# Reduced to Order Alone, the Fraction of Related Pairs Still Gives the Dimension and the Longest Chain Gives Proper Time

— The fraction in a causal diamond depends on the dimension alone, and solving it backwards returns 2, 3 and 4 to within 0.01. It does not move by a single digit under a boost and does move under a stretch in time — the separator is order or number — [Paper 1041]

---

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
September 2026

*— From thinking to syncing. —*

## Abstract

**Strip a spacetime of length and of time, leaving only which event comes first.This paper counts what can still be read from that, and what cannot.No new theorem and no new law is claimed. First, and this is the core. Order alone gives the dimension——sprinkling 1200 points into a causal diamond and counting the fraction of pairs that are causally related gives 0.499642479844, 0.227827587805 and 0.099355852099 in 2, 3 and 4 dimensions (standard errors 0.001391357911, 0.001105522973 and 0.000783958515). These lie within 4σ of the formula values 0.500000000000, 0.228571428571 and 0.100000000000, and more than 10σ from the neighbouring dimensions**(Section 3). **Second, solving the fraction backwards returns the dimension——the values read off are 2.000953678, 3.004029042 and 4.007677109, within 0.01 of the true ones. Changing the number of points from 300 to 2400 leaves the fraction where it was (0.226927157149 at 2400)**(Section 3). **Third, the fraction does not move under a boost and does move under a stretch——Lorentz transforming the same set of points at β = 0.3 or 0.8 changes not one digit (the difference is exactly zero). Stretching by a factor two in time moves it from 0.480535043805 to 0.719702753442. What order measures is a structure that allows stretching**(Section 4). **Fourth, the longest chain is a ruler for proper time——in two dimensions with 10000 points the longest chain is 192.750000, a ratio of 1.005061642 to the finite-size prediction 191.779282, and it follows the square-root law with coefficient two. That is the same law as for the longest increasing subsequence of a random permutation, Ulam's problem**(Section 5). **Fifth, the straight path is the longest——on a path bent through the null coordinates (0.8, 0.2) the chain is 150.700000, that is 0.781841764 of the straight one, while the proper time along that path is 0.8 of the straight one, and the finite-size prediction is 0.770871072. The twin paradox comes out of counting points alone**(Section 5). **Sixth, the separator is order or number——order alone fixes no scale. Quadrupling the density of points doubles the chain in two dimensions, and in d dimensions the chain grows as the number of points to the power one over d (a factor 2.519842100 for sixteen times as many points when d = 3). Where Paper 111 wrote that what remains is the scale, the number of points goes in**(Section 6). **Keywords**: causal set, partial order, dimension, proper time, longest increasing subsequence, conformal structure.

**Scope of this paper (scope note)**: **No new theorem and no new law is claimed——that the fraction of causally related pairs depends only on the dimension, that the formula is written with gamma functions, that the longest chain grows as the number of points to the power one over d, that the coefficient is two in two dimensions, and that the longest increasing subsequence obeys the same square-root law are all known (Myrheim 1978, Bombelli and others 1987, Brightwell and Gregory 1991, Baik and others 1999).Quantum gravity is not discussed——neither the correctness of the causal set programme nor the discreteness of spacetime is claimed; only what can be read from order is counted.It is not claimed that the dimension must be an integer——the formula is continuous and the values read off are not integers, merely numbers like 2.000953678.The finite-size coefficient is not derived——the known correction is borrowed for comparison and nothing is said about why it takes that value.The two roads of Paper 111 are not rebuilt ——the differential-geometric road and the order-theoretic road were counted there.The values are examples——300 to 40000 points, dimensions 2, 3 and 4, and 60 repetitions.Relation to earlier papers：Paper 111 showed that light determines geometry only up to conformal structure, and that what remains, the scale, must not be bundled into one word——this paper counts the sequel from the side of order: the fraction is a quantity that is unmoved by a boost and moved by a stretch, and into the remaining scale goes the number of points.Paper 122 counted how much the four clocks know about the dimension they are in——here an order with no clock at all is shown to know the dimension.Paper 123 showed that what differs between the twins is not a lagging clock but the length of a worldline ——that length appears here as a number of points.Paper 234 showed that distance is six things, one of which satisfies the triangle inequality backwards——the longest chain is of that type: a bent path is shorter than a straight one.Paper 1039 showed that a matter-free expanding universe is flat spacetime resliced by proper time——there proper time is measured by a continuous geometry, here by counting points.**What is added is **reading the dimension backwards with an error bar**, **setting a boost against a stretch as a control**, **matching the chain on a bent path against the ratio of proper times**, and **showing with a change of density that the scale enters through the number**.

## 1. Notation and the Question

**Setting**: **points are sprinkled uniformly into a $d$-dimensional causal diamond, the region between the origin and one event in its future.Take $c = 1$.Between two points only the relation "which comes first" is kept; length and time are discarded.A pair is counted as related when the two are timelike separated.There are $1200$ points and $60$ repetitions.**

**Question**: **what can be read from order alone, and what cannot?**

## 2. The Formula for the Fraction

**Fact 2.1**: **for uniformly sprinkled points the fraction of related pairs depends only on the dimension, and equals $\Gamma(d+1)\Gamma(d/2)/(2\Gamma(3d/2))$ (Myrheim 1978).**

| $d$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| value | 0.500000000000 | 0.228571428571 | 0.100000000000 | 0.042624042624 | 0.017857142857 |

It is exactly $1/2$ at $d = 2$, $8/35$ at $d = 3$ and exactly $1/10$ at $d = 4$.The fraction falls as the dimension rises——because the light cone grows narrower.

## 3. Order Alone Gives the Dimension

**Computation 3.1**: points were sprinkled, the fraction counted, and compared with the formula.

| $d$ | fraction | standard error | formula | dimension read off |
|---|---|---|---|---|
| 2 | 0.499642479844 | 0.001391357911 | 0.500000000000 | 2.000953678 |
| 3 | 0.227827587805 | 0.001105522973 | 0.228571428571 | 3.004029042 |
| 4 | 0.099355852099 | 0.000783958515 | 0.100000000000 | 4.007677109 |

The departures are $-0.256958$, $-0.672841$ and $-0.821661$ standard deviations, while the neighbouring dimensions lie more than 10 standard deviations away.

**Computation 3.2**: at $d = 3$, changing the number of points to 300, 600 and 2400 gives $0.228179487179$, $0.234262659989$ and $0.226927157149$, scattered about the formula value $0.228571428571$.

> **Observation 3.3**
>
> not one length was measured, and the dimension comes out.All that was counted is how many pairs are related——and a value of $2.000953678$ is returned. The dimension sits inside the structure called order.

## 4. Unmoved by a Boost, Moved by a Stretch

**Computation 4.1**: the same set of points was Lorentz transformed. At $\beta = 0.3$ and at $0.8$ the fraction stayed at $0.484202127660$, the difference being exactly $0$.

**Computation 4.2**: as a control, stretching by a factor two in time moved the fraction from $0.480535043805$ to $0.719702753442$.

**Observation 4.3**

**what order remembers is a structure that allows stretching.A boost does not move the light cone, so which pairs are related does not change——a stretch in time opens the cone and creates relations. In the vocabulary of Paper 111, what order holds is the conformal structure.**

## 5. The Longest Chain as a Ruler for Proper Time

**Computation 5.1**: **in two dimensions the longest chain that can be traced point by point was counted.**

| points | longest chain | $2\sqrt{N}$ | finite-size prediction | ratio |
|---|---|---|---|---|
| 2500 | 93.450000 | 100.000000 | 93.475212 | 0.999730283 |
| 10000 | 192.750000 | 200.000000 | 191.779282 | 1.005061642 |
| 40000 | 389.500000 | 400.000000 | 389.642544 | 0.999634166 |

**The prediction is $2\sqrt{N} - 1.7711 N^{1/6}$ (Baik and others 1999).This $2\sqrt{N}$ is the same law that gives the length of the longest increasing subsequence of a random permutation.**

**Computation 5.2**: **the path was bent. Through the null coordinates $(0.5, 0.5)$ the proper time equals the straight one and the chain is $188.350000$ (ratio $0.977172503$, prediction $0.974820750$). Through $(0.8, 0.2)$ the proper time is $0.800000000000$ of the straight one and the chain is $150.700000$ (ratio $0.781841764$, prediction $0.770871072$).**

**Observation 5.3**

**the straight path is the longest.A bend that keeps the proper time keeps the chain, and a bend that cuts it to $0.8$ cuts the chain to about $0.78$——the twin paradox comes out with no clock, from counting points. The length of a worldline that Paper 123 named is, here, a number.**

## 6. The Separator: Order or Number

**Observation 6.1**

**order alone fixes no scale.Keeping the same order type and quadrupling the density doubles the chain in two dimensions——in $d$ dimensions it grows as the number of points to the power one over $d$: sixteen times as many points gives $4.000000000$ at $d = 2$, $2.519842100$ at $d = 3$ and $2.000000000$ at $d = 4$.**

**Observation 6.2**

**into the place Paper 111 left open, the number goes.That paper wrote that light determines geometry up to conformal structure and that what remains is the scale——seen from the side of order, what remains is one number: how many points are in there. The dimension is known to the order; the scale is known to the number.**

## 7. What Is Not Said

**Observation 7.1**

**quantum gravity is not discussed.Neither the correctness of the causal set programme nor the discreteness of spacetime is claimed——only what can be read from order was counted.Nor is it claimed that the dimension must be an integer——the formula is continuous and the values read off are not.The finite-size coefficient is not derived either——it was borrowed for comparison.**

## 8. Census and Closing

1. **fractions $0.499642479844$, $0.227827587805$, $0.099355852099$** (Computation 3.1): **within 4σ of the formula.**
2. **dimensions read off $2.000953678$, $3.004029042$, $4.007677109$** (Computation 3.1): **order knows it.**
3. **$0.226927157149$ from 300 to 2400 points** (Computation 3.2): **independent of number.**
4. **boost leaves a difference of 0** (Computation 4.1): **a conformal quantity.**
5. **stretch moves $0.480535043805$ to $0.719702753442$** (Computation 4.2): **the control.**

6. **chain $\mathbf{192.750000}$, prediction $\mathbf{191.779282}$** (Computation 5.1): **the square-root law.**
7. **bent path $\mathbf{150.700000}$, ratio $\mathbf{0.781841764}$** (Computation 5.2): **straight is longest.**
8. **sixteen times the density gives $\mathbf{2.519842100}$ at $d = 3$** (Observation 6.1): **the scale enters by number.**
9. **order or number** (Observation 6.2): **the separator; Paper 111, Paper 122, Paper 123, Paper 234 and Paper 1039.**

**Closing**: **Reduced to order alone, the fraction of related pairs still gives the dimension and the longest chain gives proper time.In a causal diamond the fraction is 0.499642479844, 0.227827587805 and 0.099355852099 in 2, 3 and 4 dimensions, within 4σ of the formula and more than 10σ from the neighbouring dimensions; read backwards it returns 2.000953678, 3.004029042 and 4.007677109. The fraction does not change by a single digit under a Lorentz transformation, and a stretch by two in time moves it from 0.480535043805 to 0.719702753442——the longest chain is 192.750000 for 10000 points, on the square-root law, and 150.700000 on a bent path, 0.781841764 of the straight one, close to that path's proper-time ratio of 0.8.The separator is order or number——the dimension is known to the order, the scale to the number.Placed among the earlier papers——Paper 111 wrote that light determines geometry up to conformal structure and that what remains is the scale, and Paper 122 counted how much a clock knows about its dimension. Here an order with no clock knows the dimension, and the length of a worldline named in Paper 123 appears as a count of points.To be honest——the formula and the law were published between 1978 and 1999. What is added is reading the dimension backwards, setting a boost against a stretch, and matching a bent path's chain against its proper time.**

## References


1. Baik, J., Deift, P., & Johansson, K. (1999). On the Distribution of the Length of the Longest Increasing Subsequence of Random Permutations. *Journal of the American Mathematical Society*.
2. Bombelli, L., Lee, J., Meyer, D., & Sorkin, R. D. (1987). Space-Time as a Causal Set. *Physical Review Letters*.
3. Brightwell, G., & Gregory, R. (1991). Structure of Random Discrete Spacetime. *Physical Review Letters*.
4. Myrheim, J. (1978). *Statistical Geometry*. CERN preprint.
5. Yamagishi, Y. (2026). *How Far Does Light Determine Geometry?* [Paper 111]. Zenodo. 10.5281/zenodo.21961685
6. Yamagishi, Y. (2026). *How Much Do the Four Clocks Know About the Dimension They Are In* [Paper 122]. Zenodo. 10.5281/zenodo.22054748
7. Yamagishi, Y. (2026). *What Differs Between the Twins Is Not a Lagging Clock but the Length of a Worldline* [Paper 123]. Zenodo. 10.5281/zenodo.22055231
8. Yamagishi, Y. (2026). *"Distance" Is Six Things, and One of Them Satisfies the Triangle Inequality Backwards* [Paper 234]. Zenodo. 10.5281/zenodo.22128226
9. Yamagishi, Y. (2026). *A Matter-Free Expanding Universe Is Flat Spacetime Resliced by Proper Time* [Paper 1039]. Zenodo. 10.5281/zenodo.22860855

*— From thinking to syncing. —*

**Yuuki YAMAGISHI (山岸 勇輝)**

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)