The Time Difference for Light Going Once Round a Rotating Loop Is Set by the Enclosed Area Alone, Not by the Perimeter

— A square, an equilateral triangle, a rectangle with 2.125 times the perimeter and a regular 64-gon of the same area all give $4A\Omega/c^2$, and a figure of eight gives zero because its signed area is zero — the separator is first order or second — [Paper 1037]

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— From thinking to syncing. —

Abstract

Send light both ways round a rotating loop and the two beams return at different times.This paper solves, edge by edge, the motion of mirrors that move while the light is on its way, and counts what fixes that difference.No new theorem and no new law is claimed. First, and this is the core. Only the enclosed area matters; the perimeter does not——for a square, an equilateral triangle, a long thin rectangle with 2.125 times the perimeter and a regular 64-gon, all of area 1, the time difference divided by 4AΩ is 0.9999999999998333, 1.00000000000077, 0.9999999999986615 and 1.000000000000317. The perimeter spans a factor 2.39684243451, from 3.54633240701 to 8.5, and the difference does not move(Section 3). **Second, the centre of rotation may be anywhere ——the same square displaced to (3,2) still gives 1.000000000025833, and quadrupling the area exactly quadruples the difference**(Section 3). **Third, what acts is the signed area——for a figure of eight made of two loops of opposite sense, the absolute areas sum to 4 while the signed area is 0, and the time difference is exactly 0. Tilting the loop leaves the area of its projection onto the plane perpendicular to the rotation axis, and a loop stood on edge gives no difference at all**(Section 4 and Section 5). **Fourth, half of the difference is the failure of clock synchronisation to close around the loop——it is $2A\Omega/c^2$, and synchronising clocks all the way round the equator of the Earth leaves a mismatch of 207.386107 ns at the starting point. The east-west difference for light is twice that, 414.772213 ns, and at latitude 35.68 degrees the projected area is 0.659810449 times smaller, giving 136.835520 ns**(Section 6 and Section 7). **Fifth, the separator is first order or second——"area alone" is a statement at first order in Ω. Raising Ω a hundredfold to expose the second-order term, the departure is negative, -1.66682×10^{-5}, for the square centred on the axis and positive, 0.00258821, for the displaced one: even the sign differs. Both fall by a factor 100 when Ω is divided by 10 (ratios 100.0094043 and 100.1870602), confirming that the departure is second order**(Section 8). **Sixth, that area is what counts means that winding helps——winding 1000 m of optical fibre into loops 0.10 m across gives 3183.098862 turns and a total area of exactly 25 m², one quarter of length times diameter. The rotation of the Earth alone then gives a time difference of 8.113572 times 10 to the minus 20 seconds and, at a wavelength of 1.55 µm, a phase difference of 9.860092 times 10 to the minus 5 radians**(Section 7). **Keywords**: Sagnac effect, rotating frame, simultaneity, optical interference, enclosed area, fibre-optic gyroscope.

Scope of this paper (scope note): **No new theorem and no new law is claimed——the Sagnac effect, the proportionality of the time difference to the enclosed area, the failure of synchronisation to close in a rotating frame, and the use of this effect in fibre-optic gyroscopes are all standard (Sagnac 1913, Post 1967, Ashby 2003).The Ehrenfest paradox is not resolved——whether a disk can be spun up as a rigid body is not entered; the rotating loop is simply watched from the static frame.No formula for the second-order coefficient is derived——the second-order quantity is measured and the fact that its coefficient differs in sign between figures is recorded; why it takes that sign is not claimed.Gravity is not included——only the rotating coordinates of flat spacetime are used, and the gravitational rate shift of clocks (the side of Paper 66) is left out.No instrument performance is claimed——the fibre example is there to display the accounting of area; noise, resolution and thermal effects are not treated.The values are examples——the angular velocity 10 to the minus 6 with c＝1, the figures of unit area and the representative values for the Earth are for comparison.Relation to earlier papers：The scope note of Paper 1027 stated that whether a disk can be spun up rigidly, and how simultaneity around the circumference is to be fixed, are not its subjects——this paper takes the seat left by the second of these and counts that going once round leaves synchronisation short by $2A\Omega/c^2$(no earlier occupant).Paper 24 showed that the one-way speed of light cannot be measured and that the round trip is c under any synchronisation convention——what is measured here is the difference in arrival times of two beams at the same point, a quantity that needs no one-way speed.Paper 33 showed that the Wigner angle is an area, Paper 903 that the angle a Foucault pendulum turns in a day is a solid angle, and Paper 906 that carrying a polarisation round a closed path shifts its phase by half the enclosed solid angle——the time difference here has the same shape, what is left after a circuit is what was enclosed, but here the enclosure is a plane area and not a solid angle, so no common root is claimed.Paper 402 showed that the fourth satellite is for the clock and not for the position——the 207.386107 ns obtained here for the equator is the same quantity that appears when such clocks are set across the globe.**What is added is **tabulating the ratio while shape, perimeter and centre are varied**, **showing with a figure of eight that it is the signed area that acts**, **counting that tilting leaves the projected area**, and **measuring that the second-order coefficient differs in sign between figures**.

1. Notation and the Question

Setting: **mirrors are placed at the vertices of a closed polygon and the whole figure is rotated at angular velocity Ω .Light leaving one vertex for the next arrives after the mirror has moved, so the crossing time δt satisfies $|R(\Omega(t+\delta t))P_{k+1} - R(\Omega t)P_k| = c\delta t$.Solving this edge by edge gives the time for a full circuit, and Δt is the difference between the two senses.Below $c = 1$ and $\Omega = 10^{-6}$, with the arithmetic carried to 40 digits (in double precision the second-order effect and the rounding are mixed together).A is the signed area, by the shoelace formula.**

Question: **what fixes Δt: the shape, the perimeter, or the enclosed area?**

2. Self-Checks

Computation 2.1: with no rotation ($\Omega = 0$) the difference between the two senses is 0.

Computation 2.2: for a circle the static frame allows a closed solution: the light runs $2\pi r$ while the mirror escapes by $r\Omega t$, so $t_{\pm} = 2\pi r/(c \mp r\Omega)$ and the difference is $4A\Omega/(c^2 - v^2)$. At radius 0.5 and $\Omega = 0.01$ the exact value is 0.031416711953696774804 against $4A\Omega = 0.031415926535897932385$, a ratio of 1.0000250006250156254 that matches $1/(1 - v^2)$ within 10^{-25}.As a control, raising Ω to 0.2 gives a ratio of 0.993087007497, which departs from $4A\Omega$.

3. Shape, Perimeter and Centre Do Not Act

Computation 3.1: four figures were built with projected area 1 and rotated at $\Omega = 10^{-6}$.

figure	area	perimeter	difference $\div\, 4A\Omega$
square	1	4.0	0.9999999999998333
equilateral triangle	1	4.55901411391	1.00000000000077
long rectangle 4×0.25	1	8.5	0.9999999999986615
regular 64-gon	1	3.54633240701	1.000000000000317
square displaced to $(3, 2)$	1	4.0	1.000000000025833
square of area 4	4	8.0	0.9999999999993333

> **Observation 3.2**
>
> the perimeter spans a factor 2.4, from 3.54633240701 to 8.5, and the difference does not move.The long rectangle has 2.125 times the perimeter of the square, yet its ratio departs from 1 by only 10^{-11}——had the effect gone with the perimeter, these two would have differed by a factor 2.125.

Observation 3.3

the centre of rotation may lie outside the figure.Displacing the same square to $(3, 2)$ raises the rim speed to 4.3011626×10^{-6}, and the ratio is still 1.000000000025833——the residue of 2.6×10^{-11} is the second-order term, measured in Section 8.

4. It Is the Signed Area That Acts

Computation 4.1: a figure of eight was rotated, with vertices $(0, 0)$, $(1, 1)$, $(2, 0)$, $(1, -1)$, $(0, 0)$, $(-1, 1)$, $(-2, 0)$, $(-1, -1)$. The absolute areas sum to 4 and the perimeter is 11.313708499, but the signed area is 0 and so is the time difference.The same two loops with matching sense would give $4A\Omega = 1.6 \times 10^{-5}$.

Observation 4.2

to enclose is a signed act.What the right loop gains the left loop returns, and the beams come home together——it is the signed area, not the absolute area, that acts.

5. Tilting Leaves the Projected Area

Computation 5.1: the square of area 1 was tilted with respect to the rotation axis.

tilt	projected area	time difference
0 degrees	1	$3.9999999999993 \times 10^{-6}$
30 degrees	0.866025403784	$3.4641016151372 \times 10^{-6}$
60 degrees	0.5	$1.9999999999997 \times 10^{-6}$
90 degrees	0	0

Observation 5.2

what acts is the shadow cast on the plane perpendicular to the rotation axis.A loop stood on edge gives no difference——its own area is still 1, so what vanished was not the area but its orientation.

6. On a Rotating Loop, Synchronisation Does Not Close

Computation 6.1: for the square of area 1 the circuit difference is $3.999999999999333 \times 10^{-6}$ and its half $1.999999999999667 \times 10^{-6}$, matching $2A\Omega$ within 10^{-9}; the regular 64-gon gives the same.

> **Observation 6.2**
>
> half the difference is the failure to close.Setting neighbouring clocks on the rotating loop by the round trip of light and walking once around leaves the starting clock short by $2A\Omega/c^2$——every single step was set correctly and the circuit still does not close. That is why the scope note of Paper 1027 set aside the question of how simultaneity around the circumference is to be fixed.

7. The Earth, and a Wound Fibre

Computation 7.1: with the angular velocity of the Earth 7.292115×10^{-5} rad/s and the equatorial radius 6378137 m, the equatorial loop encloses 1.278020×10^{14} m², so $2\Omega A/c^2$ is 207.386107 ns and the east-west difference for light is twice that, 414.772213 ns. At latitude 35.68 degrees the projected area is 0.659810449 times smaller (the square of the cosine of the latitude), giving 136.835520 ns.

Computation 7.2: winding 1000 m of fibre into loops 0.10 m across gives 3183.098862 turns and a total area of 25.000000 m², exactly one quarter of length times diameter. The rotation of the Earth alone then gives a time difference of 8.113572×10^{-20} s and, at 1.55 μm, a phase difference of 9.860092×10^{-5} radians.

> **Observation 7.3**
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> if area is what counts, then winding helps.For a fixed length of fibre the total area is one quarter of length times diameter, so it falls only in proportion to the diameter——which is the trade between making the instrument small and winding it long.

8. The Separator: First Order or Second

Computation 8.1: Ω was raised to see how far the ratio departs from 1.

Ω	ratio, square	departure	ratio, displaced	departure
0.01	0.99998333175011	-1.66682×10^{-5}	1.0025882145481	0.00258821
0.003	0.99999849998718	-1.50001×10^{-6}	1.0002325394804	0.000232539
0.001	0.99999983333318	-1.66667×10^{-7}	1.0000258338207	2.58338×10^{-5}

Dividing Ω by 10 divides the departure by 100 (ratios 100.0094043 and 100.1870602), so the departure is second order.

> **Observation 8.2**
>
> at second order the area no longer decides.The square centred on the axis departs downward and the displaced one upward: the sign itself differs——for a circle the form $1/(c^2 - v^2)$ makes it positive. At second order, the distance from the centre of rotation to the loop comes back into play alongside the enclosed area.Why the coefficient takes that sign in each figure is not claimed——only what was measured is written.

> **Observation 8.3**
>
> the separator is first order or second.The statement "only the area acts" is true at first order in Ω——at first order shape, perimeter and centre all drop out; at second order the distance from the centre returns.

9. What Is Not Said

> **Observation 9.1**
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> the Ehrenfest paradox is not resolved.Whether a disk can be spun up rigidly is not entered; the rotating loop is watched from the static frame——the spatial curvature of the rotating frame belongs to Paper 1027.Gravity is not included either——every figure for the Earth is flat-spacetime rotation, with no gravitational rate shift.No instrument performance is claimed——the fibre example displays the accounting of area.

10. Census and Closing

1. **ratios 0.9999999999998333, 1.00000000000077, 0.9999999999986615, 1.000000000000317** (Computation 3.1): **shape does not act.**
2. **2.125 times the perimeter, same difference** (Observation 3.2): **nor does the perimeter.**
3. **displaced, still 1.000000000025833** (Computation 3.1): **nor the centre.**
4. **figure of eight gives 0** (Computation 4.1): **the signed area acts.**
5. **tilt 90 degrees gives 0** (Computation 5.1): **the projected area acts.**
6. **half the difference is $2A\Omega$** (Computation 6.1): **synchronisation does not close.**
7. **207.386107 ns, 414.772213 ns, 136.835520 ns** (Computation 7.1): **the Earth.**
8. **departures -1.66682×10^{-5} and 0.00258821, ratios 100.0094043 and 100.1870602** (Computation 8.1): **second order, opposite signs.**
9. **first order or second** (Observation 8.3): **the separator; Paper 24, Paper 33, Paper 402, Paper 903, Paper 906 and Paper 1027.**

Closing: **The time difference for light going once round a rotating loop is set by the enclosed area alone, not by the perimeter.For a square, an equilateral triangle, a rectangle with 2.125 times the perimeter and a regular 64-gon, all of area 1, the difference divided by 4AΩ is 0.9999999999998333, 1.00000000000077, 0.9999999999986615 and 1.000000000000317, and displacing the centre of rotation to (3,2) still leaves 1.000000000025833. What acts is the signed area: a figure of eight cancels to zero, and tilting the loop leaves the area of its projection on the plane perpendicular to the rotation axis——half the difference, 2AΩ/c², is what remains when clocks are set in turn around the rotating loop, and for the equator of the Earth that is 207.386107 ns.The separator is first order or second——at second order the sign of the departure differs between figures and the distance from the centre returns.Placed among the earlier papers——the scope note of Paper 1027 set aside simultaneity around the circumference. This paper takes that seat and writes in numbers why it was set aside: a circuit does not close. Paper 33, Paper 903 and Paper 906 have the same shape, what is left after a circuit is what was enclosed, but all three enclose a solid angle while this one encloses a plane area.To be honest——the formula dates from 1913; what is added here is a measurement of what does not act, across shape, perimeter, centre, sense and tilt.**

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