

Two Kinds of Subtracting One: One Counts Holes, the Other Counts Dimensions

— The partitions in a row fall one short of the cells because of the Euler characteristic, which drops to 0 for a ring and to -1 for a figure eight; the exponent of a shell falls one short of the dimension because of codimension, and stays at 2.000000000 for a sphere, a torus and a two-holed torus alike. Codimension one holds in both cases and yet fixes neither count — the separator is whether it moves when the topology is changed — [Paper 1052]

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September 2026

— *From thinking to syncing.* —

Abstract

A tridiagonal matrix has $n - 1$ entries off the diagonal, and in d dimensions the density exponent for which a shell's mass does not depend on the radius is $d - 1$. Both look like “one less”. This paper asks whether the two share a root. No new theorem and no new law is claimed. First, and this is the core. The two have different sources—one of the ones is the Euler characteristic, that is, a count of holes, and the other is codimension, that is, the fact that a surface is one dimension lower than what it bounds (Section 5). Second, codimension alone does not produce “one less”—in a lattice of any dimension the partitions have codimension one, yet the count is cells minus one only in one dimension. A three by three lattice has 9 cells and 12 partitions, not 8; a three by three by three lattice has 27 cells and 54 partitions (Section 2). Third, the one in a row is the Euler characteristic—the edges equal the vertices minus the Euler characteristic, which is 1 for a tree, 0 for a ring, -1 for a figure eight and -2 for three rings. The number being subtracted itself moves. In matrix terms the shortfall equals the number of edges, 5 for a row and 6 for a ring (Section 3). Fourth, the exponent of a shell does not respond to topology at all—for a sphere (Euler characteristic 2), a cube, a box, a torus (0) and a two-holed torus (-2), the exponent of the surface stays at 2.000000000. Only dimension moves it: from 2 to 6 the exponent runs 1, 2, 3, 4, 5 (Section 4). Fifth, the two are blind in opposite directions—the one in a row sees topology and not dimension, the one in a shell sees dimension and not topology. In the labels of Paper 86 the former is topological necessity and the latter dimensional uniqueness, two different roots among the four (Section 5). Sixth, the agreement of the numbers is an accident— $n - 1$ and $d - 1$ take the same value only when the number of points equals the dimension, and the two that actually occur in this body of work are 5 and 2. The separator is whether it moves when the topology is changed (Section 6).

Keywords: Euler characteristic, codimension, distinct roots, dimensional analysis, graphs, spherical shells.

Scope of this paper (scope note): No new theorem and no new law is claimed—the relation among edges, vertices and the Euler characteristic of a graph, the counting of faces in a lattice, the fact that the area of a self-similar surface grows as the $d - 1$ power of a length, and the condition for a spherically symmetric shell's mass to be independent of the radius are all standard. The reading in terms of a codimension-one boundary is not called wrong—the partitions in a row and the surface of a ball are both of codimension one. What is said here is only that this by itself does not fix a count. No unification over general complexes is attempted—only graphs, lattices and self-similar surfaces are treated, with no generalisation to complexes of arbitrary dimension. No permanent separation is claimed—the criterion of Paper 300 means that the responses differ under the measurements available now. That the two might connect at a deeper level is not denied. The area of the two-holed torus was set by hand—only a coefficient was given, as a family grown self-similarly, rather than the area of an actual surface being computed. It serves to show that the exponent does not move when the topology changes. No physical claim is made—the shell of mass is a computation in geometry and is not about any star or any material. Relation to earlier papers: Paper 300 showed that whether two things share a root is decidable—that criterion is used here: a coincidence of numbers is not taken as a shared root, and what is examined is what makes each move. Paper 1051 counted the unknowns of a tridiagonal matrix as $2n - 1$ —where that $n - 1$ comes from is examined here. Paper 01 showed that in three dimensions the exponent for which a shell's mass does not depend on the radius is 2 and only 2—where that 2, as $d - 1$, comes from is examined here. Paper 116 showed that the exponent sitting in the denominator is $d - 1$ —here it is counted that this $d - 1$ does not respond to topology. Paper 224 showed that the angular defects of a polyhedron sum to 4π , and to 0 on a torus—it is cited as a case where the Euler characteristic fixes the value. Paper 86 sorted the reasons why something is fixed into four—the two found here fall under different labels among those four. Paper 1050 counted that the four labels are not enough—this paper is not about the side that is missing but gives one case where the labels do their work. What is added is showing by a count of lattices that codimension one, common to both, fixes neither count, showing that the one in a row is the Euler characteristic by moving it to 0 and to -1 with a ring and a figure eight, showing that the exponent of a shell does not respond to topology at all, with Euler characteristics 2, 0 and -2, and lining up in one table the fact that the two are blind in opposite directions.

1. Notation and the question

Setting: there are two places in this body of work where something is “one less”.

A: a symmetric tridiagonal matrix has n unknowns on the diagonal and $n - 1$ off it, because n points in a row are joined by $n - 1$ links.

B: in d dimensions the density exponent for which the mass of a spherical shell does not depend on the radius is $d - 1$. In three dimensions it is 2, which is the uniqueness of Paper 01.

The question: do the two share a source?The reading offered was that both are boundaries of codimension one and are therefore the same—that where there are n things inside, the skins between them are always one fewer.

The criterion is that of Paper 300: a coincidence of numbers is not taken as a shared root.What is examined is what makes each move, and if the responses differ the roots are taken to differ.Three things were prepared to move: topology, shape and dimension.

2. Codimension alone does not produce “one less”

The reading is tested head on.Take a lattice in d dimensions with m cells along each side.There are m^d cells and $dm^{d-1}(m - 1)$ interior partitions.Every partition is a face one dimension lower than a cell—that is, every one of them has codimension one.

Dimension	Side	Cells	Partitions	Against cells minus one
1	3	3	2	exactly
1	10	10	9	exactly
2	3	9	12	4 more
2	4	16	24	9 more
3	3	27	54	28 more
3	4	64	144	81 more

In two dimensions with three cells a side there are **9** cells and **12** partitions, not the **8** that “cells minus one” would give.In three dimensions with three a side there are **27** cells against **54** partitions.

Codimension one holds in every dimension, and yet “one less” holds only in one dimension.So codimension is not what produces “one less”.

3. The one in a row is the Euler characteristic

What does produce it?Measure on graphs.With V vertices and E edges the Euler characteristic is $\chi = V - E$, so that $E = V - \chi$.

Shape	Vertices	Edges	Euler characteristic
A row (path)	6	5	1
A branching tree	7	6	1
A star-shaped tree	5	4	1
A ring	6	6	0
Two rings (figure eight)	5	6	-1
Three rings	4	6	-2

The number being subtracted itself moves through 1, 0, -1 and -2 . It is 1 only for a tree, and it is 1 for a branching tree and a star alike. So being in a row is not what matters——what matters is having no ring.

In matrix terms: the unknowns are V on the diagonal and E off it, while there are only V eigenvalues, so the shortfall equals E . It is **5** for a row and **6** for a ring. On a ring, “one less” disappears.

4. The exponent of a shell does not respond to topology

Now move the other side in the same way. Line up surfaces grown self-similarly and read the exponent from how the area grows when a length is multiplied by ten.

Shape	Euler characteristic	Exponent of the surface
Sphere	2	2.000000000
Cube	2	2.000000000
Box (1 to 2 to 3)	2	2.000000000
Torus	0	2.000000000
Two-holed torus	-2	2.000000000

The Euler characteristic runs 2, 2, 2, 0, -2 , and yet the exponent stays at **2.000000000** and does not move at all. The area of the two-holed torus was set by hand as a family grown self-similarly——no actual surface area was computed; it serves only to show that the exponent does not move when the topology changes.

Only dimension moves it. Taking d from 2 to 6, the exponent of the surface runs 1, 2, 3, 4, 5, and the exponent of the volume is always one greater. That one is exactly codimension—a surface is one dimension lower than what it bounds.

5. The two side by side

What is moved	A (the one in the edge count)	B (the one in the exponent)
Topology	1 to 0, -1, -2 (moves)	stays at 2 (does not move)
Shape	1 for any tree (does not move)	2 for a cube too (does not move)
Dimension	partitions cease to be cells minus one	1, 2, 3, 4, 5 (moves)

The responses are wholly different. A sees topology and not dimension; B sees dimension and not topology—the two are blind in opposite directions. By the criterion of Paper 300 these are different roots.

Placed against the labels of Paper 86, A is topological necessity and B is dimensional uniqueness, two different ones among the four. The fact shown in Paper 224, that the angular defects of a polyhedron sum to 4π and to 0 on a torus, sits on the same side as A—there too the value is fixed by the Euler characteristic.

The reading offered was not wrong. The partitions in a row and the surface of a ball are indeed both of codimension one. But being of codimension one does not fix the count—in a two-dimensional lattice the partitions have codimension one as well, and yet the count is not cells minus one.

6. When do the numbers agree

Finally, how far do the two numbers agree at all. $n - 1$ and $d - 1$ take the same value only when the number of points n equals the dimension d .

Placing side by side the two that actually occur in this body of work: the tridiagonal matrix of Paper 1051 has $n = 6$ and a shortfall of 5, while the shell of Paper 01 has $d = 3$ and an exponent of 2. The values themselves differ. What looked alike was not the value but the way of writing it, as “subtract one”.

7. Census and closing

Closing: “one less” comes in two kinds, one counting holes and the other counting dimensions. Codimension one is common to both and yet fixes neither count—in a lattice of any dimension the partitions have codimension one,

but the count is cells minus one only in one dimension, a three by three lattice having 9 cells against 12 partitions. The one in a row is the Euler characteristic—the edges equal the vertices minus the Euler characteristic, which is 1 for a tree, 0 for a ring, -1 for a figure eight and -2 for three rings, so the number subtracted itself moves; the matrix shortfall is 5 for a row and 6 for a ring. The exponent of a shell does not respond to topology at all—with Euler characteristics running 2, 0 and -2 the exponent stays at **2.000000000**, and only dimension moves it, running 1, 2, 3, 4, 5 as d goes from 2 to 6. The two are blind in opposite directions—the one in a row sees topology and not dimension, the one in a shell sees dimension and not topology; in the labels of Paper 86 they fall under topological necessity and dimensional uniqueness, two different roots among the four. The separator is whether it moves when the topology is changed—under one and the same way of writing, one of them moves when a ring is made and the other does not move even with two holes. Plainly—that both are of codimension one is correct, and what is said here is only that this does not fix a count; and the claim is that the responses differ under the measurements available now, not that the separation is permanent. What is added—showing by a count of lattices that codimension one fixes neither count, showing that the one in a row is the Euler characteristic by means of a ring and a figure eight, showing that the exponent of a shell does not respond to topology with Euler characteristics 2, 0 and -2, and lining up in one table that the two are blind in opposite directions.

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