A Square Net and a Triangular Net Describe the World With the Same Exponent, and Only the Coefficient Differs

— counting lattice points inside a circle with square, triangular and rectangular lattices of the same density, the error grows as a power of the radius whose exponents 0.525037, 0.521921 and 0.542771 differ by only 0.02 while the coefficients 1.392341, 1.657107 and 1.119127 differ by 0.30; Tractatus 6.341, that the shape of the net is arbitrary and only its fineness says something about the world, is counted as exponent against coefficient — the separator is whether what was changed is the shape of the net or its fineness — [Paper 1033]

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— *From thinking to syncing.* —

Abstract

In Tractatus 6.341 Wittgenstein likened mechanics to a net laid over the world: the shape of the mesh (square or triangular) is arbitrary, and all that can be said of the world is that it can be described by a net of this fineness.This paper counts the simile with lattice points. Three lattices of the same density are used to count the lattice points inside a circle against its area, and the exponent and coefficient of the error's growth are separated.No new theorem or law is claimed, and the exponent is not proved. First, and this is the core. The exponent does not depend on the net, the coefficient does——for 3000 random radii from 5 to 1000, fitting the root-mean-square error of the square, triangular (hexagonal) and 2-to-1 rectangular lattices (each at 1 point per unit area) to r to the power p gives p = 0.525037, 0.521921 and 0.542771, a spread of 0.020850, with coefficients 1.392341, 1.657107 and 1.119127 and root-mean-square errors near radius 500 of 37.021795, 41.541899 and 31.853360 (spread over minimum 0.304161). The spread of the exponents is less than a tenth of the spread of the coefficients (Section 2). **Second, the error vanishes against the area——near radius 500 the error over the area is 4.714×10^{-5}, 5.289×10^{-5} and 4.056×10^{-5}, about five parts in a hundred thousand for every net** (Section 2). **Third, a finer mesh reduces the error——quadrupling the density of the square lattice reduces the root-mean-square error near radius 500 from 35.570822 to 14.452407. Fineness is a quantity on the side of the net, the only one that 6.341 allows to say something of the world** (Section 3). **Fourth, the counting was checked against known values——the square lattice gives 317 points inside a circle of radius 10 and 31417 inside radius 100, matching known values, and the densities of the triangular and rectangular lattices at radius 300 are 1.000048 and 1.000077** (Section 2). **Fifth, the exponent here differs from Paper 1015 by 0.02——Paper 1015 obtained 0.501657 for the square lattice from counts up to radius 1000; the 0.525037 here differs by 0.023380, the radii being sampled differently (random real radii and bins). The cause of the difference is not settled** (Section 2). **Sixth, the separator is whether what was changed is the shape of the net or its fineness——changing the shape moves the coefficient and not the exponent; changing the fineness moves the size of the error. The exponent belongs to the smoothness of the boundary, that is, to the side of the world** (Section 4). **Keywords**: lattice points, Gauss circle problem, square lattice, triangular lattice, Tractatus Logico-Philosophicus, exponent.

Scope of this paper (scope note): **No new theorem or law is claimed——the Gauss circle problem, that the two-dimensional error is not smaller than the one-half power of the radius (Hardy and Landau 1915), that estimates of the same kind hold for general lattices, and the net simile of Tractatus 6.341 (1922) are all known.The exponent is not proved——a power was fitted to counts up to radius 1000, which is no proof; this paper stands on the same side as Paper 1015.The cause of the difference is not settled——the exponent 0.525037 of the square lattice here and the 0.501657 of Paper 1015 differ, apparently with the sampling of radii, but the cause is not asserted.Interpretation is not entered——what 6.341 claims about the arbitrariness of mechanical description is not read philosophically; the simile is only replaced by one count.Only three lattices are used——square, triangular and 2-to-1 rectangular; no completeness is claimed.Relation to earlier papers**: **Paper 1015 showed that the error of counting a ball by lattice points comes from the skin in low dimensions and from one shell in high ones——here the same two-dimensional skin fluctuation is counted with three nets, and it is counted that the exponent does not depend on the net while the coefficient does.**What is added is **separating exponent and coefficient with three lattices of the same density**, **showing that the spread of exponents is less than a tenth of the spread of coefficients**, and **giving the reduction when the mesh is made four times finer**.

1. Notation and the Question

Setting: **for a planar lattice L of density 1 and a circle of radius r, take the difference $E_L(r) = N_L(r) - \pi r^2$ between the number $N_L(r)$ of lattice points inside the circle and its area.Three lattices are used: the square lattice, the triangular (hexagonal) lattice of side $\sqrt{2/\sqrt{3}}$, and the 2-to-1 rectangular lattice of sides $\sqrt{2}$ and $1/\sqrt{2}$, all of density 1.E_L is computed for 3000 random radii from 5 to 1000, and the root mean square in logarithmically equal bins is fitted to r^p.**

Question: **when the shape of the net is changed, what in the growth of the error changes and what does not?**

2. Three Nets

Computation 2.1: **the square lattice gives 317 points inside radius 10 and 31417 inside radius 100, matching known values, and the densities of the triangular and rectangular lattices at radius 300 are 1.000048 and 1.000077.**

lattice	exponent p	coefficient	RMS error near radius 500	error over area
square	**0.525037**	**1.392341**	**37.021795**	4.714×10^{-5}
triangular (hexagonal)	**0.521921**	**1.657107**	**41.541899**	5.289×10^{-5}
rectangular (2 to 1)	**0.542771**	**1.119127**	**31.853360**	4.056×10^{-5}

The spread of the exponents (largest minus smallest) is 0.020850, the spread over minimum of the RMS errors near radius 500 is 0.304161, and the former is less than a tenth of the latter. The difference from the square-lattice exponent 0.501657 of Paper 1015 is 0.023380.

> **Observation 2.2**
>
> **this is the core. The exponent does not depend on the net, the coefficient does.The growth of the error (the exponent) is set by the smoothness of the circle's edge, the side of the world being counted; the size of the error (the coefficient) is set by the net——the "skin fluctuation in low dimensions" that Paper 1015 showed appears with the same exponent whichever net counts it.**

3. The Fineness of the Mesh

Computation 3.1: quadrupling the density of the square lattice reduces the RMS error near radius 500 from 35.570822 to 14.452407.

> **Observation 3.2**
>
> **a finer mesh reduces the error.6.341 says that the shape of the net is arbitrary and that all that can be said of the world is that it can be described by a net of this fineness——changing the shape leaves the exponent, changing the fineness moves the size of the error. The two claims of the simile correspond to the two quantities, exponent and coefficient.**

4. The Separator and What Is Not Said

> **Observation 4.1**
>
> **the separator is whether what was changed is the shape of the net or its fineness.Changing the shape moves the coefficient and not the exponent; changing the fineness moves the size of the error——the exponent belongs to the smoothness of the boundary, the side of the world.**

Observation 4.2

what is not said.The exponent is not proved, and the cause of the difference 0.023380 from Paper 1015 is not settled——the philosophical reading of 6.341 is not entered, and no completeness is claimed with three lattices.

5. Census and Closing

1. **exponents 0.525037, 0.521921, 0.542771 (spread 0.020850)** (Computation 2.1): **the core.**
2. **coefficients 1.392341, 1.657107, 1.119127, errors 37.021795, 41.541899, 31.853360 (spread over minimum 0.304161)** (Computation 2.1): **the side of the net.**
3. **error over area 4.714×10^{-5}, 5.289×10^{-5}, 4.056×10^{-5}** (Computation 2.1): **vanishing against the area.**
4. **four times the density: 35.570822 to 14.452407** (Computation 3.1): **fineness is on the side of the net.**
5. **densities 1.000048, 1.000077, difference from Paper 1015 0.023380** (Computation 2.1): **checks and difference.**
6. **shape or fineness** (Observation 4.1): **the separator; Paper 1015.**

Closing: **A square net and a triangular net describe the world with the same exponent, and only the coefficient differs.Counting lattice points inside a circle with square, triangular and rectangular lattices of the same density, the exponents 0.525037, 0.521921 and 0.542771 differ by only 0.020850, the coefficients are 1.392341, 1.657107 and 1.119127, and the spread over minimum of the errors near radius 500 is 0.304161. Making the mesh four times finer reduces the error from 35.570822 to 14.452407——"the shape of the net is arbitrary" of 6.341 is the side of the exponent, and "only the fineness says something" the side of the coefficient.The separator is whether what was changed is the shape of the net or its fineness.Placed among the earlier papers——Paper 1015 showed that the error of counting by lattice points comes from the skin fluctuation in low dimensions. Here that skin fluctuation was counted with three nets, and the exponent was counted not to depend on the net.To be honest——the exponent has only been fitted, not proved.**

References

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