The Correlation of Timing Residuals Between Two Pulsars Is a Function of the Angle Between Them Alone

— Neither distance nor position on the sky enters; the curve crosses zero at 49.317267601 and 121.788897909 degrees and reaches its minimum of −0.151897312761 at 82.483969242 degrees — the separator is whether the response is quadrupolar or monopolar or dipolar — [Paper 1047]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
September 2026

— From thinking to syncing. —

Abstract

A background of gravitational waves arriving from all over the sky shakes the arrival times of distant pulsars.Compare the timing residuals of two of them and the correlation falls on a single curve.This paper obtains that curve by two routes and matches them.No new theorem and no new law is claimed. First, and this is the core. Only the angle between the two enters——the all-sky integral and the closed form agree, giving 0.211633634877 against 0.211627828168 at 30 degrees, −0.144858678228 against −0.144860385420 at 90, and 0.250000000000 against 0.250000000000 at 180, with a largest difference over the five angles of 1.154×10 to the minus 5(Section 3). **Second, the normalisation is not fitted to the curve——averaging the square of a single pulsar's response over the whole sky gives 0.333333333333, one third, differing by 3.442×10 to the minus 15, and that fixes the normalisation at three halves. The five angles then agree on a scale set elsewhere, not fitted**(Section 2). **Third, the curve crosses zero twice——the crossings are at 49.317267601 and 121.788897909 degrees and the minimum is −0.151897312761 at 82.483969242 degrees. That minimum sits where x equals e to the power minus five sixths (0.434598208507), not at the midpoint of the two crossings (85.553082755 degrees)**(Section 4). **Fourth, the shape is the fingerprint of the kind of wave——replacing only the response in the same integral, a response independent of direction (monopole) gives a correlation independent of angle, 1.000000000000, and a response equal to the dot product with the direction of travel (dipole) gives the cosine divided by three (0.288675134595 at 30 degrees, −0.333333333333 at 180). The number of zeros changes from 0 to 1 to 2**(Section 5). **Fifth, neither distance nor position enters——placing the same pair elsewhere on the sky changes the correlation by at most 1.024×10 to the minus 5, less than the 1.004×10 to the minus 4 produced by changing the angle by 0.05 degrees, and less than a thousandth of the value itself**(Section 6). **Sixth, a measurement of time yields a function of angle——what is measured is nothing but the drift of clock ticks, and what comes out is a function of an angle on the sky. The separator is whether the response is quadrupolar or monopolar or dipolar**(Section 5). **Keywords**: pulsar timing, gravitational-wave background, Hellings-Downs curve, quadrupole, correlation function, isotropy.

Scope of this paper (scope note): **No new theorem and no new law is claimed——that the correlation between two pulsars under an isotropic background is a function of their angular separation alone, and that its shape has a closed form, is standard (Hellings and Downs 1983).No observational statistics——significance, cosmic variance, the pulsar term and noise models are not entered.No source of the background——whether it comes from supermassive binaries or from the early universe is not the subject.No alternative theory of gravity——the monopole and the dipole here are responses substituted as controls, not theories.Nothing about the coefficient of the quadrupole formula——how to read G and the fifth power of c is the work of Paper 42.The values are examples——the fineness of the grid, the angles and the placements are given numbers.Relation to earlier papers：Paper 42 showed that the G and the fifth power of c in the quadrupole formula cannot be separated——here the same quadrupole appears not as a coefficient but as the shape of a curve.Paper 111 counted how far light determines geometry, as two roads that stop at conformal structure ——here gravitational waves and clocks determine a function of an angle on the sky.Paper 118 showed that coordinate time, proper time and entropy are three different logical types——what is measured here is only the drift of clock ticks, and what emerges is geometry.Paper 906 showed that carrying a polarisation around a closed path shifts its phase by half the enclosed solid angle——there an area survives one circuit, here an angle survives an average over the whole sky.**What is added is **matching the all-sky integral against the closed form at five angles**, **fixing the normalisation from a point unrelated to the curve, namely that the mean square response is one third**, **giving the minimum and the two zeros in numbers and showing that the minimum is not the midpoint**, **substituting monopole and dipole responses and watching the number of zeros change**, and **showing that moving the pair on the sky leaves the value unchanged, measured against its sensitivity to the angle**.

1. Notation and the question

Setting: **consider a background of gravitational waves arriving isotropically from the whole sky.The response of the timing residual of a pulsar in direction $\hat{p}$, for a wave travelling along $\hat{\Omega}$ with polarisation A, is $F^A = \frac{1}{2} p^i p^j e^A_{ij}(\hat{\Omega})/(1+\hat{\Omega}\cdot\hat{p})$.The correlation of two pulsars is this product averaged over the sky and over the two polarisations.The integration used 400 Gauss-Legendre points in the cosine of the direction and 800 uniform points in azimuth.**

Question: **what determines the correlation between two clocks?**

2. Fixing the normalisation apart from the curve

Computation 2.1: **the square of a single pulsar's response, averaged over the whole sky, is 0.333333333333, differing from $1/3$ by 3.442×10^{-15}.Adopting the convention that the correlation at zero separation is $1/2$, the normalisation is 1.500000000000, three halves.**

> **Observation 2.2**
>
> the scale comes from outside the curve.This three halves is fitted to no point of the curve——it follows from a quantity defined for a single pulsar. The agreement at five angles in the next section is therefore not the result of a fit.

3. The all-sky integral and the closed form

Computation 3.1: integral and closed form were compared at five separations.

Separation	All-sky integral	Closed form	Difference
30 degrees	0.211633634877	0.211627828168	5.807×10^{-6}
60 degrees	−0.082371923584	−0.082360385420	1.154×10^{-5}
90 degrees	−0.144858678228	−0.144860385420	1.707×10^{-6}
120 degrees	−0.011143630232	−0.011142331508	1.299×10^{-6}
180 degrees	0.250000000000	0.250000000000	8.327×10^{-17}

The closed form is $\frac{1}{2} - \frac{x}{4} + \frac{3}{2} x \ln x$ with $x = (1 - \cos\theta)/2$.The largest difference over the five is $\mathbf{1.154 \times 10^{-5}}$, and at opposition the value is exactly $1/4$.

> **Observation 3.2**
>
> only the separation enters.Neither the distance to the pulsars nor their position on the sky enters the formula or the integral——what enters is one dot product between two directions.

4. The shape of the curve: the minimum and the two zeros

Computation 4.1: differentiating with respect to x gives $\ln x = 1/6 - 1$, so the minimum sits at $x = \mathbf{0.434598208507}$, e to the power $-5/6$, at a separation of $\mathbf{82.483969242}$ degrees with value $\mathbf{-0.151897312761}$.Searching a grid of 400001 points gives $\mathbf{82.484131040}$ degrees and $\mathbf{-0.151897312757}$, differing by 0.000162 degrees.

Computation 4.2: the zeros are at **49.317267601** and **121.788897909** degrees.The integral changes sign there too, giving **0.002893865042** at 49 degrees, **−0.006151489754** at 50, **−0.004934658415** at 121 and **0.007615786702** at 123.

> **Observation 4.3**
>
> the minimum is not at the midpoint.The midpoint of the two zeros is **85.553082755** degrees, while the minimum is at **82.483969242**——the curve is not symmetric. It is the term in $x \ln x$ that breaks the symmetry, and that asymmetry is part of the shape.

5. The shape is the fingerprint of the kind of wave

Computation 5.1: only the response was substituted in the same integral.

Separation	Monopole	Dipole	Quadrupole (the true response)
30 degrees	1.000000000000	0.288675134595	0.211633634877
60 degrees	1.000000000000	0.166666666667	−0.082371923584
90 degrees	1.000000000000	0.000000000000	−0.144858678228
120 degrees	1.000000000000	−0.166666666667	−0.011143630232
180 degrees	1.000000000000	−0.333333333333	0.250000000000

The dipole agrees with $\cos\theta/3$ to within 10^{-9}.The number of zeros is 0 for the monopole, 1 for the dipole (at 90 degrees) and 2 for the quadrupole.

> **Observation 5.2**
>
> the curve counts the kind of wave.The curve says nothing about how strong the background is or where it came from, but it does say what multipole the response is——the number of zeros, and whether the value returns to positive at opposition, are its fingerprint. The quadrupole of Paper 42 appears here not as a coefficient but as a shape.

6. Neither distance nor position on the sky

Computation 6.1: **a pair separated by 75 degrees was placed three further ways on the sky.The dot product stayed at 0.258819045103 throughout, and the correlations were −0.144450506396, −0.144460743845, −0.144450219375 and −0.144449434177.The largest spread is 1.024×10^{-5}, smaller than the 1.004×10^{-4} produced by changing the separation by 0.05 degrees, and smaller than a thousandth of the value itself.**

> **Observation 6.2**
>
> **the residual spread belongs to the grid.Moving the pair puts the point where the denominator of the response becomes small at a different place relative to the grid, so the accuracy of the quadrature shifts a little——it is not that the curve depends on position. The spread is smaller than changing the separation by 0.05 degrees.**

7. What is not claimed

> **Observation 7.1**
>
> **no observational statistics.Significance, cosmic variance, the pulsar term and noise models are not entered——nor is the origin of the background, whether supermassive binaries or the early universe.No alternative theory of gravity either——the monopole and the dipole are responses substituted as controls, not theories.Nor the coefficient of the quadrupole formula——how to read G and the fifth power of c is the work of Paper 42.**

8. Census and closing

1. **mean square response 0.333333333333** (Computation 2.1): **the scale comes from outside.**
2. **normalisation 1.500000000000** (Computation 2.1): **not a fit.**
3. **0.211633634877 against 0.211627828168 at 30 degrees** (Computation 3.1): **two routes agree.**
4. **largest difference 1.154×10^{-5}** (Computation 3.1): **over five angles.**
5. **0.250000000000 at opposition** (Computation 3.1): **exactly $1/4$.**
6. **minimum −0.151897312761 at 82.483969242 degrees** (Computation 4.1): **e to the power $-5/6$.**
7. **zeros at 49.317267601 and 121.788897909 degrees** (Computation 4.2): **two crossings.**

8. **midpoint $\mathbf{85.553082755}$ degrees** (Observation 4.3): **not where the minimum is.**
9. **dipole from $\mathbf{0.288675134595}$ to $\mathbf{-0.333333333333}$** (Computation 5.1): **one zero.**
10. **spread $\mathbf{1.024 \times 10^{-5}}$** (Computation 6.1): **less than $\mathbf{0.05}$ degrees of separation.**
11. **quadrupolar, or monopolar or dipolar** (Observation 5.2): **the separator; Paper 42, Paper 111, Paper 118 and Paper 906.**

Closing: **The correlation of timing residuals between two pulsars is a function of the angle between them alone.The all-sky integral and the closed form give 0.211633634877 against 0.211627828168 at 30 degrees, −0.144858678228 against −0.144860385420 at 90, and exactly 0.250000000000 at opposition, the largest difference over five angles being 1.154×10 to the minus 5. The scale was not fitted to the curve but fixed at three halves by the mean square response of a single pulsar, 0.333333333333——the curve crosses zero at 49.317267601 and 121.788897909 degrees and reaches its minimum of −0.151897312761 at 82.483969242 degrees, which is not the midpoint of the crossings, 85.553082755 degrees.The separator is whether the response is quadrupolar or monopolar or dipolar——substitute the response and the monopole gives 1.000000000000 at every angle and the dipole the cosine divided by three, with the number of zeros running 0, 1, 2.Placed among the earlier papers——the quadrupole of Paper 42 appears here as a shape rather than a coefficient. What is measured is only the drift of clock ticks, and what comes out is a function of an angle on the sky.To be honest——this curve dates from 1983. What is added is to fix the normalisation from a point unrelated to the curve and match two routes, to show in numbers that the minimum is not the midpoint of the zeros, and to substitute responses and watch the number of zeros change.**

References

1. Agazie, G., et al. (NANOGrav Collaboration) (2023). The NANOGrav 15 yr Data Set: Evidence for a Gravitational-wave Background. *The Astrophysical Journal Letters*.
2. Hellings, R. W., & Downs, G. S. (1983). Upper limits on the isotropic gravitational radiation background from pulsar timing analysis. *The Astrophysical Journal Letters*.
3. Yamagishi, Y. (2026). *The G and the c^5 of the Quadrupole Formula Cannot Be Separated* [Paper 42]. Zenodo. 10.5281/zenodo.21601988
4. Yamagishi, Y. (2026). *How Far Does Light Determine Geometry?* [Paper 111]. Zenodo. 10.5281/zenodo.21961685
5. Yamagishi, Y. (2026). *Two Clocks and One Scale* [Paper 118]. Zenodo. 10.5281/zenodo.22036237
6. Yamagishi, Y. (2026). *Carrying a Polarisation Around a Closed Path Shifts Its Phase by Half the Enclosed Solid Angle* [Paper 906]. Zenodo. 10.5281/zenodo.22803812

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)