There Are Three "Impossible Colours", and the Reasons They Are Impossible Are Different

— reddish green is impossible because the coordinates of colour space are two opponent axes, transparent white because scattering and transmission sum to at most 1, and luminous grey because lightness is a ratio to a reference white; of the three opposite pairs of the colour octahedron, only white and black have colours in between — the separator is whether the reason is coordinates, conservation or reference — [Paper 1030]

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— From thinking to syncing. —

Abstract

In Remarks on Colour Wittgenstein wrote that there is no reddish green, no transparent white and no luminous grey, and called these matters of "grammar" rather than of experience.This paper counts the three in the terms of this corpus and shows that the reasons fall into three, coordinates, conservation and reference.No new theorem or law is claimed, and the debate over grammar versus experience is not entered. First, and this is the core. The three impossible colours have different roots——reddish green is impossible because the coordinates of colour space are two opponent axes (geometry), transparent white because scattering and transmission sum to at most 1 (conservation), and luminous grey because lightness is a ratio to a reference white (a relative quantity) (Sections 2 to 4). **Second, the red-green axis takes only one value——in the same cone model as Paper 832, the red-green axis crosses zero at 552.500000 nm and the blue-yellow axis at 495.582593 nm, once each; among 301 monochromatic lights (148 reddish, 153 greenish) and 5673 two-light mixtures, the axis is positive and negative at once in 0 cases. Of the 6 vertices and 15 pairs of the colour octahedron, 12 are edges (adjacent, with a name for the mixture) and 3 are opposite pairs (white-black, red-green, yellow-blue), of which only white-black has colours in between (the greys)** (Section 2). **Third, transparent white lies outside R + T ≤ 1——in the Kubelka-Munk formulas, without absorption the reflectance R and transmittance T sum to 1 (1.000000000000 at SX = 0.1, 1, 9 and 99), and with absorption to less than 1 (0.916421 at K = 0.1, SX = 9). "Transparent white" has both R and T equal to 1, a sum of 2, outside the reachable region R + T ≤ 1. Whiteness R = 0.9 takes SX = 9, at which T = 0.1** (Section 3). **Fourth, luminous grey lacks a reference white——the CIELAB lightness L* is fixed by Y/Yn with Yn a reference white, and a light that is its own reference has Y/Yn = 1 and L* = 100 (white). A grey of L* = 50 takes Y/Yn = 0.184187 (a surround 5.429279 times brighter), L* = 75 takes 0.482781 (2.071332 times). A light with no brighter white around it cannot be grey** (Section 4). **Fifth, within the octahedron only white-black is different——of the three opposite pairs, the middle of the red-green and yellow-blue axes is the value 0 (achromatic, not a mixed colour), while the middle of the white-black axis is the series of greys as Y/Yn runs from 0 to 1. In one figure, two axes are opponent and one is lightness** (Sections 2 and 4). **Sixth, the separator is whether the reason is coordinates, conservation or reference——what Wittgenstein called "grammar" in one word splits into three in the terms of this corpus. None of the three is refuted by experience, but the roots are not the same** (Section 5). **Keywords**: opponent colours, colour octahedron, Kubelka-Munk formulas, lightness, Wittgenstein, colour space.

Scope of this paper (scope note): **No new theorem or law is claimed——opponent-colour theory (Hering) and its psychophysics (Hurvich and Jameson 1957), the Kubelka-Munk formulas (1931), the definition of CIELAB lightness L*, and Wittgenstein's Remarks on Colour (1977) and the colour octahedron of the Philosophical Remarks are all known.The philosophical debate is not entered——whether these are matters of grammar or of experience, and how Wittgenstein is to be read, is not judged; the three "impossibles" are only counted in the corpus's quantities.The cones are a model——Gaussians at 440, 540 and 565 nm with width 40 nm are the same representative values as in Paper 832, not real colour-matching functions; the weights of the opponent axes are representative too.Real colour-vision data are not compared——measured wavelengths of unique hues and experimental values of adaptation and contrast are not cited.The octahedron is only counted by its vertices——the details of the figure in the Philosophical Remarks are not entered; the pairs among the 6 vertices white, black, red, green, yellow and blue are counted.Brown is not counted separately——that there is no brown light has the same root as luminous grey (a reference white), so it is included in the third.Relation to earlier papers**: **the scope note of Paper 832 said that appearance is not discussed and that colour names, language, culture, adaptation and contrast are not the subject; this paper sits in that seat and counts the impossibility of luminous grey by the presence or absence of the reference white against which contrast is taken** (no earlier occupant).**Paper 323 showed that the "three" in three primaries is a property of the receiver, not of light——here it is counted that when those three receptors are recombined into two opponent axes, opposite colours cannot mix.Paper 442 showed that white is not a colour but scattering made by a difference of refractive index——here it is counted that this scattering shares a sum of 1 with transmission, so that whiteness and transparency are the two ends of one knob.**What is added is **splitting the three impossible colours into the three roots of coordinates, conservation and reference**, **counting the zeros of the opponent axes and the cases of positive-and-negative in a model**, **counting that only one of the three opposite pairs of the octahedron has a middle**, and **placing R + T ≤ 1 and the reference of L* in numbers**.

1. Notation and the Question

Setting: **the three impossible colours are seen through three quantities.(a) The red-green axis $L - M$ and the blue-yellow axis $S - (L + M)/2$ (L, M, S the cone responses). (b) The reflectance R and transmittance T of a layer by the Kubelka-Munk formulas (scattering S, absorption K, thickness X). (c) The CIELAB lightness $L^* = 116 f(Y/Y_n) - 16$ (Y_n the reference white).**

Question: **are the reasons that reddish green, transparent white and luminous grey are impossible the same, or different?**

2. Reddish Green: Coordinates

Computation 2.1: with the same cone model as Paper 832 (Gaussians peaking at 440, 540 and 565 nm, width 40 nm), the two axes were computed for 301 monochromatic lights from 400 to 700 nm. The red-green axis crosses zero at 552.500000 nm and the blue-yellow axis at 495.582593 nm, once each. The red-green axis is positive (reddish) for 148 lights and negative (greenish) for 153, and positive and negative at once for 0. Among 5673 additive mixtures of two lights (pairs at 5 nm steps with weights 0.25, 0.5 and 0.75) the count is again 0. The hue angles of the monochromatic lights run from 90.187939° to 328.581259°, leaving a purple gap of 121.606681°.

Computation 2.2: placing the vertices of the colour octahedron at white $(0, 0, 1)$, black $(0, 0, -1)$, red $(1, 0, 0)$, green $(-1, 0, 0)$, yellow $(0, 1, 0)$ and blue $(0, -1, 0)$, there are 15 pairs, 12 edges at distance $\sqrt{2}$, and 3 opposite pairs at distance 2 (white-black, red-green, yellow-blue).

opposite pair	middle of the axis	colour in between
white and black	lightness Y/Y_n from 0 to 1	the greys (exist)
red and green	opponent axis at 0	achromatic (no mixed colour)
yellow and blue	opponent axis at 0	achromatic (no mixed colour)

> **Observation 2.3**
>
> the red-green axis takes only one value."Reddish green" would be an axis value both positive and negative, and a point has one value——recombining the three receptors of Paper 323 into two opponent axes, opposite colours cannot mix and only adjacent ones (the 12 edges) can be called "-ish". Of the three axes of the octahedron two are opponent and one (white-black) is lightness, so only white-black has a middle.

3. Transparent White: Conservation

Computation 3.1: for a non-absorbing layer ($K = 0$) the Kubelka-Munk formulas give $R = SX/(1 + SX)$ and $T = 1/(1 + SX)$, with $R + T = 1.000000000000$ at $SX = 0.1$, 1, 9 and 99. The general formulas with absorption return to these within 10^{-6} as $K \to 0$; at $K = 0.1$ and $SX = 9$, $R = 0.845772$, $T = 0.070648$ and $R + T = 0.916421$, and at $K = 1$, $R + T = 0.634494$.

SX ($K = 0$)	reflectance R	transmittance T	$R + T$
0.1	0.090909	0.909091	1.000000000000
1	0.500000	0.500000	1.000000000000
9	0.900000	0.100000	1.000000000000
99	0.990000	0.010000	1.000000000000

Observation 3.2

transparent white lies outside $R + T \leq 1$.Whiteness is R, transparency is T, and their sum cannot exceed 1. "Transparent white" has both equal to 1, a sum of 2——Paper 442 showed that white is scattering made by a difference of refractive index. Raising the scattering SX brings R to 0.9 but drops T to 0.1. Whiteness and transparency are the two ends of one knob.

4. Luminous Grey: Reference

Computation 4.1: **L^* is 100.000000 at $Y/Y_n = 1$, 76.069261 at 0.5 and 37.842430 at 0.1. The Y/Y_n needed for $L^* = 50$ is 0.184187 (a surround 5.429279 times brighter) and for $L^* = 75$ it is 0.482781 (2.071332 times); substituting back gives 50.000000 and 75.000000.**

Observation 4.2

luminous grey lacks a reference white.Grey is $Y/Y_n < 1$: grey only when a brighter white is around. A self-luminous surface has no reference brighter than itself——brown (a dark orange) has the same root, and without a brighter surround it is orange.

5. The Separator and What Is Not Said

Observation 5.1

this is the core. The three impossible colours have different roots.Coordinates (an opponent axis takes one value), conservation ($R + T \leq 1$), and reference (L^* divides by Y_n)——what Wittgenstein called "grammar" in one word splits into three in the terms of this corpus. None of the three is refuted by experience, but the roots are not the same.

Observation 5.2

the separator is whether the reason is coordinates, conservation or reference.Changing the cone model does not change the number of axes, changing the scattering formula does not change $R + T \leq 1$, and changing the lightness formula does not change the division by a reference——what does not move is in a different place each time.

Observation 5.3

what is not said.The debate over grammar and experience is not entered, the cones are representative values of a model, and no real colour-vision data are compared——the octahedron is counted by its vertices only, without the details of the figure.

6. Census and Closing

1. **three roots: coordinates, conservation, reference** (Observation 5.1): **the core.**
2. **axis zeros 552.500000 nm and 495.582593 nm, positive-and-negative 0 lights and 0 mixtures, octahedron pairs 15, edges 12, opposite 3, with a middle 1** (Computations 2.1 and 2.2): **coordinates.**
3. $R + T$ = 1.000000000000 **(no absorption),** 0.916421 **(K = 0.1)** (Computation 3.1): **conservation.**
4. Y/Y_n **for** $L^* = 50$**:** 0.184187**, surround** 5.429279 **times; for** $L^* = 75$**:** 0.482781**,** 2.071332 **times** (Computation 4.1): **reference.**
5. **only white-black has a middle** (Computation 2.2): **a different kind of axis.**
6. **coordinates, conservation or reference** (Observation 5.2): **the separator; Papers 832, 323 and 442.**

Closing: **There are three impossible colours, and the reasons they are impossible are different.Reddish green is impossible because the coordinates of colour space are two opponent axes (in the model the axes cross zero once each, at 552.500000 nm and 495.582593 nm, and positive-and-negative occurs 0 times), transparent white because scattering and transmission sum to at most 1 (R + T = 1.000000000000 without absorption), and luminous grey because lightness is a ratio to a reference white (L* = 50 takes a surround 5.429279 times brighter)——of the three opposite pairs of the colour octahedron, only white and black have colours in between.The separator is whether the reason is coordinates, conservation or reference.Placed among the earlier papers——Paper 323 showed that the three of three primaries is a property of the receiver, and Paper 442 that white is scattering made by a difference of refractive index. Here, on the side of contrast that Paper 832 did not enter, the three impossibles were split into three roots.To be honest——nothing has been said about whether Wittgenstein was right.**

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