The Spatial Curvature of a Rotating Disk Is $-3\Omega^2/c^2$, and Its Argument Differs from the $-1/c^2$ of Velocity Space

— the Gaussian curvature of the spatial metric of a rotating frame is $-3\Omega^2/c^2$ at the centre and vanishes when Ω is set to 0; for a disk whose rim moves at half the speed of light the circumference over the diameter is $\pi\gamma = 3.627599$ and the area is 1.071797 times the Euclidean one, growing less than the circumference; the curvature $-1/c^2$ of velocity space contains no Ω and has different units — the separator is whether what bends is rotation or the composition of velocities — [Paper 1027]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
September 2026

— *From thinking to syncing.* —

Abstract

The circumference of a rotating disk is longer than π times its diameter (Ehrenfest 1909). Einstein used this disk as a clue that geometry under gravity would not be Euclidean.This paper obtains the Gaussian curvature of the spatial metric of a rotating frame by two routes, symbolic computation and numerical differentiation, counts the growth of circumference and area, and sets the result beside the negative curvature $-1/c^2$ that this corpus has seen in velocity space.No new theorem or law is claimed, and the Ehrenfest paradox is not said to be solved. First, and this is the core. The spatial curvature of a rotating disk is $-3\Omega^2/c^2$, and it vanishes when the rotation stops——the Gaussian curvature of the metric $dl^2 = dr^2 + r^2 d\varphi^2/(1 - \Omega^2 r^2/c^2)$ is $K(r) = -3\Omega^2 c^2/(c^2 - \Omega^2 r^2)^2$, equal to $-3\Omega^2/c^2$ at the centre and 0 at $\Omega = 0$. Symbolic computation and numerical differentiation agreed within 3×10^{-8} at six points (Section 2). **Second, the circumference grows by γ——integrating the circumference over the diameter numerically for a disk whose rim moves at β times the speed of light agrees with $\pi\gamma$ within 10 to the −9: 3.157419, 3.627599 and 7.207308 at β = 0.1, 0.5 and 0.9 (1.005038, 1.154701 and 2.294157 times π)** (Section 3). **Third, the area grows less than the circumference——the area over πR^2 is $2(1 - \sqrt{(1 - \beta^2)})/\beta^2$, that is 1.002513, 1.071797 and 1.392864 at β = 0.1, 0.5 and 0.9, each greater than 1 and less than γ** (Section 3). **Fourth, the radius of curvature is set by the rim speed——the central radius of curvature $c/(\sqrt{3}\,\Omega)$ is 1.154701 times the disk radius when the rim moves at β = 0.5 and 5.773503 times at β = 0.1, and equals the disk radius when the rim moves at $\beta = 1/\sqrt{3} = 0.577350$. The angles of a small geodesic triangle of area 0.01 sum to 0.007500 less than π (hyperbolic)** (Section 4). **Fifth, it differs from the $-1/c^2$ of velocity space in argument and in units, though the denominator is the same——the curvature of velocity space contains no Ω and has units of 1 over speed squared. The curvature of the disk has units of 1 over length squared and is proportional to Ω^2. There is no equation tying them, so by the criterion of Paper 300 they cannot be said to share a root** (Section 5). **Sixth, the separator is whether what bends is rotation or the composition of velocities——what bends the disk is rotation; what bends velocity space is the composition of velocities. Of two negative curvatures, one vanishes when stopped and the other does not** (Section 5). **Keywords**: rotating disk, Gaussian curvature, Ehrenfest paradox, metric of a rotating frame, velocity space, hyperbolic geometry.

Scope of this paper (scope note): **No new theorem or law is claimed——that the circumference of a rotating disk exceeds π times its diameter (Ehrenfest 1909), the spatial metric of a rotating frame (Landau and Lifshitz), Einstein's use of this disk on the way to general relativity (1916), and Born's condition of rigidity are all known.The Ehrenfest paradox is not solved——whether the disk can be spun up as a rigid body and how simultaneity along the rim is to be fixed are not the subject; the spatial metric of the rotating state is taken as a premise.The field equations are not entered——the rotating frame is a coordinate transformation of flat spacetime, and no gravitational field equation is used; what is curved is the spatial metric, not spacetime.The curvature of velocity space is not derived——$-1/c^2$ is cited from Papers 60 and 66.The values are examples——rim speeds β = 0.1, 0.5 and 0.9 are values for comparison; no real rotating body is claimed.Relation to earlier papers**: **Paper 60 showed that c is the radius of the ball of velocity space——here a negative curvature appearing elsewhere is separated by its argument although the c^2 in the denominator is the same.Paper 66 showed that the curvature of velocity space is written with the local c and is a different root from the coordinate c of gravitational redshift——here a third c^2, the denominator of the disk's curvature, is set beside them and counted to vanish with Ω.Paper 33 showed that the Wigner angle is an area in velocity space——here it is counted that the angle deficit of a small triangle on the disk is the product of curvature and area.Paper 300 showed that whether two things share a root is decidable——here the absence of a tying equation is the reason not to call them one root.**What is added is **matching the curvature by two routes**, **matching the growth of circumference and area by numerical integration against closed forms**, **giving the rim speed at which the radius of curvature equals the disk radius**, and **separating the disk's curvature from that of velocity space by argument and units**.

1. Notation and the Question

Setting: **in a frame rotating at angular velocity Ω, the spatial metric in the plane of the disk is**

$$dl^2 = dr^2 + \frac{r^2\,d\varphi^2}{1-\Omega^2 r^2/c^2}$$

(Landau and Lifshitz). With $G(r) = r^2/(1-\Omega^2 r^2/c^2)$ the Gaussian curvature is $K = -\frac{1}{\sqrt{G}}\frac{d^2\sqrt{G}}{dr^2}$.Let R be the rim radius, $\beta = \Omega R/c$ the rim speed and $\gamma = 1/\sqrt{1-\beta^2}$.

Question: **what is the curvature of this space and what fixes it? Is it the same as the $-1/c^2$ this corpus has seen in velocity space?**

2. Curvature: Two Routes

Computation 2.1: **symbolically, $K(r) = -3\Omega^2 c^2/(c^2-\Omega^2 r^2)^2$, $K(0) = -3\Omega^2/c^2$ at the centre, and 0 at $\Omega = 0$ .Compared with numerical differentiation (step 10^{-4}) at six points ($c = R = 1$):**

rim speed β	r/R	symbolic K	numerical K	Kc^2/Ω^2
0.1	0.2	-0.030024014	-0.030024014	-3.002401
0.1	0.5	-0.030150564	-0.030150582	-3.015056
0.1	0.9	-0.030491969	-0.030491978	-3.049197
0.5	0.2	-0.765228038	-0.765228053	-3.060912
0.5	0.5	-0.853333333	-0.853333362	-3.413333
0.5	0.9	-1.179233695	-1.179233702	-4.716935

The two routes differ by at most 3×10^{-8}; at the rim ($\beta = 0.5$) $Kc^2/\Omega^2 = -5.333333$.

> **Observation 2.2**
>
> this is the core. The spatial curvature of a rotating disk is $-3\Omega^2/c^2$, and it vanishes when the rotation stops.The curvature is negative and grows from the centre to the rim——the c^2 in the denominator is the same as in the curvature of velocity space, but Ω^2 multiplies the numerator.

3. Circumference and Area

Computation 3.1: the circumference $\oint \sqrt{G}\,d\varphi$ and the area $\int 2\pi r\,dr/\sqrt{1-\Omega^2 r^2/c^2}$ were integrated numerically.

β	circumference over diameter	$\pi\gamma$	area over πR^2	$2(1-\sqrt{1-\beta^2})/\beta^2$
0.1	3.157419	3.157419	1.002513	1.002513
0.5	3.627599	3.627599	1.071797	1.071797
0.9	7.207308	7.207308	1.392864	1.392864

The circumference over the diameter agrees with $\pi\gamma$ within 10^{-9} and the area ratio with the closed form within 10^{-6}; all three area ratios lie above 1 and below γ (1.005038, 1.154701, 2.294157).

Observation 3.2

the circumference grows by γ, but the area grows less.The radial rod is not contracted; only the tangential one is——this is the disk's form of a negatively curved surface, whose circumference exceeds what its radius would suggest.

4. Radius of Curvature and Triangles

Computation 4.1: **the central radius of curvature $1/\sqrt{|K(0)|} = c/(\sqrt{3}\Omega)$ is 1.154701 times R when the rim moves at $\beta = 0.5$, 5.773503 times at $\beta = 0.1$, and equals R when the rim moves at $\beta = 1/\sqrt{3} = 0.577350$.A small geodesic triangle of area 0.01 near the centre has an angle sum smaller than π by 0.007500 at $\beta = 0.5$ (the product of $K(0)$ and the area).**

Observation 4.2

the radius of curvature is set by the rim speed.Paper 33 showed that the Wigner angle is an area in velocity space——here it is counted that the angle deficit of a triangle on the disk is the product of curvature and area, with negative sign.

5. Comparison with Velocity Space, the Separator, and What Is Not Said

Observation 5.1

it differs from the $-1/c^2$ of velocity space in argument and in units, though the denominator is the same.Paper 60 showed that c is the radius of the ball of velocity space, and Paper 66 that its curvature is written with the local c——the curvature of velocity space contains no Ω and has units of 1 over speed squared; the disk's curvature has units of 1 over length squared, is proportional to Ω^2, and vanishes when the rotation stops. No equation ties the two, and by the criterion of Paper 300 they cannot be called one root.

Observation 5.2

the separator is whether what bends is rotation or the composition of velocities.What bends the disk is rotation; what bends velocity space is the composition of velocities——of two negative curvatures, one vanishes when stopped and the other does not.

Observation 5.3

what is not said.Whether the disk can be spun up rigidly and how simultaneity along the rim is fixed are not the subject, and the paradox is not said to be solved——the rotating frame is a coordinate transformation of flat spacetime, and no field equation is used.

6. Census and Closing

1. $K(0) = -3\Omega^2/c^2$**, two routes within** $\mathbf{3 \times 10^{-8}}$ (Computation 2.1): **the core.**
2. **circumference over diameter** $\mathbf{3.157419}$, $\mathbf{3.627599}$, $\mathbf{7.207308} = \pi\gamma$ (Computation 3.1): **the circumference grows by γ.**
3. **area ratios** $\mathbf{1.002513}$, $\mathbf{1.071797}$, $\mathbf{1.392864}$ (Computation 3.1): **less than the circumference.**
4. **radius of curvature** $\mathbf{1.154701}$ **R,** $\mathbf{5.773503}$ **R, equality at rim speed** $\mathbf{0.577350}$**, angle deficit** $\mathbf{0.007500}$ (Computation 4.1): **set by the rim speed.**
5. **the −1/c² of velocity space contains no Ω** (Observation 5.1): **different argument and units.**
6. **rotation or composition of velocities** (Observation 5.2): **the separator; Papers 60, 66, 33 and 300.**

Closing: **The spatial curvature of a rotating disk is −3Ω²/c², and its argument differs from the −1/c² of velocity space.The Gaussian curvature of the spatial metric of a rotating frame is K(r) = −3Ω²c²/(c² − Ω²r²)², matched by symbolic computation and numerical differentiation, and it vanishes at Ω = 0. For a disk whose rim moves at half the speed of light the circumference over the diameter is πγ = 3.627599 and the area is 1.071797 times the Euclidean one, growing less than the circumference; the central radius of curvature equals the disk radius when the rim moves at 1/√3 = 0.577350——the curvature of velocity space contains no Ω, has different units, and no equation ties the two.The separator is whether what bends is rotation or the composition of velocities.Placed among the earlier papers——Paper 60 showed that c is the radius of the ball of velocity space, Paper 66 that its curvature is written with the local c, and Paper 33 that the Wigner angle is an area. Here a third negative curvature, made by rotation, was set beside them and separated by its argument.To be honest——not one step has been taken into how the disk is spun up.**

References

1. Ehrenfest, P. (1909). Gleichförmige Rotation starrer Körper und Relativitätstheorie. *Physikalische Zeitschrift*.
2. Einstein, A. (1916). Die Grundlage der allgemeinen Relativitätstheorie. *Annalen der Physik*.
3. Landau, L. D., & Lifshitz, E. M. (1975). *The Classical Theory of Fields*. Pergamon Press.
4. Yamagishi, Y. (2026). *The Wigner Angle Is an Area* [Paper 33]. Zenodo. 10.5281/zenodo.22090872
5. Yamagishi, Y. (2026). ***c*** *as the Ball of Velocity Space* [Paper 60]. Zenodo. 10.5281/zenodo.21598012
6. Yamagishi, Y. (2026). *Velocity-Space Curvature and Gravitational Redshift* [Paper 66]. Zenodo. 10.5281/zenodo.21616607
7. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914

— *From thinking to syncing.* —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)