Light Is Gathered by a Gravitational Lens, Yet the Curvature of the Geometry It Travels In Is Negative

— The deflection is the curvature integrated over the region outside the ray, by the Gauss-Bonnet theorem: tracing the ray gives 4.011823809924e−3 and the outside integral 4.000000000000e−3. Integrating inside diverges — the separator is whether the region lies inside or outside the ray — [Paper 1044]

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— From thinking to syncing. —

Abstract

Gravity bends light and gathers it as a lens does.Yet measure the curvature of the very geometry that light travels in, and it is negative.This paper counts that discrepancy as a difference in the region of integration.No new theorem and no new law is claimed. First, and this is the core. The geometry of light is negatively curved——in a static gravitational field, light travels as in a medium of refractive index 1+2m/r (m being the mass written as a length), and in that geometry, the optical metric, the Gaussian curvature is −0.060948481913 by finite differences against −0.060948441945 from the closed form at m＝0.05 and the point (1, 0.3). It is negative everywhere(Section 2). **Second, and still the light bends inward——by the Gauss-Bonnet theorem the deflection is the curvature integrated over the region outside the ray. At m/b＝10 to the minus 3, tracing the ray gives 4.011823809924 times 10 to the minus 3 and the outside integral 4.000000000000 times 10 to the minus 3, both within one percent of 4m/b**(Section 3). **Third, the departure vanishes at first order in m——dividing m by ten divides the departure from 4m/b by 10.032726651. That ratio shows this is a statement about weak fields**(Section 3). **Fourth, for the Sun it is 1.75 arcseconds——for light grazing the limb, tracing the ray gives 1.751201387 arcseconds against 1.751190439 from 4m/b. The Newtonian estimate is exactly half, 0.875595220 arcseconds**(Section 4). **Fifth, the curvature contributes mostly nearby, but with a tail——out to five times the closest approach lies 0.705908430 of the whole, out to twenty times 0.922710482, and out to a hundred times 0.984342001**(Section 3). **Sixth, the separator is whether the region lies inside or outside the ray——outside, the integral converges without a cutoff and gives 4m/b; inside, it diverges. Lowering the cutoff radius from 0.5 to 0.05 raises the value from 2.141593 to 30.415927 times the outside one**(Section 5). **Keywords**: gravitational lensing, optical metric, Gauss-Bonnet theorem, Gaussian curvature, Fermat’s principle, bending of light.

Scope of this paper (scope note): **No new theorem and no new law is claimed——that by Fermat's principle a light path in a static field is a geodesic of the optical metric, that this metric has negative Gaussian curvature, that the deflection can be written by the Gauss-Bonnet theorem as the curvature integrated outside the ray, and that it is 4m/b in a weak field are all known (Gibbons and Werner 2008).Strong fields are not treated——the photon sphere and multiple images are not entered; only first order in a weak field.No lens equation is built——image positions, magnification and critical curves are not the subject.Time delay is not entered——the optical metric is used as a geometry of space, and the Shapiro delay is left out.The split of the 4 is not repeated——that it factors as 2 times 2 was counted in Paper 134.The values are examples——m/b from 10 to the minus 3 down to minus 5, and representative values for the Sun.Relation to earlier papers：Paper 134 showed that the 4 in the deflection factors as 2 times 2 and that the second factor is a measured quantity——here the same 4m/b is obtained twice, once by tracing the ray and once by integrating curvature outside it.Paper 485 showed that Michell's radius was right for the wrong reason and that the Newtonian deflection is exactly half the relativistic one——the 0.875595220 and 1.751190439 arcseconds here are that half and that whole.Paper 487 showed that a stationary point comes in three kinds——Fermat's principle selects such a point, and this paper looks at the geometry that path lives in.Paper 1036 showed that the radius of the slice of a light sphere is the Doppler factor——that was a way of cutting flat spacetime; this is the geometry of light in a curved one.Paper 1043 showed that the universe has three edges——both are of the type in which the region of integration decides the answer.**What is added is **obtaining the curvature by finite differences and by closed form and confirming it is negative**, **obtaining the deflection by the ray and by the curvature integral and matching them**, **showing with a ratio that the departure vanishes at first order**, **giving the contribution by distance**, and **placing the divergent inside integral as a control**.

1. Notation and the Question

Setting: **take a static gravitational field of mass M and write $m = GM/c^2$ (for the Sun, 1476.625134072 m).In a weak field light travels as in a medium of refractive index $n(r) = 1 + 2m/r$, along geodesics of that medium's geometry, the optical metric $n^2(dx^2 + dy^2)$ (Fermat's principle).Write b for the closest approach.**

Question: **if light is gathered, is that geometry positively curved?**

2. The Optical Metric Has Negative Curvature

Computation 2.1: **the Gaussian curvature was obtained by finite differences from the Laplacian of the logarithm of the index, and compared with the closed form.**

m	point (r)	finite differences	closed form
0.05	$(1, 0.3)$ (1.044031)	−0.060948481913	−0.060948441945
0.05	$(2, -1)$ (2.236068)	−0.007508319191	−0.007508325595
0.05	$(5, 0)$ (5.000000)	−0.000739081571	−0.000739076341
0.20	$(1, 0.3)$ (1.044031)	−0.096043399142	−0.096043414991
0.20	$(5, 0)$ (5.000000)	−0.002352075629	−0.002352095529

All agree to within a relative 10^{-4}, and every value is negative.As controls, a constant index gives curvature 0 and Maxwell's fish-eye index $2/(1 + r^2)$ gives 1.000000022, which is positive.

> **Observation 2.2**
>
> the geometry of light is hyperbolic.The index is larger nearer the mass, so paths passing there are slower than others——and that geometry bends in the direction that pulls neighbouring paths apart.

3. The Deflection Is Curvature Accumulated Outside

Fact 3.1: by the Gauss-Bonnet theorem, for a ray that returns asymptotically to a straight line, the deflection equals the integral $-\iint K dA$ over the region outside the ray (Gibbons and Werner 2008).

Computation 3.2: tracing the ray (integrating the Schwarzschild light path) was compared with integrating the curvature outside.

m/b	closest approach	by the ray	outside integral	$4m/b$
10^{-3}	0.998998495987	$4.011823809924 \times 10^{-3}$	$4.000000000000 \times 10^{-3}$	$4.000000000000 \times 10^{-3}$
10^{-4}	0.999899984996	$4.001178524078 \times 10^{-4}$	$4.000000000000 \times 10^{-4}$	$4.000000000000 \times 10^{-4}$
10^{-5}	0.999989999850	$4.000117813918 \times 10^{-5}$	$4.000000000000 \times 10^{-5}$	$4.000000000000 \times 10^{-5}$

The ratios are 1.002955952481, 1.000294631019 and 1.000029453480 for the ray, and 1.000000000000 throughout for the curvature integral.Dividing m by ten divides the departure from $4m/b$ by 10.032726651, so it vanishes at first order.

Observation 3.3

the curvature is negative and the deflection positive.On the ray ($r = b$) the curvature is $-1.984079681116 \times 10^{-3}$ while the deflection is $+4.011823809924 \times 10^{-3}$——a negative quantity integrated giving a positive angle is a matter of orientation and of which region is taken, and the sign in Gauss-Bonnet fixes both.

Computation 3.4: the contribution was counted by cutting off the distance.

out to how many times b	contribution	fraction of the whole
2	$1.369709426188 \times 10^{-3}$	0.342427357
5	$2.823633718693 \times 10^{-3}$	0.705908430
20	$3.690841926685 \times 10^{-3}$	0.922710482
100	$3.937368004203 \times 10^{-3}$	0.984342001

Observation 3.5

mostly nearby, but with a tail.Seven tenths lie within five times the closest approach, and even at a hundred times, 1.6 percent remains——because the curvature falls as the cube of the distance while the area grows as the square.

4. The Sun

Computation 4.1: with $m = GM/c^2 = 1476.625134072$ m and a radius of 6.957×10^{8} m, $b/m = 471141.919467$. Tracing the ray gives a deflection of $8.490063906130 \times 10^{-6}$ radians, that is 1.751201387 arcseconds, against 1.751190439 from $4m/b$, agreeing to the order of 10^{-5} arcseconds.The Newtonian estimate (coefficient 2) is 0.875595220 arcseconds, a ratio of exactly 2.000000000.

Observation 4.2

even the Sun is a weak field.With b/m near half a million, the difference between the traced ray and the first-order formula is only of order 10^{-5} arcseconds——so the value the curvature integral gives at first order is quite enough.

5. The Separator: Inside or Outside

Computation 5.1: **the same curvature was integrated over the region inside the ray, nearer the mass.**

inner cutoff radius	value	ratio to the outside value
0.5	$8.566370606359 \times 10^{-3}$	2.141593
0.2	$2.741592651590 \times 10^{-2}$	6.853982
0.05	$1.216637060636 \times 10^{-1}$	30.415927

Outside, the integral converges without a cutoff to $4.000000000000 \times 10^{-3}$, which is $4m/b$.

Observation 5.2

the separator is whether the region lies inside or outside the ray.The curvature is the same in both, yet outside it gives a finite $4m/b$ and inside it grows without bound as the cutoff falls——because the curvature diverges at the mass, and only the outside region, which excludes it, returns an answer.

6. What Is Not Said

Observation 6.1

strong fields are not treated.Neither the photon sphere nor multiple images is entered; only first order in a weak field——no lens equation is built either.Time delay is not entered——the optical metric was used as a geometry of space.Nor is the split of the 4 repeated——that was the work of Paper 134.

7. Census and Closing

1. **curvature -0.060948481913 and -0.060948441945** (Computation 2.1): **negative.**
2. **ray 4.011823809924 times 10 to the minus 3, outside 4.000000000000 times 10 to the minus 3** (Computation 3.2): **two routes agree.**
3. **departure ratio 10.032726651** (Computation 3.2): **first order.**
4. **curvature on the ray -1.984079681116 times 10 to the minus 3** (Observation 3.3): **negative, yet bending inward.**
5. **0.705908430 at five times, 0.984342001 at a hundred** (Computation 3.4): **a tail.**
6. **Sun: 1.751201387 against 1.751190439 arcseconds** (Computation 4.1): **a weak field.**
7. **Newtonian 0.875595220 arcseconds** (Computation 4.1): **exactly half.**
8. **inside: 2.141593 to 30.415927 times** (Computation 5.1): **divergent.**
9. **inside or outside** (Observation 5.2): **the separator; Paper 134, Paper 485, Paper 487, Paper 1036 and Paper 1043.**

Closing: **Light is gathered by a gravitational lens, yet the curvature of the geometry it travels in is negative.The Gaussian curvature of the optical metric of index 1+2m/r is negative at every point, by finite differences and by closed form alike (−0.060948481913 and −0.060948441945 at m = 0.05 and the point (1, 0.3)). The deflection is nonetheless positive, and comes out of integrating that curvature over the region outside the ray: at m/b = 10 to the minus 3, tracing the ray gives 4.011823809924e−3 and the outside integral 4.000000000000e−3, both equal to 4m/b, with the departure vanishing at first order in m (a factor 10.032726651 for a tenfold reduction)——at the limb of the Sun this is 1.751201387 arcseconds, exactly twice the Newtonian value.The separator is whether the region lies inside or outside the ray——outside it converges without a cutoff, inside it grows as the cutoff falls.Placed among the earlier papers——Paper 134 split the same 4 into 2 times 2, and Paper 485 showed the Newtonian value to be exactly half. Here the same 4m/b is obtained from the side of the ray and from the side of the curvature.To be honest——this reading is in a paper of 2008. What is added is counting curvature and deflection twice each, and showing the difference the region of integration makes.**

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