A Matter-Free Expanding Universe Is Flat Spacetime Resliced by Proper Time

— The slice is a hyperboloid of curvature $-1/(c\tau)^2$, and the position of a galaxy is its rapidity. The redshift is $\exp(\chi)$, the luminosity distance $z+z^2/2$, and the product of Hubble constant and age is exactly 1 — the separator is cutting by proper time or by coordinate time — [Paper 1039]

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— *From thinking to syncing.* —

Abstract

One and the same spacetime looks like a different universe once it is cut a different way.This paper counts what comes out when flat Minkowski spacetime is cut by the surface on which proper time from the origin is equal.No new theorem and no new law is claimed. First, and this is the core. A slice of flat spacetime is an expanding universe of negative curvature——the surface of equal proper time is a hyperboloid, and distance along it equals the comoving coordinate χ. The circumference is 1.175201193644 times 2π times the distance at χ = 1 and 1.813430203924 at χ = 2; the volume is 1.220145305885 and 2.183429737231 times the Euclidean ball. Extracting the curvature from the excess circumference converges to 1.000000050766 as χ shrinks, so the curvature is $-1/(c\tau)^2$(Section 3). **Second, there is no centre——the hyperboloid is unmoved by a Lorentz transformation, so bringing any particle to the origin gives the same picture (boosting at 0.6 leaves a residue of 10 to the minus 15). A flat surface of constant t does move under a boost**(Section 2). **Third, Hubble's law comes out exactly ——the density is zero, yet the rate of change of proper distance equals H times proper distance for every particle, and Hτ is exactly 1 (for a dust universe it would be 0.666666666667). Far away the recession speed exceeds the speed of light (at χ = 1.5 it is 1.500000000000) while the particle's own speed reaches only 0.905148253645** (Section 4). **Fourth, the redshift is one number with three faces——the ratio of scales τ0/τe, the relativistic Doppler factor, and exp(χ). At β = 0.5 all three are 1.732050807569 and at β = 0.9 all three are 4.358898943541**(Section 5). **Fifth, the luminosity distance agrees along three routes——the cosmological expression (1+z)sinh χ, the closed form $z+z^2/2$, and the special-relativistic route (distance to the point of emission times the square of the Doppler factor) all return 9.000000000000 at β = 0.9**(Section 6). **Sixth, there is no particle horizon, and the space is the same shape as velocity space——conformal time keeps growing as one goes back, reaching 69.077552789821 at proper time 10 to the minus 30, whereas in a dust universe it stops at 3.000000000000. The curvature −1 of the τ = 1 surface is the same number as the $-1/c^2$ of velocity space in Paper 60 and Paper 66, and composing collinear velocities is a translation on that surface**(Section 7 and Section 8). **Keywords**: Milne universe, hyperbolic geometry, comoving coordinate, rapidity, redshift, cosmological horizon.

Scope of this paper (scope note): **No new theorem and no new law is claimed——that the Milne universe is a coordinate transformation of flat spacetime, that its space has negative curvature, that the product of Hubble constant and age is 1, that the luminosity distance is z+z²/2, and that velocity space is hyperbolic are all standard (Milne 1935, Rindler 2006).No claim is made about the real universe——this is a model of zero density and is not compared with the observed expansion history; neither matter, radiation nor a cosmological constant is included.No solution of general relativity is constructed——only a coordinate transformation of flat spacetime is used; the field equations are not solved.The cosmological constant is not treated——this paper stands with Paper 907, Paper 1001 and Paper 1013.Reading the curvature term as an energy is not entered——that was the work of Paper 907; here only the geometric side is examined.The values are examples——c = 1, four values of β from 0.1 to 0.99, four values of χ from 0.25 to 2, and the slice τ = 1.Relation to earlier papers： The scope note of Paper 907 stated that neither the cosmological constant nor the geometry of spatial curvature is treated, the curvature term being read only as an energy——this paper takes the seat left by the geometry of spatial curvature. The seat left by the cosmological constant is occupied by Paper 1001; the geometric one has no earlier occupant.Paper 60 showed that c is the radius of the ball of velocity space and Paper 66 that local c and coordinate c are distinct roots——here a hyperboloid of that same curvature appears as the space of a universe.Paper 118 showed that coordinate time is a label with no domain of its own, becoming a clock only once attached to a congruence of world lines——here changing the congruence alone turns the same spacetime from a flat space into an expanding one.Paper 1036 showed that cutting a light sphere by another frame's simultaneity gives an ellipse——there a light cone was cut by a flat surface, here spacetime is cut by a hyperboloid: both are of the type in which the cut decides the shape.Paper 779 showed that there are four distances in an expanding universe——the luminosity distance obtained here is one of those four, in closed form for zero density.Paper 1027 showed that the −3Ω²/c² of a rotating disk and the −1/c² of velocity space differ in their argument ——here is the case in which even the argument coincides.**What is added is **measuring the curvature through circumference, area and volume**, **obtaining that the redshift is also exp(χ)**, **matching the luminosity distance along three independent routes**, and **counting conformal time to show there is no horizon, against a dust universe**.

1. Notation and the Question

Setting: **from the origin of Minkowski spacetime, particles of every speed set out at once.Take $c = 1$.The event at which a particle of speed β has proper time τ is $t = \gamma\tau$, $x = \beta\gamma\tau$, satisfying $t^2 - r^2 = \tau^2$.This hyperboloid is read as the space of a single instant, with comoving coordinate $\chi = \operatorname{artanh}\beta$, the rapidity.**

Question: **what space comes out when flat spacetime is cut by proper time?**

2. The Slice Is a Hyperboloid, and It Has No Centre

Computation 2.1: **position and $t^2 - x^2$ were obtained for four particles at proper time 1.**

β	time t	position x	χ	$t^2 - x^2 - 1$
0.10	1.005037815259	0.100503781526	0.100335347731	0
0.50	1.154700538379	0.577350269190	0.549306144334	2.220×10^{-16}
0.90	2.294157338706	2.064741604835	1.472219489583	-8.882×10^{-16}
0.99	7.088812050083	7.017923929583	2.646652412362	-7.105×10^{-15}

The time equals $\cosh \chi$ and the position $\sinh \chi$, in each case within 10^{-12}.

Computation 2.2: boosting at 0.6 leaves all four particles on the same hyperboloid (largest residue 5.329×10^{-15}). As a control, a flat surface $t = 1$ moves by more than 0.1 under the same boost.

> **Observation 2.3**
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> there is no centre.Bringing any particle to the origin gives the same picture——because the hyperboloid is invariant under Lorentz transformations, and that single fact is what plays the part of the cosmological principle.

3. The Space Has Negative Curvature

Computation 3.1: on the $\tau = 1$ surface, distance from the centre, circumference, area of a sphere and volume were obtained.

χ	distance	circumference $\div 2\pi d$	area $\div 4\pi d^2$	volume $\div$ Euclidean
0.25	0.250000000000	1.010449267233	1.021007721651	1.012574663700
0.50	0.500000000000	1.042190610987	1.086161269630	1.051207161863
1.00	1.000000000000	1.175201193644	1.381097845542	1.220145305885
2.00	2.000000000000	1.813430203924	3.288529104502	2.183429737231

The distance was accumulated along the hyperboloid with the Minkowski metric and agrees with χ within 10^{-5}.

Computation 3.2: extracting the curvature from the excess circumference, $(\text{circumference} \div 2\pi d - 1) \times 6 \div \chi^2$ is 1.000500119064 at $\chi = 0.1$, 1.000005000020 at 0.01 and 1.000000050766 at 0.001.As a control, a surface of positive

curvature built with sin makes the same quantity negative.

> **Observation 3.3**
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> the curvature is $-1/(c\tau)^2$.It is -1 on the $\tau = 1$ surface, -0.250000000000 at $\tau = 2$ and -0.040000000000 at $\tau = 5$——flatter as time goes on, because τ is itself the scale factor.

4. Hubble's Law Comes Out Exactly

Computation 4.1: on the surfaces $\tau = 1$, 2 and 5, the proper distance $\tau\chi$ and its rate of change were obtained. For every particle the rate equals χ, and matches H times the proper distance with $H = 1/\tau$ within 10^{-12}.$H\tau$ is 1.000000000000 on every surface (for a dust universe it would be 0.666666666667).

> **Observation 4.2**
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> it expands with zero density.Not one particle of matter is present, and still the proper distance grows exactly in proportion to H——what expands here is not matter but the way the particles are spread out.

> **Observation 4.3**
>
> the recession speed exceeds the speed of light; the particle does not.The particle at $\chi = 1.5$ has proper distance 1.500000000000 and rate of change 1.500000000000——its own speed β reaches only 0.905148253645. The two are different quantities.

5. Three Faces of the Redshift

Computation 5.1: the arrival time at the origin was obtained for light emitted at proper time 1.

β	arrival time	ratio of scales	Doppler factor	$\exp(\chi)$
0.10	1.105541596785	1.105541596785	1.105541596785	same
0.50	1.732050807569	1.732050807569	1.732050807569	same
0.90	4.358898943541	4.358898943541	4.358898943541	same
0.99	14.106735979666	14.106735979666	14.106735979666	same

The three quantities agree within 10^{-12} at every β.

> **Observation 5.2**
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> "redshift by expansion" and "Doppler effect" are here one and the same number.Neither wording changes the value——and the third face, $\exp(\chi)$, is the sign that the comoving coordinate reads both as a distance and as a speed.

6. The Luminosity Distance Agrees Along Three Routes

Computation 6.1: the cosmological expression $(1+z)\sinh\chi$, the closed form $z+z^2/2$ and the special-relativistic route were compared.

β	z	cosmological	closed form
0.10	0.105541596785	0.111111111111	0.111111111111
0.50	0.732050807569	1.000000000000	1.000000000000
0.90	3.358898943541	9.000000000000	9.000000000000
0.99	13.106735979666	99.000000000000	99.000000000000

The special-relativistic route returns the same values, within 10^{-9}.

> **Observation 6.2**
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> the same quantity can be written in the vocabulary of cosmology or of special relativity.One comes from an integral and a curvature, the other from a speed and the fall-off of brightness——they agree because both are readings of the same flat spacetime.

7. There Is No Particle Horizon

Computation 7.1: going back, the conformal time $\int d\tau/\tau = \ln \tau$ reaches 6.907755278982 at proper time 10^{-3}, 23.025850929940 at 10^{-10} and 69.077552789821 at 10^{-30}.In the dust universe used as a control the same quantity stops at 3.000000000000.

> **Observation 7.2**
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> everything is visible.Conformal time diverges logarithmically, so the range from which light can arrive keeps widening as one goes back——in contrast with the dust universe, where it stops at a finite value.

8. The Same Hyperboloid as Velocity Space

Computation 8.1: the curvature of the $\tau = 1$ surface is -1, the same number as the $-1/c^2$ of velocity space in Paper 60 and Paper 66 when $c = 1$.For the four values of β, the distance in velocity space (the rapidity) and the proper distance on the $\tau = 1$ surface agree within 10^{-15}.Composing $\beta = 0.6$ with 0.8 collinearly gives 0.945945945946, whose rapidity equals the sum 1.791759469228 of 0.693147180560 and 1.098612288668.

> **Observation 8.2**
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> the position of a galaxy is its velocity.The comoving coordinate χ is the rapidity itself, so the map of this universe and the map of velocity space are one sheet——Paper 1027 showed that the curvature of a rotating disk and that of velocity space differ in their argument; here even the argument coincides.

9. The Separator and What Is Not Said

> **Observation 9.1**
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> **the separator is cutting by proper time or by coordinate time.Cut by coordinate time t and one gets a flat, infinite, non-expanding space of zero density——cut by proper time τ and one gets a space of negative curvature that begins at a point and obeys Hubble's law. The spacetime is one.In the vocabulary of Paper 118, which face appears is decided by the choice of label.**

> **Observation 9.2**
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> **what is not said.No claim is made about the real universe——this is a model of zero density and is not compared with observation.No solution of general relativity is built either——only a coordinate transformation of flat spacetime was used.The cosmological constant is not touched——this paper stands with Paper 907, Paper 1001 and Paper 1013.**

10. Census and Closing

1. **circumference ratio $\mathbf{1.175201193644}$, volume ratio $\mathbf{1.220145305885}$** (Computation 3.1): **negative curvature.**
2. **curvature converges to $\mathbf{1.000000050766}$** (Computation 3.2): $-1/(c\tau)^2$**.**
3. **invariant under a boost** (Computation 2.2): **no centre.**
4. $H\tau = 1$ **(dust would give $\mathbf{0.666666666667}$)** (Computation 4.1): **Hubble's law.**
5. **recession $\mathbf{1.500000000000}$, particle speed $\mathbf{0.905148253645}$** (Observation 4.3): **different quantities.**
6. $\mathbf{1.732050807569}$ **as ratio, Doppler factor and** $\exp(\chi)$ (Computation 5.1): **three faces.**
7. **luminosity distance $\mathbf{9.000000000000}$ by three routes** (Computation 6.1): $z + z^2/2$**.**
8. **conformal time $\mathbf{69.077552789821}$ (dust: $\mathbf{3.000000000000}$)** (Computation 7.1): **no horizon.**
9. **proper time or coordinate time** (Observation 9.1): **the separator; Paper 60, Paper 66, Paper 118, Paper 779, Paper 907, Paper 1027 and Paper 1036.**

Closing: **A matter-free expanding universe is flat spacetime resliced by proper time.The hyperboloid that results is unmoved by a boost, so it has no centre, and distance along it equals the comoving coordinate. The circumference is 1.175201193644 times 2π times the distance at $\chi = 1$, the volume 1.220145305885 times the Euclidean ball, and the curvature extracted from the excess circumference converges to 1.000000050766. The product of Hubble constant**

and age is exactly 1, the redshift wears three faces, ratio of scales, Doppler factor and exp(χ), and the luminosity distance agrees at 9.000000000000 along three routes——conformal time keeps growing as one goes back, so there is no particle horizon.The separator is cutting by proper time or by coordinate time.Placed among the earlier papers——the scope note of Paper 907 set aside the geometry of spatial curvature; this paper takes that seat. The $-1/c^2$ that Paper 60 and Paper 66 obtained as the curvature of velocity space appears here as the curvature of the space of a universe, and the position of a galaxy is its rapidity.To be honest——not one formula is new. What is new is matching the same numbers along three routes and gathering into one paper that only the cut decides the face.

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