What Numbers Can Say About "Does 777 Occur in π" Is That the Waiting Time Is Set by the Word’s Self-Overlap

— in the decimals of π, 777 first occurs at digit 1589, its expected waiting time is 1110, and that of 123 is 1000; the mean first occurrence of the 1000 three-digit words is 991.994000 against an expected mean of 1002, indistinguishable from random digits — the separator is whether the word overlaps itself or not — [Paper 1032]

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— From thinking to syncing. —

Abstract

In Philosophical Investigations §516 Wittgenstein asked whether 777 occurs in the expansion of π, and whether the question has a sense before a proof.This paper counts only the part of the question that numbers can answer: where 777 occurs in the digits of π, the expected waiting time, the distribution of first occurrences of the 1000 three-digit words, and the comparison with random digits.No new theorem or law is claimed, and π is not claimed to be normal. First, and this is the core. The waiting time is set by the word's self-overlap——in random digits the expected wait until a word first occurs is the sum of 10 to the k over the lengths k at which a prefix of the word equals a suffix (Conway's leading number): 10 + 100 + 1000 = 1110 for 777, 1000 for 123, 1010 for 121. Among words of the same length, the more a word overlaps itself the longer it makes one wait (Section 2). **Second, in π, 777 occurs at digit 1589 ——in the first 300000 decimals, 777 first occurs at digit 1589 (1.431532 times its expectation), 123 at 1924 (1.924000 times), 121 at 710, 000 at 601, 999 at 762, 314 at 2120 and 159 at digit 3** (Section 2). **Third, the first occurrences of the 1000 words follow the expected distribution——all 1000 three-digit words occur by digit 8553 (the last is 483); the mean first occurrence is 991.994000 against an expected mean of 1002.000000 (ratio 0.990014), with 365 words later than expected and 635 earlier. By type the means are 981.471111 for the 900 words expected at 1000, 1022.855556 for the 90 at 1010, and 1661.300000 for the 10 at 1110** (Section 3). **Fourth, they cannot be told from random digits——over 20 strings of random digits the mean first occurrence of all words is 998.342200 with standard deviation 24.333544, and the mean for π sits at −0.260883 of that. By type the random means are 997.218389 (type 1000), 1003.056111 (type 1010) and 1057.060000 (type 1110, standard deviation 350.670369), so the 1661.3 of π for the 1110 type lies within the scatter of a mean of only 10 words** (Section 4). **Fifth, the 1589 of 777 is an ordinary delay——in 42 of 200 random strings (0.210000) the first 777 came after digit 1589, close to the exponential approximation exp(−1589/1110) = 0.238943; among the 20 strings, 4 were later than π** (Section 4). **Sixth, the separator is whether the word overlaps itself or not——non-overlapping words (900) expect 1000, words overlapping by one digit (90) expect 1010, words overlapping by two (10) expect 1110, and in π they come later in that order. What answers the question is not π but the shape of the word** (Section 5).

Keywords: digits of pi, pattern waiting time, Conway's leading number, self-overlap, normal number, Wittgenstein.

Scope of this paper (scope note): **No new theorem or law is claimed——that the waiting time for a pattern in a random sequence is given by Conway's leading number (Guibas and Odlyzko 1981, Li 1980), that the digits of π are statistically indistinguishable from random digits, and Wittgenstein's question in §516 (1953) are all known.π is not claimed to be normal——this is open, and the 300000 digits here say only that the statistic cannot tell them from random digits; this stands on the same side as the scope note of Paper 252.The sense of the question is not entered——the philosophical question of whether the question has a sense before a proof is not entered; only the part numbers can answer is counted.The random samples are limited——20 and 200 strings; no precision of the shares is claimed.The leading-number formula is only cited——its derivation (by martingales) is not entered.Relation to earlier papers**: **Paper 252 showed that information is four quantities needing different things, and wrote that the digits of π are not claimed to be normal——here, for the same digits, what is counted is not normality but the waiting time word by word.Paper 176 showed that "small" has two senses, in measure and in category, which do not agree——here the gap between "occurs" for almost all numbers and "occurs" for the one number π is seen through the distribution of waiting times.Paper 1021 showed that continued-fraction statistics cannot tell known transcendentals from unknown ones——here it is counted by first occurrences that digit statistics cannot tell π from random digits.**What is added is **counting the first occurrences of the 1000 three-digit words in π and their means by leading-number type**, **comparing type by type with 20 strings of random digits**, and **placing the 1589 of 777 among 200 random strings**.

1. Notation and the Question

Setting: **take 300000 decimals of π and count the first occurrence of a word w (a string of digits) by the decimal digit at which it starts.The expected wait until a word first occurs in random digits (Conway's leading number) is the sum of 10^k over the lengths k at which a prefix and a suffix of the word coincide.For three-digit words, non-overlapping words (900) expect 1000, words overlapping by one digit (the form aba, 90) expect 1010, and words overlapping by two (the form aaa, 10) expect 1110.**

Question: **how much of "does 777 occur in π" can numbers answer?**

2. Named Words

Computation 2.1: **the first occurrences in the decimals of π and the expected waits are as follows.**

word	first occurrence (digit)	expected	ratio
777	1589	1110	1.431532
123	1924	1000	1.924000
121	710	1010	0.702970
000	601	1110	0.541441
999	762	1110	0.686486
314	2120	1000	2.120000
159	3	1000	0.003000

Observation 2.2

this is the core. The waiting time is set by the word's self-overlap.Because 777 overlaps itself by one and by two digits, after a near miss the next candidate starts one digit later, and the expectation stretches to 1110 ——123 does not overlap itself, so its expectation is 1000. The 1589 of π is 1.431532 times the expectation of 777, while the 1924 of 123 is 1.924000 times, so in π it was 123 rather than 777 that came "late".

3. The 1000 Words

Computation 3.1: **all 1000 three-digit words occur by digit 8553 (the last is 483). The mean first occurrence is 991.994000, the expected mean 1002.000000, ratio 0.990014; 365 words came later than expected and 635 earlier.**

type	words	expected	mean first occurrence in π	median
no overlap	900	1000	981.471111	676.0
overlap by one	90	1010	1022.855556	734.0
overlap by two	10	1110	1661.300000	1643.5

Observation 3.2

the first occurrences follow the expected distribution.The medians are below the means because the waiting-time distribution has a long right tail: a few words come late and many come early——the 1661.3 of the 10 doubly overlapping words is the mean of only 10, which the next section compares with random digits.

4. Comparison with Random Digits

Computation 4.1: **20 strings of 300000 random digits were generated and the first occurrences of the same 1000 words counted. The mean over all words is 998.342200 with a standard deviation over the 20 strings of 24.333544; the 991.994000 of π sits at -0.260883 of that. By type the random means are 997.218389 (type 1000), 1003.056111 (type 1010) and 1057.060000 (type 1110, standard deviation 350.670369). In 4 of the 20 strings 777 came later than in π.**

Computation 4.2: **in 42 of 200 random strings (0.210000) the first 777 came after digit 1589, close to the exponential approximation $\exp(-1589/1110) = 0.238943$.**

Observation 4.3

the digits of π cannot be told from random digits.Paper 1021 showed that continued-fraction statistics cannot tell known transcendentals from unknown ones——here too, digit statistics do not tell π from random digits. The 1589 of 777 is an ordinary delay that one random string in five exceeds.

5. The Separator and What Is Not Said

Observation 5.1

the separator is whether the word overlaps itself or not.Non-overlapping words expect 1000, words overlapping by one 1010, words overlapping by two 1110, and in π they come later in that order——what answers the question is not π but the shape of the word; π is one example in which that shape returns the same answer as random digits.

Observation 5.2

what is not said.π is not claimed to be normal; standing on the same side as the scope note of Paper 252, only "indistinguishable at 300000 digits" is said——in the words of Paper 176, "occurs" for almost all numbers and "occurs" for π are different, and this paper does not close that gap, nor enter the philosophy of whether the question has a sense before a proof.

6. Census and Closing

1. **expectations 1110, 1000, 1010** (Computation 2.1): **the core.**
2. **777 at 1589 (1.431532 times), 123 at 1924 (1.924000 times)** (Computation 2.1): **it was 123 that was late.**
3. **mean of 1000 words 991.994000, expected 1002.000000, ratio 0.990014, last at digit 8553** (Computation 3.1): **as the distribution says.**
4. **means by type 981.471111, 1022.855556, 1661.300000** (Computation 3.1): **later in order of overlap.**
5. **random 998.342200 ± 24.333544, π at −0.260883, type 1110 1057.060000 ± 350.670369, share later than 1589: 0.210000 (approximation 0.238943)** (Computations 4.1 and 4.2): **indistinguishable.**
6. **overlapping or not** (Observation 5.1): **the separator; Papers 252, 176 and 1021.**

Closing: **What numbers can say about "does 777 occur in π" is that the waiting time is set by the word's self-overlap.In the decimals of π, 777 first occurs at digit 1589, its expected waiting time is 1110 and that of 123 is 1000. The mean first occurrence of the 1000 three-digit words is 991.994000 against an expected mean of 1002, by type 981.471111, 1022.855556 and 1661.300000, later in order of overlap, and the position against 20 strings of random digits is −0.260883 standard deviations, indistinguishable——the 1589 of 777 is an ordinary delay that one random string in five exceeds.The separator is whether the word overlaps itself or not.Placed among the earlier papers——Paper 252 did not claim π normal, Paper 176 split "small" into measure and category, and Paper 1021 showed that continued-fraction statistics cannot tell transcendence. Here it was counted, word by word through waiting times, that digit statistics cannot tell π from random digits.To be honest——whether 777 "must" occur has not been said at all.**

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