The Size of the Falling Box in Which Gravity Cannot Be Told Is Set Not by Physics but by the Precision of the Instrument

— the difference in gravity between the top and bottom of the box is the tidal gradient $2GM/r^3$ times the height, and the height at which the box gives gravity away is precision divided by gradient: at the Earth's surface 3.243790 mm for a precision of 10^{-8} m/s² and 3.243790 μm for 10^{-11}, at the horizon of a solar-mass black hole 9.709164×10^{-19} m, and at the horizon of a billion solar masses 0.9709164 m — the separator is whether what shrinks the box is precision or gradient — [Paper 1029]

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— *From thinking to syncing.* —

Abstract

Einstein called the idea that gravity vanishes inside a falling box the happiest thought of his life. But gravity differs between the top and bottom of the box, by the tidal gradient times the height, and with a fine enough instrument gravity can be told in any box.This paper counts that difference and the box height set by the instrument's precision at the surfaces of the Earth, the Moon and the Sun, at the surface of a neutron star, and at the horizons of black holes.No new theorem or law is claimed. First, and this is the core. What sets the size of the box is precision divided by gradient——the tidal gradient $2GM/R^3$ at the Earth's surface is 3.082813×10^{-6} per second squared, agreeing with a finite difference and within 1 percent of $2g/R = 3.078528 \times 10^{-6}$. Over a 1 m box the difference is 3.143595×10^{-7} of g, and an instrument of precision 10^{-8} m/s² tells gravity in a box of height 3.243790×10^{-3} m, one of 10^{-11} in a box of 3.243790×10^{-6} m (Sections 2 and 3). **Second, at the same precision the box moves through 20 orders of magnitude with place——at precision 10^{-8} the box is 5.351177×10^{-3} m at the Moon's surface, 1.268274×10^{-2} m at the Sun's, 4.649033×10^{-17} m at the surface of a neutron star (1.4 solar masses, radius 12 km), 9.709164×10^{-19} m at the horizon of a solar-mass black hole and 0.9709164 m at the horizon of a billion solar masses** (Section 3). **Third, the gradient at the horizon goes as the inverse square of the mass——at radius $2GM/c^2$ the gradient is $c^6/(4G^2M^2)$: 1.029955×10^{10} per second squared for a solar mass and 1.029955×10^{-8} for a billion, a ratio equal to the square of the mass ratio, 10^{18}. The heavier the black hole, the larger the box at its horizon** (Section 3). **Fourth, the box height is proportional to the precision——at precisions 10^{-6}, 10^{-8}, 10^{-9} and 10^{-11} the box at the Earth's surface is 3.243790×10^{-1}, 3.243790×10^{-3}, 3.243790×10^{-4} and 3.243790×10^{-6} m; an instrument a thousand times finer shrinks the box a thousandfold** (Section 3). **Fifth, the difference accumulates during the fall——in a 1 m box the top and bottom points separate by 1.541407×10^{-6} m during one second of free fall, and if the box were as tall as the Earth's radius the difference would be twice g** (Section 3). **Sixth, the separator is whether what shrinks the box is precision or gradient——raising the precision shrinks the box, and a place of larger gradient shrinks it too. The size of the "local" in "locally" is set by the ratio of these two and comes from no physical constant** (Section 4). **Keywords**: equivalence principle, tidal force, falling box, gravity gradient, instrument precision, event horizon.

Scope of this paper (scope note): **No new theorem or law is claimed——that the equivalence principle holds only locally, the tidal gradient $2GM/r^3$, the thought experiment of the falling box (Einstein 1907), and that the tide at a black-hole horizon goes as the inverse square of the mass are all known.Newton's gradient is used——the tidal tensor of general relativity is not used; the value at the horizon is an estimate from $2GM/r^3$ with $r = 2GM/c^2$ and depends on the choice of coordinates.Instrument precisions are representative——10^{-6}, 10^{-8}, 10^{-9} and 10^{-11} m/s² are values for comparison; no performance of any particular gravimeter is claimed.The three equivalence principles are not distinguished——the weak, Einstein and strong principles belong to Paper 165; only the size of the box is seen here.Astronomical values are representative——the mass and radius of the neutron star and the radii of the Earth, Moon and Sun are representative.Relation to earlier papers**: **Paper 165 showed that there are three equivalence principles and that no precision reaches the third——here it is counted, as a ratio with the gradient, that raising the precision shrinks the box in which the principle holds.Paper 485 showed that Michell's radius agrees with the Schwarzschild radius but not for the reason given——here it is counted that the tidal gradient at that radius goes as the inverse square of the mass.Paper 1006 showed that the throat of the Einstein-Rosen bridge closes within $2\pi GM/c^3$ and that not even light crosses——here it is counted, mass by mass, whether gravity can be told inside a box the size of that same horizon.**What is added is **giving the box height as precision over gradient at six places**, **checking with two masses that the horizon gradient goes as the inverse square of the mass**, and **giving the separation accumulated during a fall**.

1. Notation and the Question

Setting: **consider a box of height h at distance r from a body of mass M. The acceleration of gravity differs between top and bottom by the difference of $GM/(r - h/2)^2$ and $GM/(r + h/2)^2$, which for h small against r is $(2GM/r^3)h$.Let the box height at which an instrument of precision p tells gravity be $h = p/(2GM/r^3)$.**

Question: **what sets the size of the "local" region inside which gravity vanishes in a falling box?**

2. Checking the Gradient

Computation 2.1: **at the Earth's surface ($M = 5.9722 \times 10^{24}$ kg, $R = 6.371 \times 10^6$ m), $2GM/R^3 = 3.082813 \times 10^{-6}$ per second squared; a finite difference of g over 1 m gives 3.082813×10^{-6}, and $2g/R$ (with $g = 9.80665$) is 3.078528×10^{-6}. Over a 1 m box the difference is 3.143595×10^{-7} of g.**

Observation 2.2

the difference is not zero, only small.However small the box, gravity differs between top and bottom ——"cannot be told" means that the instrument is coarser than that difference.

3. Boxes at Six Places

Computation 3.1: **the box heights at which precisions of 10^{-8} and 10^{-11} m/s² tell gravity were obtained at six places.**

place	gradient (per s²)	box at 10^{-8} (m)	box at 10^{-11} (m)
Earth's surface	3.082813×10^{-6}	3.243790×10^{-3}	3.243790×10^{-6}
Moon's surface	1.868748×10^{-6}	5.351177×10^{-3}	5.351177×10^{-6}
Sun's surface	7.884734×10^{-7}	1.268274×10^{-2}	1.268274×10^{-5}
neutron-star surface	2.150985×10^{8}	4.649033×10^{-17}	4.649033×10^{-20}
horizon, one solar mass	1.029955×10^{10}	9.709164×10^{-19}	9.709164×10^{-22}
horizon, 10^9 solar masses	1.029955×10^{-8}	0.9709164	9.709164×10^{-4}

At the horizon $r = 2GM/c^2$ the gradient is $c^6/(4G^2M^2)$, 1.029955×10^{10} for a solar mass, and the ratio between the two masses is 10^{18}, the square of the mass ratio. At the Earth's surface, precisions of 10^{-6}, 10^{-8}, 10^{-9} and 10^{-11} give boxes of 3.243790×10^{-1}, 3.243790×10^{-3}, 3.243790×10^{-4} and 3.243790×10^{-6} m.

Computation 3.2: **in a 1 m box the top and bottom points separate by $(2gh/R)t^2/2 = 1.541407 \times 10^{-6}$ m during 1 second of free fall. A box as tall as the Earth's radius would have a difference of 2.000000 times g.**

Observation 3.3

this is the core. What sets the size of the box is precision divided by gradient.At the same precision the box differs by 16 orders of magnitude between the Earth's surface and the horizon of a solar mass, and by 18 between the horizons of one and 10^9 solar masses——the heavier the black hole, the weaker the tide at its horizon and the larger the box. The gradient at the radius that Paper 485 counted goes as the inverse square of the mass.

4. The Separator and What Is Not Said

> **Observation 4.1**
>
> **the separator is whether what shrinks the box is precision or gradient.Raising the precision shrinks the box, and a place of larger gradient shrinks it too——the size of the "local" is set by the ratio of the two and comes from no physical constant. The three principles that Paper 165 counted differed in what precision reaches; here precision moves the box.**

> **Observation 4.2**
>
> **what is not said.The tidal tensor is not used, and the horizon value is a coordinate-dependent estimate——precisions and astronomical values are representative, and the closing time of the throat that Paper 1006 counted is not entered.**

5. Census and Closing

1. **surface gradient 3.082813×10^{-6}, 2g/R = 3.078528×10^{-6}, 1 m difference 3.143595×10^{-7} of g** (Computation 2.1): **not zero.**
2. **box at the surface 3.243790×10^{-3} m (10^{-8}) and 3.243790×10^{-6} m (10^{-11})** (Computation 3.1): **the core.**
3. **Moon 5.351177×10^{-3}, Sun 1.268274×10^{-2}, neutron star 4.649033×10^{-17}, solar-mass horizon 9.709164×10^{-19}, 10^{9} solar masses 0.9709164 m** (Computation 3.1): **20 orders of magnitude.**
4. **horizon gradients 1.029955×10^{10} and 1.029955×10^{-8}** (Computation 3.1): **inverse square of the mass.**
5. **1.541407×10^{-6} m in one second of fall** (Computation 3.2): **the difference accumulates.**
6. **precision or gradient** (Observation 4.1): **the separator; Papers 165, 485 and 1006.**

Closing: **The size of the falling box in which gravity cannot be told is set not by physics but by the precision of the instrument.The difference between top and bottom is the tidal gradient $2GM/r^3$ times the height, 3.082813×10^{-6} m/s² per metre at the Earth's surface, and the box height at which gravity is told is precision over gradient: at precision 10^{-8}, 3.243790 mm at the Earth's surface, 9.709164×10^{-19} m at the horizon of a solar-mass black hole and 0.9709164 m at the horizon of a billion solar masses——the horizon gradient goes as the inverse square of the mass, so the heavier the hole the larger the box.The separator is whether what shrinks the box is precision or gradient.Placed among the earlier papers——Paper 165 showed that there are three equivalence principles, Paper 485 why Michell's**

radius was right, and Paper 1006 when the throat of the bridge closes. Here it was counted that the size of the "local" is set by the ratio of precision to gradient.To be honest——no box has been measured with a real instrument.

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