The Edge of a Black Hole’s Shadow Is Not the Horizon but the Image of the Circle Where Light Can Orbit

— The threshold between capture and escape lies at 2.598076211 times the horizon, the deflection grows as a logarithm, and each half turn narrows the image ring by a factor of 23.140692633, e to the power pi, so that infinitely many rings pile up — the separator is whether one names the surface of no return or the circle where light can orbit — [Paper 1046]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
September 2026

— *From thinking to syncing.* —

Abstract

The dark part of a photograph of a black hole is often called the event horizon.Yet what forms the edge, seen from far away, is not the horizon.This paper counts what that edge is.No new theorem and no new law is claimed. First, and this is the core. The edge is the image of the circle where light can orbit——the impact parameter separating capture from escape is 5.196152422707 in units of the mass expressed as a length, which is 2.598076211 times the horizon. The radius at which the numerical minimum is attained is 3.000003445, the radius at which light can travel in a circle(Section 2). **Second, the deflection grows as a logarithm near the threshold——reducing the gap to the threshold tenfold increases the deflection by a fixed amount each time. That increment rises from below towards ln 10, taking the values 2.280983015, 2.299841528, 2.302253070, 2.302546134, 2.302580628 and 2.302584710, the last differing from ln 10 (2.302585092994) by only 3.832×10 to the minus 7**(Section 4). **Third, each half turn narrows the ring by a factor of e to the power pi——every time the deflection grows by pi, the gap shrinks by a fixed ratio. That ratio falls from above towards 23.140692633, taking the values 24.633379005, 23.224766203, 23.145353940, 23.140938988 and 23.140700273. A full turn gives 535.497533146, agreeing with e to the power 2pi (535.491655525) to a relative 1.098×10 to the minus 5**(Section 5). **Fourth, so the edge is not a line but a pile of infinitely many rings——the rings with deflection from pi to 6pi occupy gaps from 3.094716916590×10 to the minus 2 down to 4.364397399037×10 to the minus 9. Inside the visible edge, images of the same circle are folded in a geometric sequence without end**(Section 5). **Fifth, far away the same integral returns to 4m over b——used at large impact parameters it approaches 4m over b, and the excess multiplied by b takes the values 3.056349394, 2.955954567 and 2.946518561, approaching $15\pi/16$ (2.945243113). At b over m equal to 1000 the value agrees with the one Paper 1044 obtained from a different form of the integral to a relative 2.080×10 to the minus 9**(Section 3). **Sixth, applied to real objects the magnitudes fit——with mass and distance entered as given numbers, the shadow diameter is 39.688144662 microarcseconds for M87* and 53.290797862 for Sagittarius A*. The latter falls within the reported ring diameter of 51.8 plus or minus 2.3 microarcseconds, whereas the horizon diameter taken at face value would be only 20.511637661 microarcseconds**(Section 6).

Keywords: photon sphere, impact parameter, black hole shadow, strong gravitational lensing, Schwarzschild spacetime, event horizon.

Scope of this paper (scope note): **No new theorem and no new law is claimed——that the photon sphere lies at 3GM/c², that the threshold impact parameter is the square root of 27 times GM/c², that the deflection diverges logarithmically near it, and that images pile up by factors of e to the power 2pi are all standard (Synge 1966; Darwin 1959; Bozza 2002).No rotating or charged holes——for Kerr and Reissner-Nordstrom the shadow is no longer a circle, but only spherical symmetry is examined here.No accretion brightness——the brightness of the ring, the Doppler boosting of one side and the emission mechanism are not the subject.No observational procedure——calibration of the interferometer and reconstruction of the image are not entered; the size of the shadow is obtained from geometry alone.No interior solution and no causal structure——Paper 1006 occupies the seat left by the scope note of Paper 485.The observed values are given numbers——mass, distance and the reported ring diameter are used as inputs and were not remeasured here.Relation to earlier papers：The scope note of Paper 1044 stated that strong fields are not treated and that the photon sphere and multiple images are not entered——this paper takes that seat and counts the photon sphere and the multiple images (no earlier occupant).Paper 1044 showed that the deflection of a gravitational lens is negative curvature accumulated outside the ray——here the same deflection is obtained from a different integral, one that divides out the root at the turning point, agreeing at b over m equal to 1000 to a relative 2.080×10 to the minus 9.Paper 485 showed that Michell's radius was right but not for the reason it gave——that radius, 2GM/c², is the horizon, and this paper counts that it is not the edge.Paper 1006 showed that the throat of the bridge opens and closes and that not even light crosses——that was the life of the interior, this is the edge seen from outside.**What is added is **the threshold impact parameter obtained both as a numerical minimum and in closed form, the logarithmic increment counted over six stages as it rises towards ln 10, the half-turn ratio counted over five stages as it falls towards e to the power pi, the width of the band in which the rings pile up, the check that the same integral reproduces the value of Paper 1044 far away, and the shadow size and the orbital time at the photon sphere for two objects.**

1. Notation and the question

Setting: **consider a spherically symmetric, non-rotating black hole (Schwarzschild spacetime), in units where the mass expressed as a length, $m = GM/c^2$, equals 1.The event horizon lies at $r = 2$.Light arriving from far away is classified by how far it would miss the centre if it went straight, the impact parameter b.The deflection was integrated numerically (200001 trapezoidal points) after dividing out the root that vanishes at the turning point.**

Question: **seen from far away, what forms the edge of the shadow?**

2. The threshold between capture and escape is $\sqrt{27}$

Computation 2.1: **the impact parameter is least at the radius where light can travel in a circle.Minimising $b(r) = r/\sqrt{1-2/r}$ numerically gives $r = \mathbf{3.000003445}$ and $b = \mathbf{5.196152422717}$.In closed form the photon sphere is $r = 3$ and the threshold is $\sqrt{27} = \mathbf{5.196152422707}$, the difference being 1.028×10^{-11}.**

Fact 2.2: light with $b < \sqrt{27}$ falls in and light with $b > \sqrt{27}$ turns back and leaves.Against a distant sky, no light arrives from the former directions, so they look dark.The edge of that dark region is $b = \sqrt{27}$.

> **Observation 2.3**
>
> the edge is the image of a circle, not a surface.The horizon has radius 2, while the edge lies at $\sqrt{27}$, a ratio of $\mathbf{2.598076211}$——what sets the edge is not the surface of no return but where light can travel in a circle. That circle, of radius 3, is what appears at $\sqrt{27}$ to a distant eye.

3. Far away, the same integral returns to $4m/b$

Computation 3.1: the deflection at large impact parameters, from the same integral.

b	Deflection	Ratio to $4/b$	Excess times b
100	$4.122253975769 \times 10^{-2}$	1.030563493942	3.056349394
1000	$4.011823818267 \times 10^{-3}$	1.002955954567	2.955954567
10000	$4.001178607425 \times 10^{-4}$	1.000294651856	2.946518561

The excess times b approaches $15\pi/16 = \mathbf{2.945243113}$, the second-order term.

Computation 3.2: the value at $b = 1000$, $\mathbf{4.011823818267} \times 10^{-3}$, agrees with the $\mathbf{4.011823809924} \times 10^{-3}$ that Paper 1044 obtained from a different form of the integral, to a relative $\mathbf{2.080} \times 10^{-9}$.

> **Observation 3.3**
>
> weak and strong field are two ends of one curve.The same integral gives $4m/b$ far away and a logarithmic divergence close in——these are not two phenomena but two ends of one curve, called by two names.

4. Near the threshold the deflection grows as a logarithm

Computation 4.1: the gap $x = b/\sqrt{27} - 1$ was reduced tenfold at a time.

x	Deflection	Approximation	Difference
10^{-2}	4.229661811851	4.204940146233	2.472×10^{-2}
10^{-3}	6.510644826616	6.507525239227	3.120×10^{-3}
10^{-4}	8.810486355018	8.810110332221	3.760×10^{-4}
10^{-5}	11.112739424920	11.112695425215	4.400×10^{-5}
10^{-6}	13.415285559010	13.415280518209	5.041×10^{-6}
10^{-7}	15.717866187336	15.717865611203	5.761×10^{-7}

The approximation is $-\ln x + \ln[216(7-4\sqrt{3})] - \pi$, with constant part -0.400230039755, and the difference shrinks by factors 7.925, 8.296, 8.546, 8.729 and 8.749 per decade.

Computation 4.2: the increment per tenfold reduction is **2.280983015**, **2.299841528**, **2.302253070**, **2.302546134**, **2.302580628** and **2.302584710**, rising from below towards $\ln 10 = \mathbf{2.302585092994}$ (final difference $\mathbf{3.832} \times 10^{-7}$).As a control, $\ln 2 = 0.693147181$ does not fit.

> **Observation 4.3**
>
> the coefficient of the logarithm is 1.That the increment is exactly $\ln 10$ means the angle follows $-\ln x$ with coefficient 1——the angle grows without bound towards the threshold, but by the slowest kind of growth there is. Any number of turns is possible, and each turn demands an exponentially closer approach.

5. Each half turn narrows the ring by a factor of e to the power π

Computation 5.1: the gap x at which the deflection is exactly $n\pi$, found by bisection.

Deflection	$x = b/\sqrt{27} - 1$	Ratio to the previous
π	$3.094716916590 \times 10^{-2}$	(the first ring)
2π	$1.256310356754 \times 10^{-3}$	24.633379005
3π	$5.409356312780 \times 10^{-5}$	23.224766203
4π	$2.337124040852 \times 10^{-6}$	23.145353940
5π	$1.009952120823 \times 10^{-7}$	23.140938988
6π	$4.364397399037 \times 10^{-9}$	23.140700273

The ratio falls from above towards e to the power π ($\mathbf{23.140692633}$), the last differing by a relative $\mathbf{3.302} \times 10^{-7}$ .A full turn, from 4π to 6π, gives $\mathbf{535.497533146}$, agreeing with e to the power 2π ($\mathbf{535.491655525}$) to a relative $\mathbf{1.098} \times 10^{-5}$.

> **Observation 5.2**
>
> the two laws are one.Since the angle follows $-\ln x$, the angle growing by π and the gap shrinking by e to the power π are the same statement read twice——the $\ln 10$ of Section 4 and the e to the power π of this section are not two findings but one slope, equal to 1, written in two bases.

> **Observation 5.3**
>
> the edge is not a line.The rings from π to 6π occupy gaps from $\mathbf{3.094716916590} \times 10^{-2}$ down to $\mathbf{4.364397399037} \times 10^{-9}$, a span of $7.090822933 \times 10^{6}$——images of the same circle are folded in a geometric sequence without end, and the visible edge is only the outermost sheet of that pile.

6. Applied to real objects

Computation 6.1: mass and distance were entered as given numbers.

Object	Mass and distance given	Shadow diameter	Time for one orbit
M87*	6.5×10^{9} solar masses, 16.8 Mpc	39.688144662	12.097925843 days
Sagittarius A*	4.3×10^{6} solar masses, 8.277 kpc	53.290797862	11.524670280 minutes

Shadow diameters are in microarcseconds.The computed 53.290797862 for Sagittarius A* falls within the reported ring diameter of 51.8 plus or minus 2.3 microarcseconds (difference 1.490797862).For the same object the horizon diameter taken at face value would be only 20.511637661 microarcseconds, 0.395977561 of the reported ring.

> **Observation 6.2**
>
> **the misreading costs a factor of 2.598076211.Reading the edge of the shadow as the horizon overstates the radius by that factor——wherever a mass is inferred from an angular size, this single factor enters.**

7. What is not claimed

> **Observation 7.1**
>
> **rotating holes are not treated.For Kerr the shadow is no longer a circle and the threshold depends on direction, but only spherical symmetry was examined——the brightness of the ring, the boosting of one side and the emission mechanism are not the subject either.Nor is the observational procedure entered——mass, distance and ring diameter were used as inputs and not remeasured here.Nor the interior and the causal structure——Paper 1006 occupies the seat left by the scope note of Paper 485.**

8. Census and closing

1. **threshold 5.196152422707, 2.598076211 times the horizon** (Computation 2.1): **the edge is an image of a circle.**
2. **numerical minimum at 3.000003445** (Computation 2.1): **the photon sphere.**
3. **increments 2.280983015 to 2.302584710** (Computation 4.2): **rising to $\ln 10$.**
4. **difference from $\ln 10$: 3.832×10^{-7}** (Computation 4.2): **coefficient 1.**
5. **ratios 24.633379005 to 23.140700273** (Computation 5.1): **falling to e to the power π.**

6. **a full turn: 535.497533146** (Computation 5.1): **e to the power 2π.**
7. **band from $3.094716916590 \times 10^{-2}$ to $4.364397399037 \times 10^{-9}$** (Observation 5.3): **folded without end.**
8. **far away 2.946518561** (Computation 3.1): **towards $15\pi/16$.**
9. **relative 2.080×10^{-9} from Paper 1044** (Computation 3.2): **a different integral agrees.**
10. **39.688144662 and 53.290797862 microarcseconds** (Computation 6.1): **the magnitudes fit.**
11. **the horizon would give 20.511637661** (Computation 6.1): **too small.**
12. **surface or circle** (Observation 2.3): **the separator; Paper 485, Paper 1006 and Paper 1044.**

Closing: **The edge of a black hole's shadow is not the horizon but the image of the circle where light can orbit.The impact parameter separating capture from escape is 5.196152422707 in units of the mass expressed as a length, which is 2.598076211 times the horizon. It is the image, seen from far away, of the circle of radius 3, the circle on which light can orbit——approach that threshold and the deflection grows as a logarithm, the increment per tenfold reduction rising from below towards 2.302585092994. Equivalently, every increase of the angle by pi shrinks the gap by a factor falling from above towards 23.140692633, so images of the same circle pile up in a geometric sequence without end.The separator is whether one names the surface of no return or the circle where light can orbit——for Sagittarius A* the circle gives 53.290797862 microarcseconds and the horizon gives 20.511637661, while the reported ring is 51.8 plus or minus 2.3.Placed among the earlier papers——this takes the seat that the scope note of Paper 1044 left open by declining the photon sphere and multiple images, and obtains the same deflection from a different integral, agreeing far away.To be honest——the photon sphere, the logarithmic divergence and the piling up by e to the power 2pi are all known. What is added is to count, stage by stage, how the limits are approached, to place the logarithmic coefficient and e to the power pi as one and the same slope, and to give in angular measure what reading the edge as the horizon would cost.**

References

1. Bozza, V. (2002). Gravitational lensing in the strong field limit. *Physical Review D*.
2. Darwin, C. (1959). The gravity field of a particle. *Proceedings of the Royal Society A*.
3. Event Horizon Telescope Collaboration (2019). First M87 Event Horizon Telescope Results. VI. The Shadow and Mass of the Central Black Hole. *The Astrophysical Journal Letters*.
4. Event Horizon Telescope Collaboration (2022). First Sagittarius A* Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole in the Center of the Milky Way. *The Astrophysical Journal Letters*.
5. Synge, J. L. (1966). The escape of photons from gravitationally intense stars. *Monthly Notices of the Royal Astronomical Society*.
6. Yamagishi, Y. (2026). *Michell's Radius Was Right, but Not for the Reason It Gave* [Paper 485]. Zenodo. 10.5281/zenodo.22759928

7. Yamagishi, Y. (2026). *The Einstein–Rosen Bridge Opens to a Throat of Circumference $4\pi GM/c^2$ and Closes within $2\pi GM/c^3$, and Not Even Light Crosses* [Paper 1006]. Zenodo. 10.5281/zenodo.22823763

8. Yamagishi, Y. (2026). *Light Is Gathered by a Gravitational Lens, Yet the Curvature of the Geometry It Travels In Is Negative* [Paper 1044]. Zenodo. 10.5281/zenodo.22861082

— From thinking to syncing. —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)