The Geoid Is the Surface on Which Clocks Run at the Same Rate

— A clock on the equator loses 103.976943 ns a day to its speed and gains exactly as much from its distance from the centre. They cancel because the Earth has taken the shape its own rotation calls for; with a spherical attraction the gravitational term would be 1.943779 times the kinematic one — the separator is same potential surface or same geocentric distance — [Paper 1038]

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— *From thinking to syncing.* —

Abstract

A clock on the equator moves faster than one at the pole.It is also farther from the centre.This paper counts how exactly these two cancel on the sea surface of a rotating Earth.No new theorem and no new law is claimed. First, and this is the core. On the sea surface, clocks run at the same rate everywhere——the equatorial clock loses 103.976943 ns a day to its speed and gains exactly as much from its distance from the centre, leaving −0.000000608 ns a day. Against the kinematic term that residue is 5.843942 times 10 to the minus 9(Section 2). **Second, this is not merely a statement about the equator and the pole——across seven points from geocentric latitude 0 to 90 degrees the combined attraction-plus-centrifugal potential spans 0.001164 m²/s². In clock rate that is 1.294748 times 10 to the minus 20, or 0.000001119 ns a day. The mean of the seven differs from the normal potential published for WGS84 by 0.000158 m²/s², which is 0.000016 m of height**(Section 3). **Third, they cancel because the Earth has taken the shape its own rotation calls for——dropping the flattening terms and using a bare GM/r raises the gravitational term to 1.943779 times the kinematic one, and widens the spread across the seven points by a factor 87721933.546989. That the sea surface is an equipotential and that clocks agree on it are one and the same fact**(Section 2 and Section 3). **Fourth, that shape is fixed by the ratio of centrifugal to gravitational pull ——the ratio m is 3.461391899 times 10 to the minus 3, and Clairaut's first-order relation f＝1.5·J2＋0.5·m returns 0.003354641 against the actual flattening 0.003352811, a ratio of 1.000545816. Dropping the centrifugal term leaves a ratio of 0.484353, which does not fit at all**(Section 4). **Fifth, a clock is therefore a ruler for height——one metre of height is 1.091137 times 10 to the minus 16 in rate, or 0.009427423 ns a day; 450 m gives 4.242341 ns a day and the height of Everest 83.421950 ns. Read the other way, a clock good to 10 to the minus 18 resolves 0.009165 m of height**(Section 5). **Sixth, the separator is whether the two points share a potential surface or a geocentric distance——put the equator and geocentric latitude 45 degrees at the same distance from the centre and the clocks differ by 100.881579 ns a day. The cancellation holds only between points on the sea surface; take "same height" in the other sense and those hundred nanoseconds come back**(Section 6). **Keywords**: geoid, equipotential surface, time dilation, gravitational redshift, flattening of the Earth, relativistic geodesy.

Scope of this paper (scope note): **No new theorem and no new law is claimed——that the geoid is a surface of constant combined potential, that atomic clocks agree on it, that the normal ellipsoid is an equipotential of the normal gravity field, and that optical lattice clocks can measure height differences are all standard (Ashby 2003, Chou and others 2010, Takamoto and others 2020).The undulation of the geoid is not treated——only the WGS84 normal ellipsoid is used; the tens of metres by which the real sea surface departs from it, together with tides, atmosphere and currents, are left out.Geodetic practice is not treated——how a height datum is fixed, how surveys are made and how clocks are transported are not entered.No reading of the 1905 paper is claimed——that it contains a sentence saying a clock at the equator must run slightly slow, and that adding the gravitational term cancels this, are set side by side; nothing is said about who knew what.No instrument performance is claimed——10 to the minus 18 is a rate for comparison, not the precision of any particular clock.The values are representative——the WGS84 GM, equatorial radius, flattening, angular velocity, J2, J4 and J6, together with the standard gravity 9.80665 m/s², are used.Relation to earlier papers：The scope note of Paper 915 stated that only the ellipsoid is used and that the undulation of the geoid, gravity, atmospheric refraction and survey methods are left out——this paper takes the seat left by gravity and counts that the ellipsoid is an equipotential and that clocks agree on it (no earlier occupant). The undulation is left out here as well, so on that point this paper stands with Paper 787 and Paper 915.Paper 787 showed that the summit of Chimborazo is lower than Everest yet farther from the centre of the Earth——there the same equatorial bulge is read as distance from the centre; here it is read as clock rate. The same 21384.685755 m acts on two different quantities.Paper 66 showed from the gravitational redshift that local c and coordinate c are distinct roots——the rate 1＋U/c² used here is the coordinate-side quantity.Paper 123 assembled the experiments that support time dilation——the nanoseconds per day obtained here recount the same quantity from the side of the shape of the Earth.Paper 402 showed that the fourth satellite is for the clock and not for the position——this paper is the ground-side account of setting those clocks.Paper 1029 showed that the size of the falling box in which gravity cannot be told is set by the precision of the instrument——here too, what decides whether a height difference can be told is the precision of the clock.Paper 1037 showed that the time difference for light round a rotating loop is set by the enclosed area alone——the same angular velocity pairs with an area there and with the square of a radius here.**What is added is **measuring the spread across seven latitudes to show the cancellation is not merely equator against pole, a control with the flattening terms removed, counting how far the cancellation depends on the shape, tying shape to rotation through Clairaut's first-order relation**, and **making the return of those hundred nanoseconds at equal geocentric distance the separator**.

1. Notation and the Question

Setting: **the WGS84 ellipsoid (equatorial radius $a = 6378137$ m, inverse flattening 298.257223563, polar radius $b = 6356752.314245$ m) is taken as the sea surface of the Earth.The attraction potential is $V = -(GM/r)[1 - \sum J_{2n}(a/r)^{2n} P_{2n}(\sin\psi)]$, the centrifugal potential is $-\frac{1}{2}(\omega r\cos\psi)^2$, and their sum is written U (ψ is geocentric latitude).The rate of a clock at rest is $1 + U/c^2$, with U negative.The values used are**

$GM = 3.986004418 \times 10^{14}$ m³/s², $\omega = 7.292115 \times 10^{-5}$ rad/s, $J_2 = 1.0826298213 \times 10^{-3}$, $J_4 = -2.370912220 \times 10^{-6}$ and $J_6 = 6.083464980 \times 10^{-9}$.

Question: does a clock at the equator run slow because it moves faster, or fast because it is farther from the centre?

2. Equator and Pole: the Two Terms Cancel

Computation 2.1: the geocentric distance is 6378137.000000 m at the equator and 6356752.314245 m at the pole, a bulge of 21384.685755 m.The kinematic term $-(\omega r)^2/(2c^2)$ is $-1.203436844 \times 10^{-12}$ (-103.976943 ns a day), the gravitational term is $+1.203436837 \times 10^{-12}$ ($+103.976943$ ns a day), and the sum is $-7.032815 \times 10^{-21}$, or -0.000000608 ns a day. Against the kinematic term that is 5.843942×10^{-9}.

Computation 2.2: dropping J_2 and beyond, so that the attraction is a bare GM/r, raises the gravitational term to $2.339214834 \times 10^{-12}$, which is 1.943779 times the kinematic one.

> **Observation 2.3**
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> the cancellation is no accident, but it does not happen for a sphere.With the same rotation and the same bulge, replacing the attraction by that of a sphere puts the two terms nearly a factor two apart——what cancels is not the rotation itself but the shape the Earth has taken to match it.

3. The Same at Every Latitude

Computation 3.1: U was computed at seven points on the ellipsoid.

geocentric latitude	distance (m)	U (m²/s²)	U without J_2 and beyond
0 degrees	6378137.000000	−62636851.714349	−62602966.660954
15 degrees	6376697.748281	−62636851.714731	−62609781.191567
30 degrees	6372770.601137	−62636851.714617	−62628416.123101
45 degrees	6367417.724967	−62636851.714274	−62653912.614302
60 degrees	6362078.314791	−62636851.714881	−62679455.786932
75 degrees	6358178.099804	−62636851.714526	−62698184.082625
90 degrees	6356752.314245	−62636851.713717	−62705045.295969

The spread of U is 0.001164 m²/s² (in clock rate 1.294748×10^{-20}, or 0.000001119 ns a day); without J_2 and beyond it is 102078.635015 m²/s² (98.131218 ns a day), a factor 87721933.546989 wider.The mean of the seven, −62636851.714442 m²/s², differs from the normal potential −62636851.7146 m²/s² published for WGS84 by 0.000158 m²/s², which is 0.000016 m of height.

> **Observation 3.2**
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> the sea surface is the surface on which clocks agree.The geodetic definition, an equipotential, and the clock-side definition, a surface where rates agree, name one and the same surface——not two points but every point on it.

4. Shape and Rotation Are Tied at First Order

Computation 4.1: the ratio of centrifugal to gravitational pull, $m = \omega^2 a^3 / GM$, is $3.461391899 \times 10^{-3}$. Clairaut's first-order relation $f = 1.5 J_2 + 0.5 m$ returns 0.003354641 against the actual flattening 0.003352811, a ratio of 1.000545816 and a fractional difference of 5.458156×10^{-4}.As a control, dropping the centrifugal term leaves $1.5 J_2$ alone, whose ratio to the actual flattening is 0.484353.

Observation 4.2

about half the bulge answers to the kinematic term.Converting the equatorial kinematic term into height gives 11029.200551 m, which is 0.515752 of the bulge 21384.685755 m——the clock-side view of the weights $0.5m$ and $1.5J_2$ standing side by side in the relation that fixes the shape.

5. A Clock Is a Ruler for Height

Computation 5.1: one metre of height is $g/c^2 = 1.091137 \times 10^{-16}$ in rate, or 0.009427423 ns a day.

height	rate	per day
1 cm	1.091137×10^{-18}	0.000094 ns
1 m	1.091137×10^{-16}	0.009427 ns
450 m	4.910116×10^{-14}	4.242341 ns
8848.86 m	9.655318×10^{-13}	83.421950 ns

Read the other way, a fractional discrepancy of 1 corresponds to a height $c^2/g = 9.164752 \times 10^{15}$ m, so a clock good to 10^{-18} resolves 0.009165 m.

Observation 5.2

a height datum can be fixed with clocks.If the sea surface is where rates agree, height above it can be read from the difference in rate——and, as Paper 1029 showed, how finely it can be told is set by the precision of the instrument, not by the physics.

6. The Separator: Same Potential Surface, or Same Geocentric Distance

Computation 6.1: placing the equatorial point and one at geocentric latitude 45 degrees at the same distance 6378137.000000 m from the centre gives a difference in U of 104939.631171 m²/s², in rate 1.167611×10^{-12}, or 100.881579 ns a day.Between the equatorial sea surface and a point 1000 m directly above it the difference is 9778.781679 m²/s², or 9.400633 ns a day.

Observation 6.2

mistake what "the same height" means and the hundred nanoseconds return.The cancellation holds only when both points are on the sea surface——under the other reading, equal distance from the centre, the equator and latitude 45 degrees differ by 100.881579 ns a day.That is the separator of this paper.

7. A Note, and What Is Not Said

Observation 7.1

a note: per year.Taking the kinematic term alone, the equatorial clock would lose -37.977579 μs a year; with the gravitational term included the sum is -0.000000222 μs——Einstein's paper of 1905 contains a sentence saying that a clock at the equator must run slightly slow. With the gravitational term, that lag does not survive.Nothing is said about who knew what.

Observation 7.2

what is not said.The undulation of the geoid is not treated——only the normal ellipsoid is used, with no departure of the real sea surface, no tides and no currents (the side of Paper 787 and Paper 915).Geodetic practice is not entered either——how a height datum is fixed and how clocks are carried are left out.No instrument performance is claimed.

8. Census and Closing

1. **kinematic -103.976943 ns, gravitational 103.976943 ns, sum -0.000000608 ns** (Computation 2.1): **they cancel.**
2. **a sphere gives 1.943779 times** (Computation 2.2): **the shape acts.**
3. **spread over seven points 0.001164 m²/s²** (Computation 3.1): **agreement over the surface.**
4. **without J_2, wider by 87721933.546989** (Computation 3.1): **the control.**
5. **mean against WGS84, 0.000158 m²/s²** (Computation 3.1): **0.000016 m of height.**
6. **Clairaut ratio 1.000545816** (Computation 4.1): **shape and rotation.**
7. **1 m gives 1.091137 times 10 to the minus 16; 450 m gives 4.242341 ns** (Computation 5.1): **a clock as ruler.**

8. **equal geocentric distance gives 100.881579 ns a day** (Computation 6.1): **the separator.**
9. **Paper 66, Paper 123, Paper 402, Paper 787, Paper 915, Paper 1029 and Paper 1037** (Section 7): **relation to earlier papers.**

Closing: **The geoid is the surface on which clocks run at the same rate.A clock on the equator loses 103.976943 ns a day to its speed and gains exactly as much from its distance from the centre, leaving −0.000000608 ns. Across seven points from geocentric latitude 0 to 90 degrees the combined potential spans 0.001164 m²/s², or 0.000001119 ns a day, and its mean differs from the normal potential of WGS84 by 0.000158 m²/s²——they cancel because the Earth has taken the shape its own rotation calls for: replace the attraction with that of a sphere and the gravitational term becomes 1.943779 times the kinematic one, while the spread widens by a factor 87721933.546989.That shape is tied to the rotation by Clairaut's first-order relation, whose ratio to the actual flattening is 1.000545816.The separator is whether the two points share a potential surface or a geocentric distance——in the latter reading the equator and latitude 45 degrees differ by 100.881579 ns a day.Placed among the earlier papers——the scope note of Paper 915 set gravity aside; this paper takes that seat. Paper 787 read the same 21384.685755 m of bulge as distance from the centre, and here it is read as clock rate.To be honest——the formula is the geodetic definition itself. What is added is to say it in the language of clocks and to count, with a control, how far the cancellation depends on the shape.**

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