The Ceiling of a Heat-Driven Refrigerator Is the Product of an Engine's Efficiency and a Refrigerator's Coefficient of Performance

— when cooling is driven by heat alone, as in the absorption refrigerator of Einstein and Szilard, the ceiling of heat removed from the cabinet per unit of heating is 0.200992 × 10.926000 = 2.196034 for heating at 100 ℃, ambient 25 ℃ and cabinet 0 ℃, and in the entropy ledger it is a ratio of drops in inverse temperature; putting in no work is not free, since the work carried by the heating is paid — the separator is whether what is put in is work or heat with a temperature difference — [Paper 1025]

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— *From thinking to syncing.* —

Abstract

In 1930 Einstein and Szilard patented an absorption refrigerator with no moving parts, driven by heat alone.This paper obtains the reversible ceiling of a refrigerator working between three reservoirs (heating, ambient and cabinet) by three routes, a closed form, the entropy balance and a ratio of inverse-temperature differences, and counts that it is the product of an engine's efficiency and a refrigerator's coefficient of performance, that more heat is dumped into the kitchen than with a work-driven machine, and that raising the heating temperature never passes a ceiling.No new theorem or law is claimed. First, and this is the core. The ceiling of a heat-driven refrigerator is the product of an engine's efficiency and a refrigerator's coefficient of performance——for heating Tg, ambient Ta and cabinet Tc, the ceiling of heat removed per unit of heating is Tc(Tg − Ta)/(Tg(Ta − Tc)); at 100 °C, 25 °C and 0 °C it is the Carnot efficiency 0.200992 times the coefficient of performance 10.926000, that is 2.196034. The closed form, the entropy balance solved simultaneously and the ratio of inverse-temperature differences agreed within 10 to the −12 for four sets of temperatures (Section 2). **Second, in the entropy ledger it is a ratio of drops in inverse temperature——the ceiling is (1/Ta − 1/Tg)/(1/Tc − 1/Ta): the heating heat drops from 1/Tg to 1/Ta, and that pays for lifting the cabinet heat from 1/Ta to 1/Tc. Work never appears in this ledger; heat with a temperature difference appears instead** (Section 3). **Third, putting in no work is not free——to remove 1 from the cabinet takes 0.455366 of heat at 100 °C, or 0.091525 of reversible work; the ratio 4.975333 equals the reciprocal of the Carnot efficiency. The heat dumped to the ambient is 1.455366 when driven by heat and 1.091525 when driven by work, so the kitchen is warmed 1.333333 times more** (Section 4). **Fourth, raising the heating temperature never passes the coefficient of performance——heating at 50, 100, 200, 300 and 500 °C gives ceilings of 0.845273, 2.196034, 4.041107, 5.242345 and 6.712604, and even infinite heating cannot exceed the work-driven coefficient 10.926000** (Section 4). **Fifth, at the two ends the ceiling goes to 0 and to infinity——with the heating only 0.001 K above the ambient the ceiling is 3.664586×10^{-5}, and with the cabinet only 0.001 K below the ambient it is 5.992543×10^{4}. Without a drop nothing can be moved; without a lift anything can** (Section 5). **Sixth, the separator is whether what is put in is work or heat with a temperature difference——the ceiling of a work-driven refrigerator is written with one coefficient of performance; that of a heat-driven one multiplies it by an engine's efficiency. "No moving parts" reads, on the ledger side, as "a temperature difference does the work of the compressor"** (Section 5).

Keywords: absorption refrigerator, three heat reservoirs, coefficient of performance, Carnot efficiency, entropy balance, Einstein-Szilard refrigerator.

Scope of this paper (scope note): **No new theorem or law is claimed——the Carnot efficiency, the coefficient of performance of a refrigerator, the ceiling of a reversible refrigerator working between three reservoirs, the entropy balance of reversible processes, and the absorption refrigerator of Einstein and Szilard (U.S. patent, 1930) are all known.No values of real machines are cited——the coefficients of performance of real absorption machines have not been verified and are not cited; all numbers are reversible ceilings.The mechanism is not entered——the working fluids (butane, ammonia, water), the replacement of the compressor by partial pressures, and the layout of generator, absorber and condenser are not the subject.Finite-time thermodynamics is not entered——ceilings at maximum power and finite heat conduction are not treated; Paper 311 covered that side.The second law is not proved——the entropy balance is used as a premise; Paper 126 treated its foundation by the route of the integrating factor.Temperatures are representative——100 °C, 25 °C, 0 °C and the rest are values for comparison, not the operating temperatures of any device.Relation to earlier papers**: **Paper 311 showed that one of the five things called "efficiency" comes to 14.6575 and that the correct bound is not the predictive one——here it is counted that adding a third temperature to the same two makes the ceiling the product of two ceilings.Paper 126 showed that the existence of an integrating factor does not give a temperature——here it is counted that the ceiling can be written with the inverse-temperature ledger alone.**What is added is **matching the three-reservoir ceiling by three routes**, **reading it as a ratio of drops in inverse temperature**, **comparing the dumped heat with the work-driven case**, and **giving the ceiling as the heating temperature is raised**.

1. Notation and the Question

Setting: **a device receives heat Q_g from a source at the heating temperature T_g, removes heat Q_c from a cabinet at T_c , and dumps $Q_a = Q_g + Q_c$ to the ambient at T_a. No work is put in.If reversible, entropy balances:**

$$\frac{Q_g}{T_g} + \frac{Q_c}{T_c} = \frac{Q_a}{T_a}.$$

The quantity seen as a ceiling is Q_c/Q_g, the heat removed per unit of heating.

Question: **what fixes the ceiling of a refrigerator with no work put in, and how does it differ from the ceiling of a work-driven one?**

2. Three Routes

Computation 2.1: **the ceiling was obtained for four sets of temperatures by (1) the closed form $T_c(T_g - T_a)/(T_g(T_a - T_c))$, (2) setting $Q_g = 1$ and solving the balance for Q_c, and (3) the ratio of inverse-temperature differences $(1/T_a - 1/T_g)/(1/T_c - 1/T_a)$.**

T_g, T_a, T_c (K)	closed form	balance	inverse-T ratio	Carnot efficiency	COP
373.15, 298.15, 273.15	2.196034	2.196034	2.196034	0.200992	10.926000
473.15, 298.15, 273.15	4.041107	4.041107	4.041107	0.369862	10.926000
373.15, 308.15, 263.15	1.018640	1.018640	1.018640	0.174193	5.847778
353.15, 298.15, 278.15	2.165971	2.165971	2.165971	0.155741	13.907500

The three routes agreed within 10^{-12} for all four sets, and in each the ceiling equals the product of the Carnot efficiency $(T_g - T_a)/T_g$ and the coefficient of performance $T_c/(T_a - T_c)$.

> **Observation 2.2**
>
> this is the core. The ceiling of a heat-driven refrigerator is the product of an engine's efficiency and a refrigerator's coefficient of performance.It equals the ceiling of a two-stage device that runs an engine on the heating heat and a refrigerator on the work——even with no parts in one device, the ledger splits into two stages.

3. The Inverse-Temperature Ledger

> **Observation 3.1**
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> the ceiling is a ratio of drops in inverse temperature.Q_g drops from $1/T_g$ to $1/T_a$ and creates entropy $Q_g(1/T_a - 1/T_g)$, which pays for lifting Q_c from $1/T_a$ to $1/T_c$ (costing $Q_c(1/T_c - 1/T_a)$)——work never appears in this ledger; only heat with a temperature difference does.

4. Heat Dumped into the Kitchen, and the Heating Temperature

Computation 4.1: to remove 1 from the cabinet, driving with heat at 100 °C takes 0.455366 of heat and dumps 1.455366 to the ambient. Driving with reversible work takes 0.091525 of work and dumps 1.091525; the ratio of dumped heats is 1.333333, and heat needed over work needed is 4.975333, equal to the reciprocal of the Carnot efficiency, 4.975333.

Computation 4.2: with ambient 25 °C and cabinet 0 °C, raising the heating temperature gives the following ceilings.

heating temperature	ceiling Q_c/Q_g	Carnot efficiency
50 °C	0.845273	0.077363
100 °C	2.196034	0.200992
200 °C	4.041107	0.369862
300 °C	5.242345	0.479805
500 °C	6.712604	0.614370

Even with infinite heating the ceiling does not exceed the work-driven coefficient of performance 10.926000.

> **Observation 4.3**
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> putting in no work is not free.Driven by heat, the device dumps 1.333333 times as much heat into the kitchen for the same cooling——the excess is the part of the heating heat that could not be turned into work.

5. Limits, the Separator and What Is Not Said

Computation 5.1: with the heating only 0.001 K above the ambient the ceiling is 3.664586×10^{-5}; with the cabinet only 0.001 K below the ambient it is 5.992543×10^{4}.

> **Observation 5.2**
>
> the separator is whether what is put in is work or heat with a temperature difference.The ceiling of a work-driven refrigerator is written with one coefficient of performance; that of a heat-driven one multiplies it by an engine’s efficiency——"no moving parts" reads, on the ledger side, as "a temperature difference does the work of the compressor".

> **Observation 5.3**
>
> what is not said.No coefficient of a real absorption machine is cited, and the working fluids and the partial-pressure mechanism are not entered——ceilings in finite time belong to Paper 311, the foundation of the second law to Paper 126.

6. Census and Closing

1. **ceiling $2.196034 = 0.200992 \times 10.926000$, three routes agree** (Computation 2.1): **the core.**
2. **ratio of drops in inverse temperature** (Observation 3.1): **work never appears.**
3. **heat needed 0.455366, work needed 0.091525, ratio 4.975333, dumped heat 1.455366 against 1.091525, ratio 1.333333** (Computation 4.1): **not free.**
4. **heating 50 to 500 °C: 0.845273, 2.196034, 4.041107, 5.242345, 6.712604** (Computation 4.2): **the ceiling is 10.926000.**
5. **end values 3.664586×10^{-5} and 5.992543×10^{4}** (Computation 5.1): **zero and infinity.**
6. **work, or heat with a temperature difference** (Observation 5.2): **the separator; Papers 311 and 126.**

Closing: **The ceiling of a heat-driven refrigerator is the product of an engine's efficiency and a refrigerator's coefficient of performance.For heating at 100 °C, ambient 25 °C and cabinet 0 °C, the ceiling of heat removed per unit of heating is 0.200992 × 10.926000 = 2.196034, and the closed form, the entropy balance and the ratio of inverse-temperature differences agree. Removing 1 from the cabinet takes 4.975333 times as much heat as work, dumps 1.333333 times as much heat into the kitchen, and even infinite heating cannot pass 10.926000——putting in no work is not free; a temperature difference is doing the work of the compressor.The separator is whether what is put in is work or heat with a temperature difference.Placed among the earlier papers——Paper 311 showed that the correct bound is not the predictive one, and Paper 126 that the existence of an integrating factor does not give a temperature. Here it was counted that adding a third reservoir makes the ceiling the product of two ceilings.To be honest——not a single number of a real refrigerator has been counted.**

References

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