

There Are Three Unit Slots Because Mechanics Uses Only Three Kinds of Quantity

— Adding electromagnetism makes four and heat makes five, and fixing the value of one constant removes exactly one slot, down to zero; eighteen triples span the three, and which of them shortens the formulas changes with the list (rank correlation -0.587454806) — the separator is whether the ruler for shortness follows convention or purpose — [Paper 1049]

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— *From thinking to syncing.* —

Abstract

The equations of mechanics are written in three slots: mass, length and time. This paper asks why there are three — the answer is that mechanics uses only three kinds of quantity. No new theorem and no new law is claimed. First, and this is the core. The number of slots is the rank of the dimensional matrix spanned by the kinds of quantity — ten mechanical quantities give rank 3, thirteen with electromagnetism give 4, fifteen with heat give 5. Length and time alone give 2, and length alone gives 1 (Section 2). Second, fixing the value of one constant removes exactly one slot — starting from rank 5 and fixing c , then the Boltzmann constant, then the elementary charge, then $\hbar$, then G , the remaining slots fall 4, 3, 2, 1, 0. Seven, three and zero are the same subtraction at different places (Section 3). Third, only an independent constant removes a slot — fixing two masses leaves the count at 4. The price is not three constants but three independent ones (Section 3). Fourth, eighteen triples span the three slots — of the 35 ways to choose three from seven constants, 18 span and 17 do not. Three masses cannot even make a length. The spanning triples return known lengths (the Planck length with exponents $1/2$, $-3/2$, $1/2$ and the Bohr radius with 2, -1, -1) (Section 4). Fifth, which triple shortens the formulas changes with the list — counting how many constant symbols disappear, the winner for a list of gravitational formulas is c , G and $\hbar$ with 9, and for a list of atomic formulas it is $\hbar$, the electric constant and the electron mass with 11. The Spearman rank correlation between the two orderings is **-0.587454806**, which is negative (Section 5). Sixth, the triple used by this body of work is never the winner — setting m to 4π gives rank 10 on the gravitational list, 15 on the atomic one and 17 on both, out of 18. It was not chosen for shortness. The separator is whether the ruler for shortness follows convention or purpose (Section 6). **Keywords:** dimensional analysis, base dimensions, dimensional matrix, systems of units, natural units, convention and fact.

Scope of this paper (scope note): No new theorem and no new law is claimed—the dimensional matrix and its rank, the Buckingham pi theorem, the fact that constants can be set to 1 in natural units, and the fact that the number of base dimensions varies with the system of units are all standard (Bridgman 1922, Buckingham 1914, Planck 1899, Duff and others 2002). No system of units is recommended—nothing is said about which is right or which should be used. Only how many symbols each triple removes is counted. The ruler for shortness is not claimed to be unique—the count of vanishing symbols is one way to measure, and another ruler could reorder the list. That is precisely the point of this paper. The lists of formulas are examples—five on the gravitational side and five on the atomic side were chosen to show that the choice of list fixes the ordering; no representativeness is claimed. No history is told—who added which unit and when is not the subject. The number of fundamental constants is not adjudicated—that there are three positions, 0, 2 and 3 (Duff and others 2002), is cited and no verdict is given. Mass is not said to be geometry—as Paper 60 corrected in its second edition, the 4 pi that appears when m is set to 4 pi is a mass, not a solid angle, and it is a convention rather than geometry. Relation to earlier papers: the scope note of Paper 306 states that the merits of systems of units are not discussed—this paper takes that seat. Paper 306 counted the ends (7 in SI, 3 in Gaussian, 0 in Planck units) but wrote neither the subtraction between them nor what the merits depend on. Paper 21 showed in its third edition that reading 4 pi c^2 as an area requires fixing the unit of time, and that the budget of three slots is already spent—here it is counted why that budget is three. Paper 39 showed that the exponents of the Planck length are fixed by the dimensional matrix—here the same solution is applied to eighteen triples, separating those that yield a length from those that do not. Paper 05 showed that conventions can combine into a quantity that does not depend on convention—here what does not depend on convention is the ordering, and what does depend is the list. Paper 117 lined up four distinct roots by which a scale-free requirement fixes an exponent—the pi theorem among them is the ground of the counting here. Paper 144 showed that an exponent is the signature of what is conserved—the exponents counted here solve the same linear system, but what is read off is the number of slots, not a conserved quantity. Paper 95 separated convention from fact—here a third standing is added: something that is neither, and depends on purpose. Paper 338 showed that being a promise and being arbitrary are not the same—the ordering here also stands on a promise without being arbitrary. Paper 116 showed that the exponent of c does not count dimensions—that paper is about what is inside the exponent, this one about how many slots there are. What is added is writing the number of slots as a subtraction and dropping it from 5 to 0 one constant at a time, showing by a control that a second, dependent constant removes nothing, counting the spanning triples as 18, ordering them by vanishing symbols and finding that the winner changes with the list and that the rank correlation is negative, and counting honestly where the triple used by this body of work stands.

1. Notation and the question

Setting: the dimension of a quantity is written as five exponents, for mass M , length L , time T , charge Q and temperature K . Energy, for instance, is $(1, 2, -2, 0, 0)$. For a collection of quantities, the dimension of the space spanned by these exponent vectors is called the rank. The rank is the number of units that must be fixed

independently, which this paper calls the number of slots. All elimination is carried out in exact fractions, with no rounding.

Question: why are there three slots?

2. The number of slots is the rank of the dimensional matrix

Computation 2.1: ten quantities of mechanics (length, time, mass, speed, acceleration, force, energy, momentum, angular momentum, pressure) were listed and the rank computed.

quantities listed	how many kinds	rank (number of slots)
mechanics only	10	3
plus electromagnetism	13	4
plus heat	15	5
length, time and speed only	3	2
length, volume and inverse area only	3	1

Observation 2.2

the number of kinds and the rank are different things. Ten kinds still give rank 3——force, energy, momentum and pressure are built out of mass, length and time.

Observation 2.3

three is not a number nature chose. With electromagnetism it is 4 and with heat it is 5——the number of slots is set by how many kinds of quantity the theory uses. Paper 306 counted 7 in SI, 3 in Gaussian units and 0 in Planck units——here the space between them is joined by a single quantity, the rank.

3. Subtraction: fixing one constant removes one slot

Computation 3.1: starting from rank 5, constants are set to 1 in turn. The slots that remain are the rank of the quantities minus the rank spanned by the fixed constants.

constant fixed	what it joins	slots remaining
c	length and time	4
k_B	temperature and energy	3
elementary charge	charge and mechanics	2
$\hbar$	action and mechanics	1
G	mass and geometry	0

Observation 3.2

each fixing removes exactly one. The counts are 4, 3, 2, 1, 0—the number of slots is a subtraction, not a floor.

Computation 3.3 (control): fixing both the electron mass and the proton mass leaves 4 slots.

Observation 3.4

only an independent constant removes a slot. Both point along mass alone, so the second removes no new direction—the price is not three constants but three independent ones.

4. The price is three independent constants

Computation 4.1: seven constants of mechanics were taken (c , G , $\hbar$, the electron mass, the proton mass, the electric constant $k_e e^2$, and the mass set to 4π) and the rank computed for all 35 triples. Exactly 18 span the three slots and 17 do not. None of the seven carries charge or temperature, so rank 3 here is the same as spanning M , L and T .

Observation 4.2

three masses do not span. From the electron mass, the proton mass and 4π neither a length nor a time can be built—the linear system for a length has no solution.

Computation 4.3: lengths built from spanning triples.

triple	how the length is built	name
$G, c, \hbar$	$G^{1/2}c^{-3/2}\hbar^{1/2}$	Planck length
$\hbar, k_e e^2$, electron mass	$\hbar^2(k_e e^2)^{-1}m_e^{-1}$	Bohr radius
electron mass, proton mass, 4π	no solution	—

Observation 4.4

one solution returns two known lengths. The way Paper 39 solved for the exponents of the Planck length was simply applied to another triple——what decides solvability is the rank, not the names of the constants.

5. Which triple shortens the formulas changes with the list

Setting 5.1: a constant whose value must be written disappears if it belongs to the chosen triple. Five formulas were taken on the gravitational side ($E = mc^2$, the Schwarzschild radius, the Planck length, gravitational potential energy, the Compton wavelength) and five on the atomic side (the Bohr radius, the Rydberg constant, Coulomb’s law, the Compton wavelength, the fine structure constant), and for each of the 18 triples the total number of vanishing symbols was counted.

list	winning triple	symbols removed
gravitational five	$G, c, \hbar$	9
atomic five	$\hbar, k_e e^2$, electron mass	11
all ten	$G, c, \hbar$ (tied with $c, \hbar$, electron mass)	15

Observation 5.2

the winner changes. The triple that wins on the gravitational list does not win on the atomic one——which is shorter depends on which formulas one wants shortened.

Computation 5.3: comparing the two orderings over the 18 triples, the Spearman rank correlation is **−0.587454806** (ties averaged).

Observation 5.4

the correlation is negative. A choice that shortens the gravitational formulas tends to lengthen the atomic ones—the two purposes do not point the same way.

Observation 5.5

the ordering itself is nonetheless not a convention. Fix the list and anyone counting gets the same order—what moves is the list, not the counting.

6. The separator, and what must be said plainly

Computation 6.1: the triple used by this body of work ($m = 4\pi, G, \hbar$) was ranked. It stands 10th on the gravitational list, 15th on the atomic one and 17th on both, out of 18, removing 5, 4 and 9 symbols respectively.

Observation 6.2

this triple was not chosen for shortness. It wins on no list—what Paper 21 called the fourth choice is a budget of three slots spent on something other than shortness.

Observation 6.3 (separator): the separator is whether the ruler for shortness follows convention or purpose. With the list fixed, the ordering follows no convention—change the list and it reorders. There is a third standing here that is neither convention nor fact.

Observation 6.4 (plainly): the ruler itself, counting vanishing symbols, is a choice. Another count, such as how many formulas end with integer exponents, could reorder the table—what is shown here is the type, that an ordering depends on purpose, and not a particular table.

7. Census and closing

Closing: There are three unit slots because mechanics uses only three kinds of quantity. The number of slots is the rank of the dimensional matrix spanned by the kinds of quantity: 3 for ten mechanical quantities, 4 with electromagnetism, 5 with heat—and fixing the value of one constant removes exactly one, so that c , the Boltzmann constant, the elementary charge, $\hbar$ and G bring it down 4, 3, 2, 1, 0. Only an independent constant removes a slot—fixing two masses leaves 4. The price is not three constants but three independent ones. Eighteen of the 35 triples

span the three slots, and the spanning ones return the Planck length and the Bohr radius by the same solution——three masses cannot even make a length. Which triple shortens the formulas changes with the list——on the gravitational formulas c , G and $\hbar$ removes 9 symbols and wins, on the atomic ones $\hbar$, the electric constant and the electron mass removes 11 and wins, and the Spearman rank correlation between the two orderings is -0.587454806 . The separator is whether the ruler for shortness follows convention or purpose——fix the list and anyone counting gets the same order; change the list and it reorders. Placed among the earlier papers——the ends counted by Paper 306, seven in SI and zero in Planck units, are joined by a single quantity, the rank. Plainly——the triple used by this body of work wins on no list, standing 10th, 15th and 17th. It was not chosen for shortness. What is added——writing the number of slots as a subtraction, counting the spanning triples, and showing by numbers that an ordering depends on purpose.

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