Four Roots of Necessity Are Not Enough, and Neither Are Six

— Reading the 64 papers whose titles claim uniqueness, 42 of them (0.656250) fall outside the four roots, and raising the count to six recovers only one, leaving 41; the papers citing the four roots stop at number 175, and the 874 papers after it never cite them — the separator is whether one tries to enumerate the roots or measures that they do not run out — [Paper 1050]

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— From thinking to syncing. —

Abstract

There are several kinds of reason behind "why is it fixed at this value".Paper 86 sorted them into four: topology, symmetry, algebra and dimension.This paper asks whether four exhaust them.No new theorem and no new law is claimed. First, and this is the core. Four are not enough——reading one by one the 64 papers whose titles claim uniqueness and assigning each a label gives topology 3, symmetry 2, algebra 10, dimension 6 and one mixture, against 42 that belong to none of the four. The fraction is 0.656250, so two thirds fall outside（Section 3）。 **Second, six are not enough either——adding causality and the information bound to make six is natural, but of the 42 that fell outside only one is recovered and 41 remain. Adding inverse problems to make seven leaves 39, and promoting every nameable family to a root still leaves 29**（Section 4）。 **Third, the information bound does not even fall in the net——numbers 73, 88 and 112 are in none of the 64. Their claims are not phrased as uniqueness, which is a property of the census unit rather than a defect of those papers**（Section 4）。 **Fourth, the framework of four roots was not refuted; it stopped being cited——46 papers cite Paper 86, but the last is number 175, and the 874 papers after it never cite it. The control, Paper 300, is cited by 334 papers, the largest being number 1027**（Section 5）。 **Fifth, whether one reduces to the other can be decided by changing the scale——the information bound rides on an area and so looks reducible to the topological root, yet on multiplying the radius by ten the ratio of the curvature integral stays 1.000000000 while the ratio of the area bound becomes 100.000000. Two quantities from the same surface, one scale-free and one not**（Section 6）。 **Sixth, this paper does not close the count——of the 42 outside the four, 13 fall into a nameable family and 29 have no name yet. The separator is whether one tries to enumerate the roots or measures that they do not run out**（Section 7）。 **Keywords**: necessity, uniqueness theorems, classification, census, Gauss-Bonnet theorem, holographic bound.

Scope of this paper (scope note): **No new theorem and no new law is claimed——the Gauss-Bonnet theorem, Noether's theorem, Galois theory, the area bound on entropy (Bekenstein 1973) and the fact that causal structure yields the Lorentz group (Zeeman 1964) are all standard.No complete classification is claimed——what is produced is "four are not enough", not "how many suffice". The measurement shows that papers remain outside however many roots are added.No new root is named so as to fix a count——causality, inverse problems, measure, computation, units and convention, and logic are headings for lining up what fell outside, and they are not claimed to exhaust it.Paper 86 is not called wrong——it distinguished four roots and never wrote that four exhaust them. What is added here is the number on the side it did not write.No uniqueness is claimed for the assignment——the labels on the 64 papers were decided by reading each full title. Another reader could label some differently, so the assignment is frozen in a table and can be reproduced.The census unit is not claimed to be the only cut——papers were taken whose titles contain any of six markers of uniqueness. Including a looser marker raises the count to 254, which is too loose, so it was excluded.No reason is asserted for the citations stopping——that they stop at number 175 was counted; why they stop was not measured.Relation to earlier papers**: **the scope note of Paper 86 states that the four roots are not to be unified into one mechanism——this paper takes that seat. Paper 86 lined up four but wrote nothing about whether four exhaust them, leaving the count open.The prior occupant of the seat of that scope note is Paper 100, which once wrote that the uniqueness of inverse problems is a distinct root——here that single sentence is turned into a count, giving how many papers fall outside.Paper 300 showed that whether two things share a root is decidable——the criterion used here is that one: a coincidence of numbers is not taken as a shared root.Paper 111 showed how far light determines geometry——that result is cited here as an instance of the causal kind of necessity.Paper 83 showed that the relativity principle alone narrows relativity to two——it is the single paper here given the mixture label, since symmetry and algebra cannot be separated in it.Paper 73 showed where the one quarter in black-hole entropy comes from——here it is counted that such a bound does not fall in the census net.Paper 112 lined up several realities that constrain the degrees of freedom——here it is noted that they are not written in the language of uniqueness.Paper 489 showed that causality forces the shape of refraction——here it is the single paper of the causal family among the 42.Paper 488 showed that solving the fall time backwards for the curve leaves only one——here it is cited as an instance of the inverse-problem family.Paper 1049 showed that the number of unit slots is the rank of the dimensional matrix——the papers in the units-and-convention family are necessities of that kind.**What is added is **cutting out the census unit mechanically and counting it as 64**, **reading each one and assigning a label**, **counting how many are recovered as roots are added**, **finding that citations of the framework stop at number 175**, and **deciding non-reducibility by a change of scale**.

1. Notation and the question

Setting: **this paper counts the body of work itself.The objects are the Japanese versions numbered 1 to 1049, of which 1048 were readable (one number is vacant).Paper 86 sorted the reasons why something is fixed into four: topological necessity (a quantity unchanged by continuous deformation decides), necessity by symmetry (a**

continuous symmetry yields a conserved quantity), algebraic necessity (the structure of a group decides solvability), and dimensional uniqueness (it holds in that dimension only).

The question: do these four exhaust the reasons?The scope note of Paper 86 says the four roots are not to be unified into one mechanism, and it never says that four exhaust them.The count is left open, so whether it closes can be measured.

First, where the names of the four roots occur.Each of the four Japanese terms occurs 2 times in 1 paper, and that paper is Paper 86.The terms for a fifth root, for causal necessity and for informational necessity occur nowhere in the 1048 papers （0 each）.As a control, the word for dimension occurs in more than 100 papers, so the scan itself is working.

2. Cutting out the census unit mechanically

The papers to be read must not be chosen by taste.They are cut mechanically: take every paper whose title, including the subtitle, contains any of six Japanese markers of uniqueness.There are 64 of them.

Including a looser marker raises the count to 254, catching a great many uses of the form “from that alone”, which is too loose.It was excluded.The cut can be checked at both ends: the paper numbered 1 (the uniqueness theorem for spherically symmetric density distributions) is inside the unit, and Paper 86 itself is not.Paper 86 does the sorting; it does not claim a uniqueness of its own.

Each of the 64 was then read in full title and assigned a label.The machine is not allowed to assign——a match of strings does not determine a kind of necessity.The assignment is frozen in a table, so the numbers do not move as further papers are written.

3. The assignment: two thirds fall outside the four

Label	Meaning	Papers
T	Topological necessity	3
S	Necessity by symmetry	2
A	Algebraic necessity	10
D	Dimensional uniqueness	6
M	A mixture of two or more	1
X	None of the four	42

The total is 64, matching the unit.Twenty-one fall into one of the four, one is a mixture, and 42 belong to none.The fraction is 0.656250, and more fall outside than inside （42 against 22）。

The contents of the labels are worth seeing.In T are the impossibility of an undistorted map (number 212), the sum of angular defects of a polyhedron (number 224), and the fact that following a wall arrives only when the wall is one connected piece (number 876).In D are the family of uniqueness theorems (numbers 1, 7, 8 and 26), the exponent of c (number 116), and the exponent in the denominator that decides whether a random walk returns (number 199).A is the largest at 10——the papers on number theory and finite fields gather there (numbers 603, 605, 606, 619, 633, 810 and 1019).S has only 2, which is few given how central Noether's theorem is to this body of work.

Only Paper 83 received the mixture label.The symmetry assumption of the relativity principle and the algebraic requirement that the transformations form a group are both at work and cannot be separated.

4. Does adding roots suffice

The natural next move is to add roots.Causality (order alone fixes the geometry) and the information bound (an area limits what can be held) make six——this proposal is tested by counting.

Number of roots	Recovered	Left outside
Four (topology, symmetry, algebra, dimension)	0	42
Six (adding causality and the information bound)	1	41
Seven (adding inverse problems)	3	39
Every nameable family promoted to a root	13	29

Even at six, only 1 paper is recovered.The causal family contains the single Paper 489 (given the shape of absorption, causality fixes the shape of refraction), and the family of the information bound contains 0.

Why zero.The papers treating an area bound (numbers 73, 88 and 112) are in none of the 64.Their claims are not phrased as "uniquely determined" or "there is only one".This is not a defect of those papers——a bound says "no more than this fits", not "it is fixed at exactly this", so there is no occasion to use the language of uniqueness.In other words, the information bound is finer than the mesh of the census.

Adding inverse problems to make seven recovers 3 (numbers 103, 488 and 489) and leaves 39.Giving headings to as much of the remainder as can be named——causality, inverse problems, measure and almost everywhere, what no procedure decides, units and convention, and logic——recovers 13 and leaves 29 without a name.

The remainder falls as roots are added （42 to 41 to 39 to 29）, but it does not reach zero.This paper stops here——it does not say how many suffice.

5. The four roots had stopped being cited

One further thing can be counted.Paper 86 is cited by 46 papers （45 excluding itself）.The smallest is number 5 and the largest is number 175.

No paper after number 175 cites Paper 86: the count is 0.And there are 874 papers after number 176.Through more than eight tenths of the body of work, the framework of four roots was never called upon.

That this is not merely citations thinning out is shown by the control: Paper 300 (whether two things share a root is decidable) is cited by 334 papers, the largest being number 1027.Of two censuses written at about the same time, the one that is a tool for deciding is still in use.

The reason for the stop was not measured.There is no trace of a refutation, and what can be counted is only the fact that the citations stopped.

6. Non-reducibility is decided by a change of scale

When adding a root, the worst danger is that it reduces to an existing one.The information bound is proportional to an area, so it looks reducible to the topological root, which is also about a surface.This can be decided by scale.

On a sphere of radius r, place two quantities side by side. One is the Gauss--Bonnet integral

$$\int_{S^2} K\,dA = \int_0^{2\pi}\!\!\int_0^{\pi} \frac{1}{r^2}\, r^2 \sin\theta\, d\theta\, d\phi,$$

and the other is the information bound set by the area （$A/4$ in Planck units）.

Integrating numerically, the integral is $\mathbf{12.566371906}$ at $r = 1$, 10 and 100 alike （$4\pi = 12.566370614$, with 2000 steps）, and does not move with the radius.The bound, by contrast, is 3.141593 at $r = 1$ and $\mathbf{314.159265}$ at $r = 10$.

Placing the ratios for a tenfold radius side by side, the integral gives $\mathbf{1.000000000}$ and the bound gives $\mathbf{100.000000}$.Two quantities drawn from the same surface behave entirely differently under a change of scale——one is scale-free and the other rides on the area.The appearance of an area is therefore not enough to reduce one to the other.

Coarsening the integration to 3 steps gives 13.159473, which misses 4π, so the agreement is not an artefact of the numerics but something that appears only as the steps are refined.

A toy estimate is appended: setting the count of the interior to $(4/3)\pi r^3$ and the bound of the skin to πr^2, the two are equal at $r = \mathbf{0.750000}$ （both $\mathbf{1.767145868}$）, with the interior smaller before that point and larger after it.At $r = 10$ the interior is $\mathbf{4188.790205}$ against a skin of $\mathbf{314.159265}$, a ratio of $\mathbf{13.333333}$.This is a toy for seeing the bookkeeping of dimensions, not an estimate of an actual count of states.

7. Census and closing

Closing: four roots of necessity are not enough, and neither are six.Reading one by one the 64 papers whose titles claim uniqueness gives topology 3, symmetry 2, algebra 10, dimension 6 and one mixture, against 42 belonging to none of the four, a fraction of $\mathbf{0.656250}$——two thirds fall outside.Adding causality and the information bound to make six recovers one and leaves 41——adding inverse problems to make seven leaves 39, and promoting every nameable family to a root still leaves 29.The information bound does not even fall in the net——numbers 73, 88 and 112 are in none of the 64, because a bound says that no more fits rather than that one value is fixed, which is not a defect of those papers.The framework of four roots was not refuted; it stopped being cited——46 papers cite Paper 86 but the last is number 175, and the 874 papers after it never cite it, while the control Paper 300 is cited by 334 papers up to number 1027.Whether one root reduces to another is decided by a change of scale——on multiplying the radius by ten the ratio of the curvature integral stays $\mathbf{1.000000000}$ while that of the area bound becomes $\mathbf{100.000000}$.The separator is whether one tries to enumerate the roots or measures that they do not run out——

looking at the same 64 papers, the former answers with a number of roots and the latter with the fraction left outside.Plainly——this paper does not close the count either. Of the 42 outside the four, 13 acquired a name and 29 did not.What is added——cutting out the census unit mechanically, reading each paper and assigning a label, counting how many are recovered as roots are added, finding that the framework stopped being cited at number 175, and deciding non-reducibility by a change of scale.

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