The 2.5 in the Viscosity of a Suspension Comes Not from the Volume of the Particles but from the Stress They Place on the Flow Outside

— for the Stokes solution of a sphere in a uniform straining flow, the surface integral of the stress over the sphere (the stresslet) is 20π/3 times viscosity, radius cubed and strain, and dividing by the particle volume 4π/3 gives 2.5; solving viscosity and diffusion for two unknowns, a change of the coefficient from 1 to 2.5 moves Avogadro's number by a factor 1.581139 — the separator is whether one counts the volume inside the particle or the stress outside it — [Paper 1024]

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September 2026

— *From thinking to syncing.* —

Abstract

A liquid with small spheres mixed in at volume fraction φ has, when dilute, (1 + 2.5φ) times its original viscosity. Einstein wrote the coefficient as 1 in his 1906 doctoral thesis and corrected it to 2.5 in 1911.This paper checks the Stokes solution for a sphere in a uniform straining flow by symbolic computation, obtains 2.5 from the surface integral of the stress over the sphere, and counts how that coefficient enters the Avogadro number determined from viscosity and diffusion.No new theorem or law is claimed. First, and this is the core. The 2.5 comes not from the volume of the particle but from the stress it places on the flow outside——integrating the stress over the sphere numerically on a 400×800 grid, with viscosity, radius cubed and strain set to 1, the diagonal component is 20.943978, agreeing with the closed form $20\pi/3$ = 20.943951 at a ratio of 1.000001. The coefficient of the increase in effective viscosity is this divided by the particle volume $4\pi/3$, $(10\pi/3)/(4\pi/3)$ = 2.500000 (Section 3). **Second, the solution used satisfies the equations and the boundary condition——symbolically, the divergence is 0, the residual of the Stokes equation is 0 in all three components, and the velocity vanishes at the surface point (3/5, 4/5, 0)a** (Section 2). **Third, the influence of the particle reaches outward as $1/r^2$——multiplying the disturbance from the uniform strain by the square of the distance and taking the far limit on the x axis, the first component tends to $-5a^3/2$. What acts on the viscosity is this stress that reaches outside** (Section 2). **Fourth, the coefficient enters Avogadro's number as a square root——the viscosity increase fixes the product of Avogadro's number and the cube of the molecular radius, the diffusion coefficient fixes the product with the first power, so solving the two together, changing the coefficient k from 1 to 2.5 moves Avogadro's number by $\sqrt{2.5}$ = 1.581139 and the radius by $1/\sqrt{2.5}$** (Section 4). **Fifth, in the dilute range the increase is proportional to the fraction——at φ = 0.01, 0.02 and 0.05 the viscosity ratios are 1.025000, 1.050000 and 1.125000** (Section 4). **Sixth, the separator is whether one counts the volume inside the particle or the stress outside it——the volume fraction alone would stop the coefficient at 1. The part of 2.5 beyond the volume is the stress the sphere places by bending the flow outside** (Section 5). **Keywords**: Einstein viscosity formula, Stokes flow, stresslet, suspension, Avogadro number, effective viscosity.

Scope of this paper (scope note): **No new theorem or law is claimed——the viscosity formula (1 + 2.5φ) for a dilute suspension (Einstein 1906, corrected 1911), the Stokes solution for a sphere in a uniform straining flow, the definition of the effective stress through the stresslet (Batchelor 1967), and the Stokes-Einstein diffusion coefficient are all known.Only the dilute limit is seen——hydrodynamic interactions between particles (the φ² term), Brownian motion, non-Newtonian behaviour, deformable or non-spherical particles are not included.The history of the correction is not entered——why the coefficient 1 of 1906 was corrected to 2.5 in 1911 is not entered; it is cited as a fact.Real suspensions are not compared——no agreement with measured viscosities is claimed; all numbers are computed from the definition of the solution.The dissipation route is not used——the 2.5 is obtained from the surface integral of the stress, the definition of the effective stress itself, and not from the increase in dissipation.Relation to earlier papers**: **Paper 673 showed that a colloid is the name of a window in size rather than of a substance, taking the Einstein-Stokes diffusion coefficient as known——here the other formula of the same thesis, the coefficient on the viscosity side, is counted.Paper 544 showed that thirteen routes returned one Avogadro number and that the number is now a definition——here it is counted, for one of those routes (viscosity and diffusion), by what factor an error in the coefficient moves the number.Paper 827 showed that the softening point of glass is a convention fixing one viscosity ——here it is counted that the coefficient of the viscosity increase is not a convention but a number fixed by the solution.**What is added is **checking the Stokes solution symbolically**, **integrating the stresslet numerically and matching 20π/3**, and **obtaining by simultaneous solution that the coefficient enters Avogadro's number as a square root**.

1. Notation and the Question

Setting: **a rigid sphere of radius a sits in a liquid of viscosity μ, and a uniform straining flow $\mathbf{u} \to \mathbf{E}\cdot\mathbf{x}$ ($\mathbf{E}$ symmetric and traceless) is imposed far away.The Stokes solution is**

$$u_i = E_{ij}x_j\left(1-\frac{a^5}{r^5}\right) - \frac{5}{2}\left(\mathbf{x}\cdot\mathbf{E}\cdot\mathbf{x}\right)x_i\left(\frac{a^3}{r^5}-\frac{a^5}{r^7}\right), \qquad p = -5\mu a^3\,\frac{\mathbf{x}\cdot\mathbf{E}\cdot\mathbf{x}}{r^5}.$$

The stresslet, the stress the sphere places on the flow outside, is the traceless part of $S_{ij} = \oint \tfrac{1}{2}(\sigma_{ik}n_k x_j + \sigma_{jk}n_k x_i)dS$, and the mean stress of a suspension with n spheres per volume is $2\mu\mathbf{E} + n\mathbf{S}$.

Question: **where does the 2.5 come from? The volume fraction φ alone would stop the coefficient at 1.**

2. Checking the Solution

Computation 2.1: **by symbolic computation (sympy), the divergence $\nabla\cdot\mathbf{u}$ is 0, the residual of $\mu\nabla^2\mathbf{u} - \nabla p$ is 0 in all three components, and $\mathbf{u}$ vanishes at the surface point $(3/5, 4/5, 0)a$.Multiplying the disturbance $u_i - E_{ij}x_j$ by x^2 and taking the far limit on the x axis, the first component is $-5a^3/2$ and the others 0.**

Observation 2.2

the influence of the particle reaches outward as $1/r^2$.The bending of the flow by a rigid sphere remains not inside the particle but in the flow outside, as the inverse square of distance——what acts on the viscosity is this stress that remains outside.

3. The Stresslet and 2.5

Computation 3.1: with $\mu = a = 1$ and $\mathbf{E} = \mathrm{diag}(1, -1/2, -1/2)$, the surface integral of the stress was computed numerically on a grid of 400 polar and 800 azimuthal points.

component	numerical integral	closed form $(20\pi/3)E_{ii}$
S_{11}	20.943978	20.943951
S_{22}	−10.471949	−10.471976
S_{33}	−10.472029	−10.471976

S_{11} divided by $(20\pi/3)E_{11}$ is 1.000001. Reading the mean stress $2\mu\mathbf{E} + n\mathbf{S}$ as $2\mu_{\text{eff}}\mathbf{E}$ gives $\mu_{\text{eff}}/\mu = 1 + n(10\pi/3)a^3 = 1 + \frac{10\pi/3}{4\pi/3}\phi$, with coefficient $(10\pi/3)/(4\pi/3) = 2.500000$.

Observation 3.2

this is the core. The 2.5 comes not from the volume of the particle but from the stress it places on the flow outside.The numerator is $10\pi/3$, the denominator the volume $4\pi/3$ of the sphere, and the ratio is $5/2$—— counting the volume fraction φ alone gives 1; the excess $3/2$ is the stress outside.

4. The Coefficient Enters Avogadro's Number as a Square Root

Computation 4.1: from the viscosity increase $\mu^*/\mu - 1 = k\phi$ with $\phi = N_A c \cdot \frac{4}{3}\pi a^3/M$, the product $N_A a^3$ is fixed. From the diffusion coefficient $D = RT/(6\pi\mu N_A a)$ the product $N_A a$ is fixed. Solving the two together, a scales as $k^{-1/2}$ and N_A as $k^{1/2}$.Solving the system with placeholder inputs, the ratio of N_A for $k = 2.5$ to that for $k = 1$ was 1.581139, agreeing with $\sqrt{2.5} = 1.581139$ within 10^{-12}.

Computation 4.2: **the viscosity ratios in the dilute range are 1.025000, 1.050000 and 1.125000 at $\phi = 0.01$, 0.02 and 0.05.**

> **Observation 4.3**
>
> **an error in the coefficient enters Avogadro's number as a square root.Two measurements fix two unknowns, so a coefficient off by 2.5 moves the answer not by 2.5 but by $\sqrt{2.5}$——this is the form in which the 1911 correction from 1 to 2.5 moved the Avogadro number of this route.**

5. The Separator and What Is Not Said

> **Observation 5.1**
>
> **the separator is whether one counts the volume inside the particle or the stress outside it.The volume fraction alone stops at 1, and only counting the stress the sphere places by bending the flow outside gives 2.5——the diffusion side that Paper 673 took as known and the viscosity side counted here are the two formulas of the same thesis.**

> **Observation 5.2**
>
> **what is not said.Only the dilute limit is seen; interactions between particles, Brownian motion and non-Newtonian behaviour are not entered——nor why the coefficient 1 of 1906 was corrected in 1911, nor any comparison with measured suspensions.**

6. Census and Closing

1. **stresslet 20.943978, $20\pi/3 = 20.943951$, ratio 1.000001, coefficient 2.500000** (Computation 3.1): **the core.**
2. **divergence 0, Stokes equation 0, velocity 0 on the sphere** (Computation 2.1): **the solution checked.**
3. **far coefficient $-5a^3/2$** (Computation 2.1): **the influence reaches out as $1/r^2$.**
4. **Avogadro ratio $1.581139 = \sqrt{2.5}$** (Computation 4.1): **a square root.**
5. **viscosity ratios 1.025000, 1.050000, 1.125000** (Computation 4.2): **the dilute range.**
6. **volume inside or stress outside** (Observation 5.1): **the separator; Papers 673, 544, 827.**

Closing: **The 2.5 in the viscosity of a suspension comes not from the volume of the particles but from the stress they place on the flow outside.Checking the Stokes solution for a sphere in a uniform straining flow symbolically and integrating the stress over the sphere numerically gives 20.943978, agreeing with the closed form $20\pi/3 = 20.943951$ at a ratio of 1.000001. Divided by the particle volume $4\pi/3$ the coefficient is 2.500000, and solving two measurements for two unknowns, a factor 2.5 in the coefficient appears as a factor 1.581139 in Avogadro's number——the volume fraction alone would stop at 1; the excess is the stress that remains outside as $1/r^2$.The separator is whether one counts the volume inside the particle or the stress outside it.Placed among the earlier papers——Paper 673 counted the window in size taking the diffusion side as known, Paper 544 showed that the agreement of thirteen routes turned into a definition, and Paper 827 showed the convention in a viscosity value. Here the coefficient of the other formula of the thesis was counted from the stress of the solution.To be honest——only one sphere in the dilute limit has been counted.**

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