

The Fine-Structure Constant Does Not Move with the Units, but It Moves with Energy

— A factor of 4π sits in its formula and leaves no trace in the value, and whether a formula in π gives 137 is decided by how loose a tolerance is allowed — the separator is whether the number moves when the units change or when the energy changes — [Paper 1056]

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— *From thinking to syncing.* —

Abstract

Paper 1055 divided numbers by whether they move when the units are changed—those that move record the history of the units, and those that do not record the world. This paper measures that the fine-structure constant falls into neither box. No new theorem and no new law is claimed.

First, and this is the core. The constant does not move with the units but moves with energy—scaling mass by 10^7 , length by 10^{-9} and time by 10^3 leaves the spread in its inverse at a relative **2.074** times ten to the minus sixteen (one spacing of double precision), whereas a one-loop vacuum polarisation gives **128.939785** at the mass of the Z boson, a shift of **8.096214** from 137.035999177, or **0.059081** in relative terms (Sections 2 and 6).

Second, a factor of 4π sits in the formula and leaves no trace in the value—the SI form and the Gaussian form, which has no such factor, agree to a relative **0.000** times ten to the zero, not differing in a single digit of double precision. As a control, removing only that factor from the SI form shifts the value by exactly **12.566370614** (Section 2).

Third, the factors of 4π this corpus places do not reach it—however the unit budget fixed in Paper 17 is moved, the inverse does not change, and of the numbers built from 4π that the corpus uses, the nearest is **124.025107**, still 9.4945 per cent away. Dividing by powers of π does not approach an integer either, the smallest relative distance being **0.008239** (Section 3).

Fourth, whether a formula in π gives 137 is decided by how loose a tolerance is allowed—among 10769 distinct expressions built from fractions and powers of π , none falls within the present precision (a relative **0.000262768**), while 81 do once the tolerance is loosened to 0.05. For forty targets of the same magnitude the averages are **0.3750** and **77.2000**, so 137 behaves exactly like the controls at every tolerance (Section 4).

Fifth, a close formula exists, but its closeness is a function of how wide a space of formulas is permitted—Wyller's expression comes within a relative **6.077** times ten to the minus seven, yet the inverse is measured to ten to the minus ten, so it remains about four thousand times away. Widening the space from 2184 to 52872 expressions raises the hits from 0 to 1, and the control average then stands at **1.0750** (Sections 4 and 5).

Sixth, the separator is whether the number moves when the units change or when the energy changes—the proton to electron mass ratio **1836.152673** moves under neither, the digits of the speed of light move under the units, and this constant moves under energy alone. A third box is needed (Section 7).

Keywords: fine-structure constant, dimensionless quantities, systems of units, vacuum polarisation, renormalization group, multiple comparisons.

Scope of this paper (scope note): No new theorem and no new law is claimed—the definition of the fine-structure constant, that SI and Gaussian units place the rationalising factor of 4π differently, that a coupling runs with energy through vacuum polarisation, and the form of the one-loop leptonic contribution are all standard. Why it is near 137 is not answered—no attempt is made to derive the inverse from theory. The road that Paper 101 treated as Eddington's slip is not entered. It is not claimed that 4π yields 137—the measurement points the other way: the factors of 4π this corpus places do not reach the constant at all. Wyler's expression is not called a derivation—its closeness is granted, but as Section 4 shows, that closeness is a function of how many forms of expression are permitted. The hadronic contribution is not computed here—the value 0.02766 is taken from the literature. Only the one-loop leptonic part is computed here, and that too is an approximation. Variation over cosmic time is not treated—whether the constant changed with epoch is the subject of Paper 314; what is measured here is the running with energy scale alone. The two are different questions. No renormalisation group is built—the general theory of running is not entered; one number for one coupling is produced. Eddington is not judged—this paper too, on the same day, wrote Wyler's expression from memory and got it wrong. A caution is not aimed only at others. Relation to earlier papers: the scope note of Paper 1055 said that the fine-structure constant would not be explained, and that its running with energy would be written only as a caution—this paper takes that seat (no earlier occupant). The running set down there as a caution is measured here. Paper 314 showed that in any system of units the constant is $1/137.035999$ and cannot be moved by redefining units—here that immobility is confirmed by actually changing the scales, and another axis along which it does move is added. Paper 101 treated as a caution that Eddington derived 136 and added to it once the measurement was known to be near 137—here that construction is actually computed, and an error of the same kind is recorded of the author. Paper 17 showed that 8π is an artefact of the units and fixed the corpus's unit budget—here it is counted that the same budget does not reach the constant. Paper 10 established that only dimensionless numbers can be fine-tuned—this paper shows that such a number can still move along another axis. Paper 15 computed the inverse independently—its digits are used here. Paper 44 showed that the power of 4π is the mass dimension—the factor here differs: it leaves neither dimension nor value. Paper 110 showed that Ricci flow is a renormalization group flow—that paper is the running of geometry, this one the running of a coupling. Paper 300 showed that whether two things share a root is decidable—following that discipline, no root is inferred from numerical nearness alone. What is added is measuring the constant along the two axes of units and of energy, naming the place inside this corpus where a factor of 4π enters a formula and leaves no trace, listing the distances from the corpus's factors of 4π to 137, counting how easily a formula in π hits 137, across tolerances and across sizes of the space of formulas, against controls, and showing that the separator of Paper 1055 needs a third box.

1. Setting and question

Setting: the inverse fine-structure constant is 137.035999177. Being dimensionless, its value does not change when the units change—Paper 314 wrote as much.

Question: Paper 1055 divided numbers by whether they move when the units are changed. Those that move record the history of the units; those that do not record the world. Does this constant simply belong in the second box?

2. The factor of 4π is in the formula and does not move the value

The formula looks different in different systems of units.

Form	Expression	Inverse
SI (with 4π)	$e^2/(4\pi\epsilon_0\hbar c)$	137.035999177
Gaussian (without)	$e^2/(\hbar c)$, with e^2 meaning ke^2	137.035999177

The two agree to a relative 0.000×10^0 —not differing in a single digit of double precision. With the first-edition values (CODATA 2018) the relative difference was 3.566×10^{-16} , 1.61 times the spacing of double precision, which was likewise the rounding of the computation itself. As a control, removing only the factor of 4π from the SI form shifts the value by exactly **12.566370614**.

Hence the factor is written there and leaves nothing in the answer. The 8π of Paper 17 moves when the units change, and the power of 4π in Paper 44 carries a mass dimension—this one is like neither.

Next, test it with the corpus’s own unit budget. Paper 17 fixed that this corpus spends its three degrees of freedom on the mass, on the gravitational constant and on the reduced Planck constant. The constant was recomputed on five sets of scales including those values.

Mass scale	Length and time scales	Inverse
1	1 and 1	137.035999177
12.566370614	1 and 1	137.035999177
1	12.566370614 and 1	137.035999177
1	1 and 0.079577472	137.035999177
10^7	10^{-9} and 10^3	137.035999177

The spread is a relative 2.074×10^{-16} —exactly one spacing of double precision, the rounding of the computation (with the first-edition values it was 0).As a control, the numerical value of the reduced Planck constant does move under the same rescaling.Hence placing 4π in three positions does not reach 137.

3. The distances from the corpus’s factors of 4π to 137

How near, then, are the numbers built from 4π that the corpus actually uses?

Number	Value	Relative distance to 137.036
$4\pi^3$	124.025107	0.094945
$16\pi^2$, that is $(4\pi)^2$	157.913670	0.152352
8π	25.132741	0.816598
4π	12.566371	0.908299
$32\pi^2$	315.827341	1.304703
$128\pi^3$	3968.803415	27.961758

The nearest, $4\pi^3$, is still **9.4945** per cent away.Dividing by powers of π does not approach an integer either.

	Value	Nearest integer	Relative distance
137.036 divided by π	43.619913	44	0.008638
137.036 divided by π^2	13.884650	14	0.008239
137.036 divided by π^3	4.419621	4	0.104905
137.036 divided by π^4	1.406809	1	0.406809

The smallest is **0.008239** (the zeroth power is excluded as a tautology).

4. Whether a formula in π gives 137 is decided by the tolerance

That finding one close formula means nothing is shown by counting. All 10769 distinct expressions formed from fractions with numerator and denominator at most 40 and powers of π from -4 to 6 were listed, and the hits near 137.035999 counted. The control is the average over forty log-uniform targets of the same magnitude, between 100 and 1000.

Tolerance (relative)	Hits for 137	Control average
0.000262768	0	0.3750
0.001000	2	1.5000
0.005000	10	7.1250
0.010000	16	15.2750
0.050000	81	77.2000

The first tolerance, **0.000262768**, is the distance by which the constant itself departs from the integer 137. There the hits number zero—at the present precision, 137 cannot be written as a fraction times a power of π . Loosen it and they appear—81 at 0.05, for instance $14\pi^2 = 138.174462$, $(27/2)\pi^2 = 133.239659$ and $(9/2)\pi^3 = 139.528245$.

And at every tolerance the count for 137 nearly matches the controls. It is neither easy nor hard to hit.

Widening the form of the expressions does the same thing. Allowing roots of powers of π and enlarging the space, the hits are governed by the size of the space.

Bound on the denominator	Expressions	Hits for 137	Control average
12	2184	0	0.0750
25	9576	0	0.2750
40	23496	0	0.4500
60	52872	1	1.0750

In the widest space one hit appears, but the control average is then **1.0750** and the ratio **0.9302**. Hence the number of hits is governed not by the target but by how many forms of expression are permitted.

5. Computing two formulas from the history

Paper 101 treated as a caution that Eddington derived 136 for the inverse and added to his reasoning once the measurement was known to be near 137. That construction computes: $(16^2 - 16)/2 + 16 = 136$. It reproduces. The present measurement is 137.035999177, not even an integer, and its distance 0.035999 from 137 is far larger than the uncertainty of measurement.

A second formula, Wyler's, was computed: $(9/(8\pi^4)) (\pi^5/(2^4 \cdot 5!))^{1/4} = 0.007297348$, whose inverse is 137.036082. The relative difference from the present value is 6.077×10^{-7} —close indeed. But the inverse is measured to ten to the minus ten, so it is still about four thousand times away. And by Section 4 that closeness is a function of how many forms were permitted.

An error of the author is recorded here. While writing this paper I set down the expression from memory as $(9/(16\pi^3)) (\pi^5/(2^4 \cdot 5!))^{1/4}$. The inverse of that form is 87.239880, a relative **0.363380** away from the true value—not close at all. A caution is not aimed only at others.

6. The decisive point: it moves with energy

The constant does not move with the units. Does it then move with nothing?—It does move. Vacuum polarisation weakens the screening, so the coupling grows when measured at shorter distances.

The one-loop leptonic contribution was computed: $\Delta\alpha_{\text{lep}}(M_Z) = \mathbf{0.031421}$. Its difference from the value in the literature, 0.031498, is **0.000077**, which is consistent for a one-loop approximation.

Where it is measured	Inverse
At low energy	137.035999177
At the mass of the Z boson, leptons only	132.730201
At the mass of the Z boson, hadrons included	128.939785

The hadronic contribution 0.02766 is taken from the literature—only the leptonic part was computed here, and that too at one loop. The conclusion does not depend on it.

The shift from 137.035999177 is **8.096214**, a relative **0.059081**. The movement under the units measured in Section 2 was a relative **2.074** $\times 10^{-16}$. Hence one and the same number is exactly immobile along one axis and moves by nearly six per cent along the other.

7. Census and closing

Closing: the fine-structure constant does not move with the units, but it moves with energy. Scaling mass by 10^7 , length by 10^{-9} and time by 10^3 leaves the spread in its inverse at a relative **2.074** times ten to the minus sixteen (one spacing of double precision), while a one-loop vacuum polarisation gives **128.939785** at the mass of the Z boson, a shift of **8.096214**, or **0.059081** relative. A factor of 4π sits in the formula and leaves no trace—the SI and Gaussian forms agree to a relative **0.000** times ten to the zero, not differing in a single digit of double precision, and removing that factor alone shifts the value by exactly **12.566370614**. The factors of 4π the corpus places do not reach it either—the nearest, $4\pi^3$, is **124.025107**, still 9.4945 per cent away, and the smallest relative distance from dividing by powers of π is **0.008239**. Whether a formula in π gives 137 is decided by the tolerance—of 10769 expressions none falls within the present precision **0.000262768** and 81 do at 0.05, against control averages of **0.3750** and **77.2000**. Wyler's expression is close but is not a derivation—it comes within a relative **6.077** times ten to the minus seven, yet remains about four thousand times farther than the precision of measurement, and widening the space from 2184 to 52872 raises the hits from 0 to 1 while the control average becomes **1.0750**. The separator is whether the number moves when the units change or when the energy changes—the proton to electron mass ratio **1836.152673** moves under neither, the digits of the speed of light move under the units, and this constant moves under energy alone. Plainly—why it is near 137 is not answered here either, the hadronic contribution 0.02766 is taken from the literature, and while writing this paper the author set down Wyler's expression from memory and got it wrong (87.239880, a relative **0.363380** away). What is added—measuring the constant along the two axes of units and of energy, naming the place in this corpus where a factor of 4π enters a formula and leaves no trace, listing the distances from the corpus's factors of 4π to 137, counting how easily a formula in π hits it across tolerances and sizes of space against controls, and showing that the separator of Paper 1055 needs a third box.

Revision Record

2nd edition (September 2026): The fine-structure constant, the vacuum permittivity and the electron and proton masses were changed from the CODATA 2018 recommended values to the CODATA 2022 recommended values (the inverse of the fine-structure constant from **137.035999084** to **137.035999177**). Only the last digits and the rounding changed, and the conclusions of this paper are unchanged: the distance of the constant from 137 (the first tolerance) went from **0.000262767** to **0.000262768**, the relative difference from Wyler's expression from **6.083** $\times 10^{-7}$ to **6.077** $\times 10^{-7}$, while the numbers of hits among the π formulas, the control averages and the running with energy (**128.939785**, a relative **0.059081**) are unchanged. In Section 2 the difference between the SI form and the Gaussian form went from a relative **3.566** $\times 10^{-16}$ in

the first edition to **0**, and the spread under the rescalings in the same section went from **0** to a relative 2.074×10^{-16} — both are rounding of no more than one spacing of double precision.

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