A Rotor Driven at Its Tips Has No Torque Reaction, but the Price Is Paid in the Kinetic Energy the Jet Carries Away

— a shaft-driven rotor spins the body the other way and needs a tail rotor taking 0.043139 of the main rotor's power; a tip-jet rotor has zero shaft torque, but the jet's propulsive efficiency is set by the ratio of jet speed to tip speed, a jet twice as fast as the tip wastes as much power as it delivers, and the price falls to zero only when the jet is 1.086279 times the tip speed — the separator is whether the force is applied at the shaft or at the rim — [Paper 1034]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
September 2026

— *From thinking to syncing.* —

Abstract

Before philosophy Wittgenstein studied aeronautics, and in 1911 he was granted a patent for a propeller driven by combustion chambers at the blade tips. It did not work for aeroplanes, but it is the type that later became the helicopter tip jet.This paper sets a shaft-driven rotor beside a tip-driven one with representative values of momentum theory and counts what is paid in exchange for the vanishing torque reaction.No new theorem or law is claimed, and no performance of real machines is cited. First, and this is the core. The torque reaction vanishes because the point of application moves from the shaft to the rim, and the price is the kinetic energy the jet carries away——with representative values of 2000 kg, radius 5 m and angular speed 40 rad/s (tip speed 200 m/s), the induced velocity in hover is 10.095954 m/s and the induced power 198.014970 kW. Driven at the shaft, a torque of 4950.374246 N·m spins the body the other way, and a tail rotor of radius 1 m at 6 m needs a thrust of 825.062374 N and an induced power of 8.542248 kW (0.043139 of the main rotor). Driven at the tips, the jet thrust needed is 990.074849 N and the shaft torque is 0, but the propulsive efficiency of the jet is set by the tip speed V and the jet speed as 2V/(V + jet speed) (Sections 2 and 3). **Second, the price of the jet exceeds ten times the price of the tail rotor——for jet speeds of 1.25, 2, 3 and 5 times the tip speed the propulsive efficiency is 0.888889, 0.666667, 0.500000 and 0.333333, and the power wasted is 0.125000, 0.500000, 1.000000 and 2.000000 times the power delivered. At twice the tip speed the price is 11.590331 times that of the tail rotor, at three times 23.180662** (Section 3). **Third, the jet that makes the price zero is almost as slow as the tip——the propulsive efficiency at which the jet's price equals the tail rotor's is 0.958645, at a jet speed of 217.255763 m/s, only 1.086279 times the tip speed (closed form and bisection agree). Every faster jet costs more than the tail rotor** (Section 3). **Fourth, the price of the tail rotor falls with area——doubling the tail rotor's radius from 1 m to 2 m halves its induced power to 4.271124 kW, 0.021570 of the main rotor. The tail rotor pays in area, the jet in the ratio of speeds** (Section 2). **Fifth, both prices come from the same formula of momentum theory——the induced power of the tail rotor is thrust to the 3/2 divided by the square root of area, and the propulsive efficiency of the jet is Froude's 2V/(V + jet speed); both are the account of the kinetic energy left in the flow for the momentum given to it** (Section 4). **Sixth, the separator is whether the force is applied at the shaft or at the rim——applied at the shaft, a torque reaction appears and a tail rotor is needed; applied at the rim, no torque reaction appears but a jet is needed. The idea of the patent is to apply it at the rim; the ledger of the price lies outside the idea** (Section 4). **Keywords**: rotor, torque reaction, tip jet, momentum theory, propulsive efficiency, tail rotor.

Scope of this paper (scope note): **No new theorem or law is claimed——the momentum theory of rotors (actuator disk), the role of the tail rotor, the propulsive efficiency of a jet (Froude), and Wittgenstein's patent (1911) with its history (Cater and Lemco 2009) are all known.This is a model with representative values——the weight, radius, angular speed, position and radius of the tail rotor, and the air density are values for comparison, and only momentum theory with uniform inflow and no losses is used; blade shape, stall and ground effect are not included.The type of jet is not distinguished——the efficiency formula differs between an air-breathing and a rocket-like jet, but this paper keeps to one formula written in the jet speed relative to the tip.No real machine is cited——the performance of machines that used tip jets has not been verified and is not cited.The details of the patent are not entered——compression by centrifugal force and the structure of the combustion chamber are not entered; only the single point of driving at the tip is used.Relation to earlier papers**: **Paper 726 showed that the trailing vortex is set by weight and span, not thrust——here too the induced power of the rotor is set by weight and disk area, and how the thrust is produced (shaft or rim) acts only on the side of the price.Paper 486 showed that the "exactly half" appearing in three fields is the vertex of a parabola——the induced velocity of momentum theory used here comes from the same actuator-disk formula as that vertex.**What is added is **setting the price of driving at the shaft (the tail rotor's induced power) beside the price of driving at the rim (the jet's propulsive efficiency) with the same representative values, giving the jet speed at which the two prices are equal**, and **separating the two knobs, tail-rotor area and jet-speed ratio**.

1. Notation and the Question

Setting: **a body of weight W is supported by a rotor of radius R. In momentum theory the induced velocity is $v_i = \sqrt{W/(2\rho A)}$ ($A = \pi R^2$) and the induced power $P_i = W v_i$.Driven at the shaft, the shaft torque $Q = P_i/\Omega$ spins the body the other way, and a tail rotor at distance L stops it with thrust $T_t = Q/L$; its induced power is $T_t^{3/2}/\sqrt{2\rho A_t}$.Driven by jets at the tips, the shaft torque is 0, a thrust $F = Q/R$ is needed at the tips, and the propulsive efficiency $\eta = 2V/(V + v_j)$ is set by the jet speed v_j relative to the tip speed $V = \Omega R$. The power wasted is $(1-\eta)/\eta$ times the power delivered.**

Question: **when the torque reaction is removed, what is paid?**

2. Driven at the Shaft

Computation 2.1: **with $W = 19613.300000$ N, $R = 5$ m, $\Omega = 40$ rad/s and $\rho = 1.225$ kg/m³, the disk area is 78.539816 m², the induced velocity 10.095954 m/s, the induced power 198.014970 kW and the tip speed 200 m/s. The torque is 4950.374246 N·m, and a tail rotor at $L = 6$ m needs a thrust of 825.062374 N.**

tail-rotor radius	induced power (kW)	share of the main rotor
1 m	8.542248	0.043139
2 m	4.271124	0.021570

> **Observation 2.2**
>
> the price of the tail rotor falls with area.Doubling the radius halves the induced power——as Paper 726 showed, vortex and power alike are set not by thrust but by weight and area.

3. Driven at the Rim

Computation 3.1: the jet thrust needed at the tips is $Q/R = 990.074849$ N and the shaft torque is 0. The propulsive efficiency and the price for each jet speed are as follows.

jet speed	ratio to tip speed	propulsive efficiency	power wasted over power delivered
200 m/s	1	1.000000	0.000000
250 m/s	1.25	0.888889	0.125000
400 m/s	2	0.666667	0.500000
600 m/s	3	0.500000	1.000000
1000 m/s	5	0.333333	2.000000

The propulsive efficiency at which the price equals the tail rotor's share (0.043139) is 0.958645, at a jet speed of 217.255763 m/s (1.086279 times the tip speed), where bisection and the closed form agree. At twice the tip speed the price is 11.590331 times the tail rotor's, at three times 23.180662.

Observation 3.2

this is the core. The torque reaction vanishes because the point of application moves from the shaft to the rim, and the price is the kinetic energy the jet carries away.A jet only slightly faster than the tip costs little, but thrust demands a fast jet, and at twice the tip speed as much power is wasted as delivered——the tail rotor's 4 percent is removed, and 50 percent is paid.

4. The Separator and What Is Not Said

Observation 4.1

both prices are the same account.The induced power of the tail rotor and the propulsive efficiency of the jet are both the kinetic energy left in the flow for the momentum given to it, from the same actuator-disk formula whose vertex Paper 486 counted——the difference is that the tail rotor pays in area and the jet in the ratio of speeds.

Observation 4.2

the separator is whether the force is applied at the shaft or at the rim.Applied at the shaft, a torque reaction appears and a tail rotor is needed; applied at the rim, no torque reaction appears but a jet is needed——the idea of the patent is to apply it at the rim; the ledger of the price lies outside the idea.

Observation 4.3

what is not said.This is a model with representative values, without blade shape, losses or the performance of real machines——the type of jet is not distinguished, and the details of the patent's combustion chamber are not entered.

5. Census and Closing

1. **induced velocity 10.095954 m/s, induced power 198.014970 kW, torque 4950.374246 N·m, tail thrust 825.062374 N, power 8.542248 kW (0.043139)** (Computation 2.1): **the price at the shaft.**
2. **jet thrust 990.074849 N, efficiencies 0.888889, 0.666667, 0.500000, 0.333333** (Computation 3.1): **the core.**

3. **equal-price jet 217.255763 m/s (1.086279 times the tip, efficiency 0.958645)** (Computation 3.1): **almost the tip speed.**
4. **11.590331 times at twice, 23.180662 at three times** (Computation 3.1): **more than ten times the price.**
5. **tail radius 2 m: 4.271124 kW (0.021570)** (Computation 2.1): **falls with area.**
6. **shaft or rim** (Observation 4.2): **the separator; Papers 726 and 486.**

Closing: **A rotor driven at its tips has no torque reaction, but the price is paid in the kinetic energy the jet carries away.For a representative rotor, driving at the shaft needs a tail rotor taking 0.043139 of the main rotor's power; driving at the tips makes the shaft torque zero, but a jet twice as fast as the tip wastes as much power as it delivers (efficiency 0.666667) and the price is 11.590331 times the tail rotor's. The price falls to zero only when the jet is 1.086279 times the tip speed——the tail rotor pays in area, the jet in the ratio of speeds.The separator is whether the force is applied at the shaft or at the rim.Placed among the earlier papers——Paper 726 showed that the trailing vortex is set by weight and span, and Paper 486 that "exactly half" is the vertex of a parabola. Here, with the same momentum theory, the price of removing the torque reaction was counted.To be honest——not one number of a real helicopter has been used.**

References

1. Cater, J., & Lemco, I. (2009). Wittgenstein's combustion chamber. *Notes and Records of the Royal Society*.
2. Johnson, W. (1980). *Helicopter Theory*. Princeton University Press.
3. Wittgenstein, L. (1911). Improvements in propellers applicable for aerial machines (British Patent 27,087). The Patent Office.
4. Yamagishi, Y. (2026). *The "Exactly Half" in Three Fields Is the Vertex of a Parabola, Not Physics* [Paper 486]. Zenodo. 10.5281/zenodo.22759952
5. Yamagishi, Y. (2026). *The Trailing Vortex Is Set by Weight and Span, Not Thrust, and on Approach It Is No Stronger Than in Cruise* [Paper 726]. Zenodo.

— *From thinking to syncing.* —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)