e* [Paper 1001]. Zenodo. 10.5281/zenodo.22823496

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