Slicing a Light Sphere by Another Frame's Simultaneity Gives an Ellipse of Eccentricity β with the Emission Point at a Focus

— The radius in each direction is exactly the Doppler factor for that direction, the two vertices are k and 1/k, the semi-major axis is γ, the semi-minor axis is cT, and the centre is where the moving observer stands — the separator is whose simultaneity does the cutting — [Paper 1036]

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— From thinking to syncing. —

Abstract

Light emitted at the origin is said to spread as a sphere in every frame.This paper counts what comes out when that sphere is cut by the simultaneity of another frame.No new theorem and no new law is claimed. First, and this is the core. The light sphere is not an invariant; the light cone is——when the points cut by the simultaneity surface of a frame S' moving at speed β are read in the original frame S, the radius is not constant: at β = 0.6 it runs from 0.500000000000 to 2.000000000000. Yet for all six values of β the points lie on the same light cone, within 10 to the minus 13(Section 2). **Second, the slice is an ellipse whose eccentricity is exactly β——three independent routes agree: a least-squares conic fit, the sum of distances from the two foci, and the polar equation. At β = 0.6 the eccentricity is 0.600000000000, the semi-major axis 1.250000000000 (= γ), the semi-minor axis 1.000000000000 (= cT) and the semi-latus rectum 0.800000000000 (= cT/γ), and one of the two foci is the emission point itself**(Section 2). **Third, the two vertices are the relativistic Doppler factors——at β = 0.6 the far vertex is 2.000000000000 = k and the near vertex 0.500000000000 = 1/k, and their product is the square of the semi-minor axis, 1.000000000000. The near vertex is the same number that Paper 1026 obtained as the advance of the receding image, (1−β)γ, and the geometric mean 0.577350269190 of the two classical formulas 0.500000000000 and 0.666666666667 of Paper 491 (at β = 0.5) sits here as well**(Section 3). **Fourth, the radius in each direction is exactly the Doppler factor for that direction——the radius r(θ) agrees with 1/(γ(1−β cosθ)) within 10 to the minus 12 for all six values of β. The ellipse is therefore a polar plot of the Doppler factor. Sideways the radius equals the semi-latus rectum cT/γ exactly, 0.800000000000 at β = 0.6, the direction in which only time dilation is left**(Section 4). **Fifth, the centre is where the moving observer stands, and the volume is γ times larger in every dimension——the centre 0.750000000000 (β = 0.6) coincides with the position βγcT of the observer in S at that time. Numerical integration gives a three-dimensional volume of 5.235987756, which is 1.250000000 times (= γ) the sphere's 4.188790205. Since only the long axis is stretched, the ratio stays γ from d = 2 to d = 5**(Section 5). **Sixth, the separator is whose simultaneity does the cutting——cut by the same frame's simultaneity the radius is constant and the figure is a sphere; cut by another frame's it is an ellipse. The simultaneity surface tilts even when one merely walks: the now at 2.5 million light years moves by 4.230357 days at 5 km per hour and by 248.338468875 years at the orbital speed of the Earth. What moved is the surface, not the light cone**(Section 6 and Section 7).

Keywords: special relativity, relativity of simultaneity, light cone, ellipse, Doppler factor, Lorentz transformation.

Scope of this paper (scope note): **No new theorem and no new law is claimed——the Lorentz transformation, the invariance of the light cone, the relativity of simultaneity, the relativistic Doppler effect and the fact that a plane section of a cone is a conic are all standard (Einstein 1905, Rindler 2006).No dynamics is imported——nothing sits at the focus. The polar equation has the same form as a Kepler orbit because both are ellipses written with a focus at the origin, not because a force is involved.Visual appearance is not discussed——what is treated here is a figure in coordinates cut by a simultaneity surface; the image formed by light that actually arrives belongs to Paper 901.The convention on the one-way speed of light is untouched——only a pair of inertial frames with standard synchrony is used, and the synchronisation convention of Paper 24 is not varied.Acceleration is not treated——two inertial frames only; no observer that turns around.The values are examples——the six values of β from 0.1 to 0.99 and cT＝1 are for comparison and no real apparatus is claimed.Relation to earlier papers：Paper 61 read c as the light sphere r＝ct and drew volume and the inverse-square law from it——here it is counted that this sphere ceases to be a sphere under another frame's simultaneity while the light cone does not change.Paper 118 showed that coordinate time dt has no domain of its own and becomes a clock only once attached to a congruence of world lines——here that choice of label is shown, in a single figure, to make the same light either a sphere or an ellipse.Paper 491 showed that the classical Doppler effect is two formulas and that relativity is their geometric mean——here that geometric mean is the near vertex of the ellipse, and the product of the vertices is the square of the semi-minor axis.Paper 1026 showed that a receding clock tower looks stopped because of light delay and that relativity rather speeds the image up by γ——here that (1−β)γ appears as the near vertex.Paper 901 showed that a moving sphere does not look contracted——that is the image made by arriving light, this is the slice by a simultaneity surface: two different quantities about the same moving sphere.**What is added is **obtaining by three independent routes that the eccentricity of the slice is exactly β and that a focus is the emission point**, **showing that the radius in each direction is exactly the Doppler factor for that direction**, **obtaining that the two vertices are k and 1/k with product equal to the square of the semi-minor axis**, **that the centre coincides with the position of the moving observer**, and **counting that the volume ratio is γ in every dimension**.

1. Notation and the Question

Setting: **light is emitted at the origin at time 0. Frame S' moves along x at speed βc relative to frame S, and the two frames are synchronised in the standard way.Below $c = 1$ and $T = 1$.The light sphere of radius cT is cut by the simultaneity surface $t' = T$ of S', and the resulting set of points is read in the coordinates of S.Write $\gamma = 1/\sqrt{1-\beta^2}$ and $k = \sqrt{(1+\beta)/(1-\beta)}$.**

Question: **what shape is that slice, and what is fixed by the cutting while what is not?**

2. The Slice Is an Ellipse of Eccentricity β

Computation 2.1: 1440 equally spaced points were taken on the simultaneity surface of S' and carried into the coordinates of S. The parameters were obtained by three independent routes: a least-squares fit of the conic $Ax^2 + Bxy + Cy^2 + Dx + Ey = 1$, the sum of the distances from the two foci, and the polar equation with a focus at the origin.

β	eccentricity	semi-major	semi-minor	centre	semi-latus rectum
0.1	0.100000000000	1.005037815259	1.000000000000	0.100503781526	0.994987437107
0.3	0.300000000000	1.048284836722	1.000000000000	0.314485451017	0.953939201417
0.5	0.500000000000	1.154700538379	1.000000000000	0.577350269190	0.866025403784
0.6	0.600000000000	1.250000000000	1.000000000000	0.750000000000	0.800000000000
0.9	0.900000000000	2.294157338706	1.000000000000	2.064741604835	0.435889894354
0.99	0.990000000000	7.088812050083	1.000000000000	7.017923929583	0.141067359797

For every β the fitted eccentricity agrees with β within 10^{-9}, the semi-major axis is γ, the semi-minor axis is 1 and the semi-latus rectum is $1/\gamma$.The sum of the distances from the two foci equals 2γ at every point (2.500000000000 at $\beta = 0.6$), and the deviation from the polar equation $r = (1/\gamma)/(1 - \beta\cos\theta)$ is below 10^{-13}.The cross term B and the linear term E are below 10^{-14} for every β, so the major axis lies along x.

> **Observation 2.2**
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> one focus is the emission point itself.Subtracting the product of the semi-major axis and the eccentricity from the centre gives 0 within 10^{-8} for every β——the point where the light was emitted sits at a focus.

> **Observation 2.3**
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> what is invariant is the light cone.Points cut by the simultaneity of S' and points cut by that of S both satisfy $x^2 + y^2 - t^2 = 0$ within 10^{-13}——two slices of the same cone. As a control, stretching the ellipse by a factor 1.0001 produced a deviation above 10^{-4}, which was detected.Read in S, the times carried by the points are not constant either: at $\beta = 0.6$ they run from 0.500000000000 to 2.000000000000——on the light cone $r = ct$, so the spread of radii and the spread of times are the same numbers.

3. The Two Vertices Are the Relativistic Doppler Factors

Computation 3.1: the far and near vertices were obtained.

β	far vertex	k	near vertex	product
0.1	1.105541596785	1.105541596785	0.904534033733	1.000000000000
0.5	1.732050807569	1.732050807569	0.577350269190	1.000000000000
0.6	2.000000000000	2.000000000000	0.500000000000	1.000000000000
0.9	4.358898943541	4.358898943541	0.229415733871	1.000000000000
0.99	14.106735979666	14.106735979666	0.070888120501	1.000000000000

Observation 3.2

the vertices are k and $1/k$, and the product is the square of the semi-minor axis.In an ellipse the product of the distances from a focus to the two vertices equals the square of the semi-minor axis, which here is cT, so the product is 1——the trivial identity $k \cdot (1/k) = 1$ appears on the geometric side as the property that the product is the square of the semi-minor axis.

Observation 3.3

the near vertex is the same number as the advance $(1-\beta)\gamma$ of the receding image obtained in Paper 1026.At $\beta = 0.5$ the two classical formulas placed side by side in Paper 491 are 0.500000000000 (observer moving) and 0.666666666667 (source moving), with geometric mean 0.577350269190——which is the near vertex. The three formulas line up as points on one ellipse.

4. The Radius Is Exactly the Doppler Factor of That Direction

Computation 4.1: for each direction θ seen from the emission point (measured in S), the radius $r(\theta)$ was compared with the Doppler factor $D(\theta) = 1/(\gamma(1-\beta\cos\theta))$. The deviation was 4.441×10^{-16} at $\beta = 0.1$, 8.882×10^{-16} at 0.6, 9.770×10^{-15} at 0.9 and 2.220×10^{-13} at 0.99.As a control, dropping γ from D raised the deviation above 0.1.

> **Observation 4.2**
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> this ellipse is a polar plot of the Doppler factor.Forward, D is large and the radius is stretched; backward it shrinks——the shape called an ellipse and the quantity called the Doppler factor are two readings of one and the same function.

Computation 4.3: the sideways point ($\cos\theta = 0$) is the point with $\cos\theta' = -\beta$ in S', where $x = 0$ and $y = 1/\gamma$. The radius is 0.866025403784 at $\beta = 0.5$, 0.800000000000 at 0.6 and 0.435889894354 at 0.9, each agreeing with the semi-latus rectum $1/\gamma$ within 10^{-15}.

> **Observation 4.4**
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> sideways, only time dilation is left.The semi-latus rectum is the radius perpendicular to the major axis at the focus, and its value is cT/γ——the same $1/\gamma$ that Paper 1026 obtained as the advance of the image at closest approach appears here as the semi-latus rectum.

5. The Centre Is the Observer's Position and the Volume Is γ Times in Every Dimension

Computation 5.1: at time $t = \gamma T$ in S the origin of S' (the moving observer) stands at $x = \beta\gamma T$. The centre of the ellipse agrees with this value within 10^{-9} for every β (0.750000000000 at $\beta = 0.6$, 2.064741604835 at 0.9).Carried back into the coordinates of S', the points have distance 1.000000000000 everywhere and time 1.000000000000 everywhere.

> **Observation 5.2**
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> for the moving observer it is still a sphere.That observer stands at the centre of the ellipse and, measuring with their own simultaneity, finds cT in every direction——it looks like an ellipse only because the same slice is read with another frame's scale.

Computation 5.3: integrating the ellipsoid of revolution in slabs gives a volume of 4.836798305 at $\beta = 0.5$, 5.235987756 at 0.6 and 9.609743789 at 0.9. Against the sphere's 4.188790205 the ratios are 1.154700538, 1.250000000 and 2.294157339, which are γ.Stretching the unit balls of $d = 2$ to 5 (volumes 3.141592654, 4.188790205, 4.934802201, 5.263789014) along the long axis alone by γ gives 3.926990817, 5.235987756, 6.168502751 and 6.579736267, a ratio of 1.250000000 in every dimension at $\beta = 0.6$.

Observation 5.4

only one axis is stretched, so the ratio does not see the dimension.The size of the slice is multiplied by γ by the choice of cut——the volume of a light sphere is not fixed until one decides whose simultaneity measures it.

6. Tilting the Simultaneity Surface Moves the Distant Now

Computation 6.1: **read in S, the simultaneity surface of S' is $t = T/\gamma + \beta x$, so at a distance d the time shifts by vd/c^2. The values below take $d = 2.5$ million light years.**

speed	β	shift of the now (years)	in days
walking 5 km/h	$4.632834655530 \times 10^{-9}$	0.011582087	4.230357
car 100 km/h	$9.265669311060 \times 10^{-8}$	0.231641733	84.607143
orbit 29.78 km/s	$9.933538755001 \times 10^{-5}$	248.338468875	90705.625757

The ratio of the exact Lorentz result to the approximation vd/c^2 is γ at every speed, and even at the orbital speed $\gamma - 1$ is only 4.933760×10^{-9}.

Observation 6.2

what moved is the surface, not the light cone.The now of someone who starts walking shifts by 4.230357 days at 2.5 million light years, yet the light arriving from there does not move by one second——the arrival is a single event on the light cone, and it is that same event in every frame.The sphere turning into an ellipse in Section 2 is a consequence of this same tilt.

7. The Separator and What Is Not Said

> **Observation 7.1**
>
> **the separator is whose simultaneity does the cutting.Cut by the same frame's simultaneity the radius is 1.000000000000 everywhere and the figure is a sphere; cut by another frame's it is an ellipse of eccentricity β——reversing the roles and reading the simultaneity surface of S in the coordinates of S' produces the same ellipse, with vertices again k and $1/k$.In the vocabulary of Paper 118, what decides between sphere and ellipse is the choice of the label called coordinate time, not any property on the side of the light.**

> **Observation 7.2**
>
> **what is not said.It is not claimed that anything sits at the focus——the polar equation has the form of a Kepler orbit because both are ellipses with a focus at the origin, not because of a force.Visual appearance is not discussed either——the image formed by arriving light belongs to Paper 901, while the figure here is a slice by a simultaneity surface.The convention on the one-way speed of light (Paper 24) is not varied——only frames with standard synchrony were used.**

8. Census and Closing

1. **eccentricity 0.600000000000, semi-major 1.250000000000, semi-minor 1.000000000000** (Computation 2.1): **the slice is an ellipse.**
2. **a focus is the emission point** (Observation 2.2): **where the light was emitted.**
3. **vertices 2.000000000000 and 0.500000000000, product 1.000000000000** (Computation 3.1): **k and $1/k$.**
4. **radius $= 1/(\gamma(1 - \beta\cos\theta))$** (Computation 4.1): **a plot of the Doppler factor.**
5. **semi-latus rectum 0.800000000000** (Computation 4.3): **sideways, time dilation alone.**
6. **centre 0.750000000000 = the observer's position** (Computation 5.1): **a sphere for the one who moves.**
7. **volume 5.235987756 / 4.188790205 = 1.250000000** (Computation 5.3): **γ in every dimension.**
8. **4.230357 days at 2.5 million light years** (Computation 6.1): **walking moves the now.**
9. **whose simultaneity** (Observation 7.1): **the separator; Paper 24, Paper 61, Paper 118, Paper 491, Paper 901 and Paper 1026.**

Closing: **Slicing a light sphere by another frame's simultaneity gives an ellipse of eccentricity β with the emission point at a focus.At β = 0.6 the semi-major axis is 1.250000000000, the semi-minor axis 1.000000000000, the semi-latus rectum 0.800000000000 and the centre 0.750000000000; the two vertices 2.000000000000 and 0.500000000000 are the relativistic Doppler factors k and 1/k, and their product is the square of the semi-minor axis, 1.000000000000. The radius in each direction is exactly the Doppler factor of that direction, the three-dimensional volume is 1.250000000 times the sphere, and this ratio is γ in every dimension——the centre is where the moving observer stands, and that observer, measuring with their own simultaneity, finds cT in every direction.The separator is whose simultaneity does the cutting.Placed among the earlier papers——Paper 61 read c as the light sphere r = ct, Paper 491 gave the two classical formulas and their geometric mean, Paper 1026 the advance (1−β)γ of the receding image, and Paper 118 that coordinate time is a label. Here those three numbers are set on one ellipse as its vertices and semi-latus rectum, and it is counted that the choice of label is what decides between sphere and ellipse.To be honest——not one formula is new; what is new is only putting three numbers on a single slice of one light cone.**

References

1. Einstein, A. (1905). Zur Elektrodynamik bewegter Körper. *Annalen der Physik*.
2. Rindler, W. (2006). *Relativity: Special, General, and Cosmological*. Oxford University Press.
3. Yamagishi, Y. (2026). *The One-Way Speed of Light Cannot Be Measured* [Paper 24]. Zenodo. 10.5281/zenodo.22090261
4. Yamagishi, Y. (2026). ***c*** *as the Light Sphere* [Paper 61]. Zenodo. 10.5281/zenodo.21598127
5. Yamagishi, Y. (2026). *Two Clocks and One Scale* [Paper 118]. Zenodo. 10.5281/zenodo.22036237
6. Yamagishi, Y. (2026). *The Classical Doppler Effect Is Two Formulas, and Relativity Is Their Geometric Mean* [Paper 491]. Zenodo. 10.5281/zenodo.22760067
7. Yamagishi, Y. (2026). *A Moving Sphere Does Not Look Contracted: Light Delay and Contraction Cancel as a Sum of Squares* [Paper 901]. Zenodo. 10.5281/zenodo.22798673
8. Yamagishi, Y. (2026). *A Receding Clock Tower Looks Stopped Because of Light Delay, and Relativity Rather Speeds the Image Up by γ* [Paper 1026]. Zenodo. 10.5281/zenodo.22860181

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