# Three Independent Quantities Do Not Always Make a Set of Base Units

— The dimension of a quantity is a point on an integer lattice and multiplication is addition; of the 106 independent triples, only 52 can serve as a basis with the exponents kept integral, and the rest produce half-integers — the separator is whether one asks for independence or for an integral basis — [Paper 1054]

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*— From thinking to syncing. —*

**Abstract**

**The dimension of a quantity is a triple of exponents.Paper 1049 counted that the number of unit slots is the rank spanned by those triples, and that fixing an independent constant removes one slot.This paper asks whether being independent is enough to serve as units.No new theorem and no new law is claimed. First, and this is the core. Being independent and being able to serve as units are different——of the 120 ways to choose three from ten quantities, 106 are independent, but only 52 can serve as a basis with the exponents kept integral. The remaining 54 produce half-integers. The fraction is 0.490566, almost exactly half** (Section 4). **Second, what separates them is the determinant——mass, length and time give 1, and energy, velocity and time also give 1, but mass, length and energy give −2. Taking the latter as a basis makes the coordinates of time one half, one and minus one half** (Section 4). **Third, the dimensions really do form a lattice——all the exponents of thirty mechanical quantities are integers, multiplication becomes addition (force plus length gives energy) and taking a power becomes scaling (length times three gives volume). The span is three; length and time alone give two, and powers of length alone give one** (Section 2). **Fourth, physics lives in a biased part of that lattice——there are 24 distinct points, occupying a fraction 0.228571 of a box of 105 cells. Twenty-nine of the thirty quantities have a mass exponent of 0 or 1 (0.966667), far more concentrated than the 0.433333 for length. Action, angular momentum and Planck's constant sit at one and the same point** (Section 3). **Fifth, the linear structure is real, but there is no length, no angle and no orthogonality ——changing the axes from mass, length and time to mass, length and energy moves the angle between force and energy from 17.715472 degrees to 45.000000, and the inner product of mass and time from 0.000000 to 0.500000, so orthogonality disappears. Yet that force plus length gives energy, and that the span is three, do not move** (Section 5). **Sixth, setting a constant to one is collapsing one direction——setting the speed of light to one makes the direction of length minus time vanish, and three dimensions fall to two. The separator is whether one asks for independence or for an integral basis** (Section 6). **Keywords**: dimensional matrix, lattice, basis, determinant, systems of units, convention and fact.

**Scope of this paper (scope note)**: **No new theorem and no new law is claimed——that the dimension of a quantity is a vector of exponents, the dimensional matrix and its rank, that a change of units is a change of basis, and that bases of an integer lattice are related by transformations of determinant 1 or $-1$ (the Smith and Hermite normal forms) are all standard (Bridgman 1922, Buckingham 1914, Smith 1861).No basis is recommended——a triple whose determinant is not 1 is not called wrong. It is only that an integral bookkeeping requires 1.Half-integer exponents are not called wrong——the exponents of the Planck length are half-integers, which only means they are not lattice points, not that the quantity is illegitimate.The list of quantities is an example——thirty mechanical quantities were laid out and the triples drawn from ten of them. A different list moves the numbers, and no representativeness is claimed.The axes of dimension space are not identified with the dimensions of spacetime——a direction in spacetime carries a dimension of length and a metric, while an axis here is a direction in a ledger of exponents, carrying neither length nor angle. Section 5 shows this by numbers.No talk of adding an orthogonal new axis——orthogonality itself depends on the basis. Mass and time are orthogonal on one choice of axes and not on another.The slot of temperature and the dimension in which the unit ball has greatest volume are not treated as the same number——both take the value five, but one counts independent kinds of quantity and the other is a dimension of space, and the agreement is one of value only.Relation to earlier papers**: **Paper 1049 counted that the number of unit slots is the rank spanned by the kinds of quantity and that each independent constant fixed removes one——here the same space is viewed as a lattice. Rank sees only whether a triple is independent; a lattice sees whether it can be written integrally.Paper 306 showed that the number of base units is a promise——here it is counted that the promise is not entirely free.Paper 116 showed what the exponent of c counts——that paper is about what is inside an exponent, this one about the exponent being an integer at all.Paper 95 separated convention from fact——here, within one and the same space, the linear structure falls on the side of fact and length and angle on the side of convention.Paper 300 showed that whether two things share a root is decidable——following that discipline, an agreement in the value five is not taken as a shared root.Paper 1052 showed that two similar-looking things separate into topology and dimension——here too, within what looks like one space, it is divided how far the structure is real.**What is added is **confirming with thirty quantities that dimensions form an integer lattice**, **counting that of 106 independent triples only 52 give an integral basis**, **showing with counterexamples that the determinant is what separates them**, **measuring by occupancy and by the skew of exponents how unevenly physics inhabits the lattice**, and **showing by numbers that length, angle and orthogonality depend on the basis, and separating them from the linear structure**.

## 1. Notation and the question

**Setting**: **a mechanical quantity can be written with three exponents, for mass, length and time.Velocity is $(0, 1, -1)$, force is $(1, 1, -2)$, energy is $(1, 2, -2)$.Paper 1049 counted the span of these triples and showed that the number of unit slots is that rank.**

**The question**: **if three quantities are independent, can they be taken as base units?In the language of rank the matter ends with “independence suffices”.It does not end there once the exponents are also required to be integers.**

## 2. The dimensions really do form a lattice

**First, confirm that this is not a metaphor.Lay out thirty mechanical quantities and see what multiplication and powers become.**

**Adding force $(1, 1, -2)$ to length $(0, 1, 0)$ gives $(1, 2, -2)$, which is energy.Energy minus time is power, mass minus volume is density, and length times three is volume.Multiplication is addition, division is subtraction, and a power is a scaling——as a control, force plus time is not energy.**

**The span is $3$.As controls, length, time, velocity and acceleration alone span $2$, and length, area, volume and wavenumber alone span $1$.So this is neither a plane nor a line but a solid.**

**And all thirty sets of exponents are integers.So this is not a continuous space but a lattice.Some quantities fall outside it——the exponents of the Planck length are $1/2$, $-3/2$ and $1/2$, which are half-integers.**

## 3. Where in that lattice physics lives

**The thirty quantities occupy $24$ distinct points** (action, angular momentum and Planck's constant sit at **one and the same point**).**The box runs from $-1$ to $1$ in mass, $-3$ to $3$ in length and $-3$ to $1$ in time, giving $105$ cells, so the occupied fraction is $0.228571$.**

| Exponent of mass | Quantities |
|---|---|
| -1 | 1 |
| 0 | 12 |
| +1 | 17 |

**Twenty-nine of the thirty have a mass exponent of $0$ or $1$, a fraction of $0.966667$.For length the same fraction is only $0.433333$.It is a solid, but physics inhabits it like a thin slab in the direction of mass.**

**Nor does a single plane hold them all——trying every coefficient from $-3$ to $3$, the best plane holds $17$ of them and never reaches $30$.It remains a solid.**

## 4. Being independent is not being able to serve as units

**This is the core.Choosing units is changing the basis of this lattice.Whether three quantities can be taken as a basis at all is decided by the determinant being nonzero——this is the "independence" of Paper 1049.**

**Whether they can be written integrally, however, is a different condition.**

| The three taken as a basis | Determinant | Result |
|---|---|---|
| Mass, length, time | +1 | integral throughout |
| Energy, velocity, time | +1 | integral throughout |
| Mass, length, energy | -2 | time becomes half-integral |
| Mass, area, time | +2 | length becomes half-integral |

**Mass, length and energy are indeed independent (the determinant is nonzero).But the determinant is $-2$, and taking these three as base units makes the coordinates of time $1/2$, $1$ and $-1/2$.The integral bookkeeping breaks.**

**How often does this happen.All $120$ ways of choosing three from length, time, mass, velocity, force, energy, momentum, pressure, density and action were tried.**

| | Count |
|---|---|
| All ways of choosing three | 120 |
| Independent (nonzero determinant) | 106 |
| Of these, an integral basis (determinant 1 or -1) | 52 |
| Independent yet not a basis | 54 |

**The fraction is $0.490566$, almost exactly half.Examples that fail are length with mass and force, length with mass and energy, length with mass and pressure (determinant $+2$ in each), and length with force and density, length with energy and density ($-2$).In every failing case one of the three basic quantities acquires a denominator in its coordinates.**

**$\Rightarrow$ Independence does not imply being able to serve as units.Looking only at rank hides half of this.**

## 5. This space has no length, no angle and no orthogonality

**Once it is seen to be a lattice, one wants to see its shape.That is where to stop.**

**For one and the same set of points, the “angle” changes with which three are taken as axes.Change the axes from mass, length and time to mass, length and energy, and measure again.**

| Two quantities | On mass, length, time | On mass, length, energy |
|---|---|---|
| Angle between force and energy | 17.715472 degrees | 45.000000 degrees |
| Angle between velocity and acceleration | 18.434949 degrees | 35.264390 degrees |
| Inner product of mass and time | 0.000000 | 0.500000 |

**The angles move by $27.284528$ and $16.829441$ degrees.And mass and time are orthogonal on one choice of axes and not on another——the inner product moves from $0.000000$ to $0.500000$.**

**Some things do not move under a change of basis: that force plus length gives energy still holds on the new axes.That the span is $3$ is also unchanged.**

**$\Rightarrow$ Points, addition and dimension are real. Length, angle and orthogonality are shadows cast by the choice of axes.So the phrase "add one orthogonal new axis" does not stand in this space.For the same reason these axes cannot be identified with directions of spacetime——a direction of spacetime carries a dimension of length and a metric, while an axis here is a direction in a ledger of exponents.**

**There is a second thing that cannot be asked——whether this lattice has fivefold symmetry.Speaking of rotation requires length and angle, and as just seen this space has neither.That a crystal lattice forbids fivefold symmetry is a statement about a lattice carrying a metric——the lattice here carries only points and addition.**

## 6. Setting a constant to one is collapsing one direction

**Finally, write the subtraction of Paper 1049 in the language of this space.**

**The dimension of the speed of light is $(0, 1, -1)$, which is length minus time.Setting the speed of light to $1$ is regarding this direction as $0$——that is, collapsing along it.The span falls from $3$ to $2$.**

**As a control, collapsing the same direction twice removes only one** (fixing two masses removes only one slot).**What Paper 1049 wrote as "three independent ones" becomes, here, a statement about directions.**

## 7. Census and closing

**Closing: three independent quantities do not always make a set of base units.Of the 120 ways to choose three from ten quantities 106 are independent, but only 52 give a basis with the exponents kept integral, and the remaining 54 produce half-integers——a fraction of $0.490566$, almost exactly half.What separates them is the determinant——mass, length and time give 1 and energy, velocity and time give 1, but mass, length and energy give $-2$, and then the coordinates of time become half-integral.The dimensions really do form a lattice——all the exponents of thirty**

**mechanical quantities are integers, multiplication is addition, a power is a scaling, and the span is three.Physics lives in a biased part of it——24 distinct points occupy a fraction 0.228571 of the box, and 29 of 30 quantities have a mass exponent of 0 or 1 (0.966667), against 0.433333 for length.Yet there is no length, no angle and no orthogonality——changing the axes moves the angle between force and energy from 17.715472 degrees to 45.000000 and the inner product of mass and time from 0.000000 to 0.500000, destroying orthogonality, while force plus length giving energy and the span being three do not move.The separator is whether one asks for independence or for an integral basis——looking at the same triples, the former answers 106 and the latter 52.Plainly——the list of quantities is an example and a different list moves the numbers; and a triple whose determinant is not 1 is not called wrong, only that an integral bookkeeping requires 1.What is added——confirming with thirty quantities that dimensions form an integer lattice, counting that only about half of the independent triples give an integral basis, showing with counterexamples that the determinant separates them, measuring how unevenly physics inhabits the lattice, and showing by numbers that length, angle and orthogonality depend on the basis.**

## References


1. Bridgman, P. W. (1922). *Dimensional Analysis*. Yale University Press.
2. Buckingham, E. (1914). On physically similar systems; illustrations of the use of dimensional equations. *Physical Review*.
3. Smith, H. J. S. (1861). On systems of linear indeterminate equations and congruences. *Philosophical Transactions of the Royal Society of London*.
4. Yamagishi, Y. (2026). *Convention and Fact* [Paper 95]. Zenodo. 10.5281/zenodo.21758621
5. Yamagishi, Y. (2026). *What Does the Exponent of* **c** *Count?* [Paper 116]. Zenodo. 10.5281/zenodo.22003794
6. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
7. Yamagishi, Y. (2026). *How Many "Base" Units There Are Is a Promise* [Paper 306]. Zenodo. 10.5281/zenodo.22172149
8. Yamagishi, Y. (2026). *There Are Three Unit Slots Because Mechanics Uses Only Three Kinds of Quantity* [Paper 1049]. Zenodo. 10.5281/zenodo.22902445
9. Yamagishi, Y. (2026). *Two Kinds of Subtracting One: One Counts Holes, the Other Counts Dimensions* [Paper 1052]. Zenodo. 10.5281/zenodo.22903834


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