The Fourth Power in the Brightness of a Moving Source Is d−1 from Solid Angle Plus 1 from Energy Plus 1 from Arrival Spacing

— The solid-angle exponent is the derivative of the aberration map itself, so in d spatial dimensions the brightness exponent is d+1. Half the photons fall inside the angle where sin θ = 1/γ — the separator is what the counting is per — [Paper 1040]

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— From thinking to syncing. —

Abstract

A moving source looks brighter ahead.That brightness is said to go as the fourth power of the Doppler factor.This paper splits the four into three.No new theorem and no new law is claimed. First, and this is the core. Four is not one number but the sum of three separate countings——d−1 from the transformation of solid angle, 1 from the energy of each photon, 1 from the arrival spacing. In d spatial dimensions the brightness exponent is d+1, which is 4 when d = 3(Sections 2 to 4). **Second, the solid-angle exponent d−1 is the derivative of the aberration map itself——differentiating cosθ = (cosθ'+β)/(1+βcosθ') gives 1/D², the ratio of sines is 1/D, and together the ratio of solid angles is 1/D raised to d−1. Against numerical differentiation at five directions in d = 2, 3 and 4 the difference is below 10 to the minus 10**(Section 2). **Third, random photons give the same answer——of 400000 photons emitted isotropically at β = 0.5, the fraction landing forward (cosθ>0) is 0.666237500, 0.750195000 and 0.804135000 for d = 2, 3 and 4. These agree within 4σ with the exponent d−1 (0.666666667, 0.750000000, 0.804498891) and lie more than 10σ from the neighbouring exponents**(Section 2). **Fourth, half the photons fall inside the angle where sin θ = 1/γ——arccos β and arcsin(1/γ) are the same angle: 60.000000000 degrees at β = 0.5, 25.841932763 at 0.9 and 8.109614456 at 0.99. In every dimension half the photons lie inside it**(Section 3). **Fifth, the all-sky mean in d = 3 has a closed form——∫D⁴dΩ/4π is 1.444444444452, agreeing with γ²(1+β²/3) to 10 to the minus 11 at β = 0.5. Per unit frequency the exponent is 3, with mean 1.154700538383; per unit wavelength it is 5, with mean 1.924500897314**(Section 5). **Sixth, the separator is what the counting is per——one and the same phenomenon goes as the square per unit solid angle, the cube with energy, the fourth power with spacing, the cube per unit frequency and the fifth power per unit wavelength. The exponent belongs to the counting, not to the phenomenon**(Section 6). **Keywords**: relativistic beaming, aberration, Doppler factor, solid angle, dimension, cosmic microwave background dipole.

Scope of this paper (scope note): **No new theorem and no new law is claimed——relativistic beaming, aberration, the fourth-power dependence of brightness on the Doppler factor, the invariance of brightness divided by the cube of frequency, and the explanation of the cosmic microwave background dipole by the motion of the solar system are all standard (Rybicki and Lightman 1979, Lind and Blandford 1985).The mechanism of radiation is not treated——why anything shines is not entered; photons emitted isotropically are simply counted.No real object is claimed——the speed and temperature used for the dipole are representative values and observational accuracy is not discussed.The history of aberration is not entered——Bradley's discovery and annual aberration belong to Paper 328, Paper 524 and Paper 909.The exponent formula of Paper 116 is not rebuilt——counting the exponent of c was done there; what is counted here is the exponent of D.The values are examples——β = 0.5 mainly, with 0.9 and 0.99 alongside; the spatial dimension is 2, 3 or 4.Relation to earlier papers：The scope note of Paper 901 stated that colour and brightness are left out: the change of colour by the Doppler effect, the change of brightness by aberration and finite exposure times ——this paper takes the seat left by brightness and counts what the exponent is made of (no earlier occupant).Paper 116 counted what the exponent of c counts, Paper 902 that the 3 in the c^3 of black-body radiation counts dimensions, and Paper 905 that the 3 in Larmor's 2/3 and in its c^3 are both the space dimension——here too part of the exponent is the space dimension, but the remaining 2 comes from energy and from spacing and counts no dimension.Paper 1026 showed that the advance of the image of a receding clock is (1−β)γ——that quantity appears here as the factor 1/D for arrival spacing, carrying one unit of the exponent.Paper 1036 showed that the radius of the slice of a light sphere is the Doppler factor of that direction——the same D appears there as a length and here as an exponent of brightness.**What is added is **splitting the exponent 4 into solid angle, energy and spacing, obtaining the solid-angle exponent d−1 from the derivative of the aberration map, matching it against random photons and against the angular integral**, and **showing that the half-angle is where sin θ = 1/γ**.

1. Notation and the Question

Setting: **a source at rest emits photons isotropically, and moves at speed β relative to the observer. Take $c = 1$.Write θ' for the direction in the source frame, θ in the observer frame, and $D = \gamma(1 + \beta\cos\theta') = 1/(\gamma(1 - \beta\cos\theta))$ .Mainly $\beta = 0.5$ is used ($\gamma = 1.154700538379$).Sideways ($\cos\theta = 0$ in the observer frame) $D = 1/\gamma = 0.866025403784$, forward $k = 1.732050807569$, backward $1/k = 0.577350269190$.**

Question: **what does the 4 in the fourth power of D count?**

2. The Solid-Angle Exponent Is $d - 1$

Computation 2.1: **the aberration map $\cos\theta = (\cos\theta' + \beta)/(1 + \beta\cos\theta')$ was differentiated numerically. In $d = 3$ at $\cos\theta' = -0.3$, $d\cos\theta/d\cos\theta' = 1.038062283715$ against $1/D^2 = 1.038062283737$, $\sin\theta/\sin\theta' = 1.018853416217$ against $1/D = 1.018853416217$, and the ratio of solid angles $d\Omega/d\Omega' = 1.038062283715$ against**

$1/D^{d-1} = 1.038062283737$. At $\cos\theta' = 0.4$ it is 0.520833333351 against $1/D^2 = 0.520833333333$.Across five directions and $d = 2$, 3 and 4 the difference stays below 10^{-10}.

> **Observation 2.2**
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> the solid-angle exponent comes from geometry alone.The ratio of sines is $1/D$, the ratio of angles is $1/D$, and the solid-angle element in d dimensions is $\sin^{d-2}\theta d\theta$, which makes $1/D^{d-1}$——no randomness and no statistics: the derivative of a map fixes it.

Computation 2.3: $\int D^p d\Omega$ divided by the full solid angle equals 1 only at $p = d - 1$.

d	$p = d-2$	$p = d-1$	$p = d$	$p = d+1$
2	1.000000000000	1.000000000000	1.154700538379	1.500000000000
3	0.951426150897	1.000000000002	1.154700538383	1.444444444452
4	0.928203230276	1.000000000000	1.154700538379	1.416666666667

For $d = 2$ the entry $p = d - 2 = 0$ is the zeroth power of D and is trivially 1; that case is separated by the next computation.

Computation 2.4: 400000 photons were emitted isotropically at $\beta = 0.5$ and the fraction landing forward in the observer frame was counted.

d	range	random	exponent $d-1$	exponent d
2	$\cos\theta > 0$	0.666237500	0.666666667	0.804498891
3	$\cos\theta > 0$	0.750195000	0.750000000	0.843750000
4	$\cos\theta > 0$	0.804135000	0.804498891	0.873415002
3	$\cos\theta > 0.8$	0.250492500	0.249999480	0.343749350

The standard error runs from 0.000627359 to 0.000745859; in every case the random value lies within 4σ of the exponent $d - 1$ and more than 10σ from the neighbouring exponents.

3. The Angle That Holds Half the Photons

Computation 3.1: the fraction with $\cos\theta > \beta$ is 0.500970000 for $d = 2$, 0.498927500 for $d = 3$ and 0.501140000 for $d = 4$ (standard error 0.000790569).That angle, $\arccos\beta$, equals $\arcsin(1/\gamma)$ within 10^{-12}: 60.000000000 degrees at $\beta = 0.5$, 25.841932763 at 0.9 and 8.109614456 at 0.99.

> **Observation 3.2**
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> "the angle of $1/\gamma$" and "$\arccos\beta$" name the same angle.If $\cos\theta = \beta$ then $\sin\theta = 1/\gamma$——and this angle does not see the dimension: in $d = 2$ as in $d = 4$, half the photons lie inside it.

4. Energy, and Arrival Spacing

Computation 4.1: each photon's energy is multiplied by D, so the energy fraction is counted with exponent d. For $d = 3$ and $\cos\theta > 0$ the random value is 0.843362752 against 0.843750000 (difference 0.000387248).

Computation 4.2: the arrival spacing was obtained geometrically. An interval of proper time 1 at the source is a coordinate time γ, during which the source moves and shortens the path, so the interval arriving is $\gamma(1 - \beta\cos\theta) = 1/D$: 0.981495457622 at $\cos\theta = 0.3$, 1.154700538379 at 0 and 1.443375672974 at -0.5, each matching $1/D$ within 10^{-12}.

> **Observation 4.3**
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> the three multiply to D^{d+1}.Solid angle gives $d - 1$, energy gives 1, spacing gives 1——for $d = 3$ that is 4. Of that 4, only 2 counts the space dimension; the other 2 comes from energy and from time.

5. All-Sky Means and a Closed Form

Computation 5.1: for $d = 3$ at $\beta = 0.5$, $\int D^4 d\Omega/4\pi$ is 1.444444444452 against the closed form $\gamma^2(1 + \beta^2/3) = 1.444444444444$, a difference of 7.994×10^{-12}.Also $\int D^3 d\Omega/4\pi = 1.154700538383$ and $\int D^5 d\Omega/4\pi = 1.924500897314$.

Observation 5.2

summing over the whole sky does not return to 1.Only the exponent $d-1$ returns to 1; the exponents that carry energy or spacing exceed it——the excess comes from the source's energy and the source's clock.

6. The Separator: What the Counting Is Per

Observation 6.1

the exponent belongs to the counting, not to the phenomenon.For $d=3$: the square per unit solid angle, the cube once energy is included, the fourth power once spacing is included——the cube per unit frequency and the fifth power per unit wavelength.All of these describe one and the same phenomenon.

Computation 6.2: the microwave background dipole was estimated. With a solar-system motion of 369.82 km/s and a temperature of 2.7255 K (representative values), $\beta = 1.233586736862 \times 10^{-3}$ and $\gamma - 1 = 7.608690 \times 10^{-7}$, so the ratio of forward to backward temperature is 1.002470220705. The first-order amplitude is 3.362140651 mK and the second-order term 0.004147492 mK, their ratio being β itself.

Observation 6.3

at slow speeds the split is invisible.To first order $D \approx 1 + \beta\cos\theta$, so every exponent gives an amplitude of the same form——the split shows only from second order, and that term is smaller than the first by a factor β.

7. What Is Not Said

Observation 7.1

the mechanism of radiation is not treated.Why anything shines is not entered; isotropically emitted photons are counted——and no real object is claimed.The history of aberration is not entered either——that belongs to Paper 328, Paper 524 and Paper 909.The exponent formula of Paper 116 is not rebuilt——that counts the exponent of c; this counts the exponent of D.

8. Census and Closing

1. **solid-angle ratio $= 1/D^{d-1}$** (Computation 2.1): **from the derivative of a map.**
2. **random 0.666237500, 0.750195000, 0.804135000** (Computation 2.4): **within 4σ of $d - 1$.**
3. **half-angle 60.000000000 and 8.109614456 degrees** (Computation 3.1): **$\sin\theta = 1/\gamma$.**
4. **arrival spacing 0.981495457622, 1.443375672974** (Computation 4.2): **$1/D$ from geometry.**
5. **the three multiply to D^{d+1}** (Observation 4.3): **4 when $d = 3$.**
6. **$\int D^4 d\Omega/4\pi = 1.444444444452$** (Computation 5.1): **matches the closed form.**
7. **cube 1.154700538383, fifth power 1.924500897314** (Computation 5.1): **moves with the counting.**
8. **dipole 3.362140651 mK, second order 0.004147492 mK** (Computation 6.2): **the split is invisible at first order.**
9. **what the counting is per** (Observation 6.1): **the separator; Paper 116, Paper 901, Paper 902, Paper 905, Paper 1026 and Paper 1036.**

Closing: **The fourth power in the brightness of a moving source is d−1 from solid angle plus 1 from energy plus 1 from arrival spacing.The solid-angle exponent is the derivative of the aberration map itself, matching numerical differentiation within 10 to the minus 10 across five directions and three dimensions. Random photons agree: at β = 0.5 the forward fraction is 0.666237500, 0.750195000 and 0.804135000 for d = 2, 3 and 4, within 4σ of the exponent d−1. Half the photons fall inside arccos β, which is the angle where sin θ = 1/γ——the all-sky mean 1.444444444452 for d = 3 matches γ²(1+β²/3), and the exponent becomes 3 per unit frequency and 5 per unit wavelength.The separator is what the counting is per——the exponent belongs to the counting, not to the phenomenon.Placed among the earlier papers——the scope note of Paper 901 set brightness aside; this paper takes that seat. Paper 116, Paper 902 and Paper 905 separated which part of an exponent of c counts dimensions; here, of the 4, only 2 does, and the rest comes from energy and from time.To be honest——the formula is in a textbook of 1979. What is added is the split of the four, and the count of how much of it is the dimension.**

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