

There Are Two Kinds of Exactness: One Is Set by the Units, the Other Is Unmoved by Continuous Deformation

— The quantum Hall resistance is the product of the two, and the fine-structure constant can be neither, so it runs — the separator is whether a quantity has a dimension, is an integer, or is neither — [Paper 1058]

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— *From thinking to syncing.* —

Abstract

Paper 1056 and Paper 1057 measured that the fine-structure constant does not move with the units but moves with energy, and why it stops at low energy. Why, then, is there a quantity built from the same charge and the same Planck constant that does not move at all? This paper answers that. No new theorem or law is claimed.

First, and this is the core. The word exact is doing two different jobs—one is that a definition of the units has fixed the value, the other is that continuous deformation does not move it. The first is available only to quantities with a dimension, the second happens only to integers (Sections 2 and 5).

Second, a quantity with a dimension can be made exact by definition—the von Klitzing constant is **25812.807459** ohms, its inverse **3.874045865**×10^{−5} siemens, the flux quantum **2.067833848**×10^{−15} weber, all from defined values alone. Moving the fine-structure constant by a relative 1×10^{−6} changes it by **0.000**×10⁰ (Section 2).

Third, the 2019 revision was a trade—the permeability of vacuum is recovered from the fine-structure constant to a relative **5.718**×10^{−12}, and moving that constant by a relative 1×10^{−6} moves it by **1.000**×10^{−6}. The departure from $4\pi\times10^{-7}$ is **5.444**×10^{−10} with the CODATA 2018 recommended value and **1.320**×10^{−10} (**0.825** times the uncertainty) with CODATA 2022, moving, even in sign, each time the recommended values are revised—that is the price of the trade (Section 3).

Fourth, a dimensionless quantity cannot be made exact by definition—across five choices of scale the spread of the inverse fine-structure constant is **0.000**×10⁰, while the control, the von Klitzing constant, has a spread of ratios of **1.000**×10¹⁴ (Section 4).

Fifth, topological exactness is not the fixing of a value—computing the Chern number of a two-band lattice model by plain integration gives **−0.999891028** and the like, and the departure from an integer shrinks as **4.358**×10^{−4}, **1.090**×10^{−4}, **2.724**×10^{−5} for meshes of 200, 400 and 800. Refining by a factor of four shrinks it by **15.996**, so it is numerical error. Meanwhile moving the parameter continuously does not move the integer (Section 5).

Sixth, the fine-structure constant can be exact in neither sense—being dimensionless it cannot be fixed by definition, and not being an integer it is not protected by topology. Its departure from the nearest integer, **0.035999178**, is **1.6419**×10⁶ times its own measurement uncertainty and does not shrink with finer computation (Section 7).

Keywords: fine-structure constant, Chern number, quantum Hall effect, definition of the units, topological invariant, dimensionless quantity, permeability of vacuum.

Scope of this paper (scope note): No new theorem or law is claimed—the quantisation of the Hall resistance, the quantisation of flux, the integrality of the Chern number and the 2019 revision of the units are all standard. It is not claimed that topology froze the value—what froze the value of the von Klitzing constant was the decision of 2019, not topology. What topology freezes is the integer. No index theorem is derived—no proof that the Chern number is an integer is given. Here it is only observed numerically that a plain integral approaches one. No mechanism of the quantum Hall effect is derived—why it quantises, and how edge states or disorder act, are not the subject. The fractional case is not treated—only the integer case is examined. Experimental accuracy is not discussed—how closely real measurements agree is not the subject; only what enters the formula is asked. Why it is 137 is not answered—following the caution of Paper 101 and Paper 1056, no explanation of the value is attempted. Relation to earlier papers: the scope note of Paper 514 says that no index theorem is derived, that the integrality of the invariant belongs to the territory of the paper numbered 70, and that it only borrows—this paper takes that seat (no earlier occupant). The integer side that was only borrowed is computed here, and shown to be exact in a different sense from the unit side. Paper 514 showed that no property of the sample enters the quantised resistance, and listed the values divided by integers—this paper does not produce that list anew but rereads it as a product of two exactnesses. Paper 270 showed that the two in the flux quantum is a count of carriers and that the 2019 revision made its value exact—here what the same revision gave up is added. Paper 10 established that only dimensionless numbers can be fine-tuned—this paper uses the converse: only a quantity with a dimension can be fixed by definition. Paper 1057 measured that in 2019 one factor of 4π ceased to be exact—here that is identified as the price of the trade. Paper 1056 showed that the fine-structure constant does not move with the units but moves with energy—this paper adds, on the side that does not move, the reason it cannot be moved. Paper 1055 divided numbers by whether they move when the units change—here the side that does not move is shown to split further into two. Paper 1049 showed that fixing an independent constant removes exactly one unit slot—the 2019 revision is an instance of that subtraction. Paper 300 showed that whether two things share a root is decidable—following that discipline, the two kinds of exactness are not called one root. What is added: separating the word exact into two different things, writing the 2019 revision as a trade in numbers, confirming by actually moving the parameter that topological exactness is exactness under continuous deformation, showing by mesh convergence that the departure from an integer is numerical error, and writing the separator that the fine-structure constant can be exact in neither sense.

1. Setting and question

Setting: the fine-structure constant α does not move when the units change, but it moves when the energy changes. Paper 1056 measured that, and Paper 1057 measured why it stops at low energy.

Question: there are quantities built from the same charge e and the same Planck constant $\hbar$ that do not move at all—the von Klitzing constant $R_K = \hbar/e^2$ and the flux quantum $\Phi_0 = \hbar/2e$. Why does one run and the other not?

The question came from outside.As it arrived, the explanation offered was that the quantum Hall quantisation is topologically exact.It is a good question, but that explanation has the order wrong——this paper corrects that first.

2. The unit side: a quantity with a dimension can be made exact by definition

Since 2019, e and h are defined values.So any quantity built from e and h alone comes out exactly by computation.

Quantity	Formula	Value
von Klitzing constant	h/e^2	25812.807459 ohms
conductance quantum	e^2/h	3.874045865×10^{-5} siemens
flux quantum	$h/2e$	$2.067833848\times10^{-15}$ weber

None of the three uses α at any point.Control: moving α by a relative 1×10^{-6} changes R_K by 0.000×10^0 ——as it must, since α is not in the formula.

What is doing the work is that these three carry a dimension.Ohm, siemens and weber are derived units whose numerical values move with the choice of scale.A quantity that moves can be made exact by deciding where to move it to.

3. The 2019 revision was a trade

Exactness was not gained for free.In exchange for e and h taking the exact seat, something left it.

Quantity	Before 2019	Now
elementary charge e , Planck constant h	measured	defined (exact)
h/e^2 , $h/2e$	measured	exact
permeability of vacuum μ_0	defined as $4\pi \times 10^{-7}$	measured
fine-structure constant α	measured	measured

That it is a trade can be seen in the formula.

$$\mu_0 = \frac{2\alpha h}{e^2 c}$$

On the right, only α is a measured quantity. Computing gives $1.2566370612628 \times 10^{-6}$, a relative difference of 5.718×10^{-12} from the recommended value. And moving α by a relative 1×10^{-6} moves μ_0 by exactly 1.000×10^{-6} ——the same relative amount.

$\Rightarrow$ the present μ_0 carries the uncertainty of α directly. Paper 1057 measured the departure from $4\pi \times 10^{-7}$: **5.444** $\times 10^{-10}$, or **3.629** times the uncertainty, with the previous recommended value (CODATA 2018), and **1.320** $\times 10^{-10}$, or **0.825** times the uncertainty, with the present one (CODATA 2022); the sign of the departure also reversed——here the fact that this number moves with each revision of the recommended values is named as the price of the trade. Paper 1049 showed that fixing an independent constant removes exactly one unit slot——the 2019 revision is that subtraction happening in practice.

4. A dimensionless quantity cannot be made exact by definition

One might ask why the same could not be done to α . The reason is visible in numbers. Five choices of scale were tried, listing the numerical values of $1/\alpha$ and of R_K .

Scale (mass, length, time)	value of $1/\alpha$	ratio for R_K
1, 1, 1	137.035999178	1.000000×10^0
12.6, 1, 1	137.035999178	7.957747×10^{-2}
1, 12.6, 1	137.035999178	6.332574×10^{-3}
1, 1, 0.0796	137.035999178	7.957747×10^{-2}
$10^7, 10^{-9}, 10^3$	137.035999178	1.000000×10^{14}

The spread of $1/\alpha$ is 0.000×10^0 ; the spread of the ratios for R_K is 1.000×10^{14} .

$\Rightarrow$ a quantity whose value moves with the scale can be made exact by choosing the scale; one that does not move cannot. Paper 10 established that only dimensionless numbers can be fine-tuned——what is used here is the converse: only a quantity with a dimension can be fixed by definition.

5. Topological exactness is a different thing

So far topology has not appeared once. What, then, does it do? A two-band lattice model is computed directly.

$$\mathbf{d}(k) = (\sin k_x, \sin k_y, m + \cos k_x + \cos k_y)$$

The Chern number counts how many times the unit vector $\hat{\mathbf{d}}$ covers the sphere. It was obtained by plain integration rather than by the lattice formula that returns an integer exactly—because approaching an integer is itself the result.

m	Chern number	departure from an integer
3.00	−0.000003809	3.809×10^{-6}
1.00	−0.999891028	1.090×10^{-4}
−1.00	0.999891028	1.090×10^{-4}
−3.00	0.000003809	3.809×10^{-6}

Rounded, these are 0, −1, +1, 0. For $|m| > 2$ it is 0, and reversing the sign of m reverses the sign.

Next, the parameter is moved continuously. This is what topological exactness means.

m	Chern number
0.40	−0.999739219
0.80	−0.999882515
1.20	−0.999884102
1.60	−0.999776813
1.90	−0.998002354

Walking m from 0.4 to 1.9, the spread is only 1.882×10^{-3} and the rounded integer stays at −1 throughout. The value is not fixed anywhere by definition—it does not move under continuous change, and that is what topological exactness is.

What is the remaining departure? The mesh was refined to find out.

Mesh	departure from an integer
200	4.358×10^{-4}
400	1.090×10^{-4}
800	2.724×10^{-5}

Refining by a factor of four shrinks the departure by **15.996**—second-order convergence, so this is numerical error and not a physical gap.

6. The two exactnesses stand side by side in a product

Paper 514 showed that no property of the sample enters the quantised resistance, and listed the values divided by integers—25812.807459 ohms for $\nu = 1$, 12906.403730 for $\nu = 2$, 8604.269153 for $\nu = 3$ and 6453.201865 for $\nu = 4$. That list is not produced anew here but reread.

$$R = \frac{R_K}{\nu}, \quad R_K = \frac{h}{e^2}$$

Two exactnesses of different origin stand in this formula. R_K is exact by the definition of the units; ν is exact by topology. The first was decided by people in 2019. The second does not move under continuous deformation.

Paper 300 showed that whether two things share a root is decidable—these two do not. A separator can be put simply as whether the exactness carries a date—the exactness of R_K carries the year 2019, that of ν carries none. Paper 70 showed that 4π and $32\pi^2$ are normalisations of the index theorem—the proof that ν is an integer lives on that side, and this paper only borrows it.

7. The fine-structure constant can be exact in neither sense

There are only two roads to exactness, and α can take neither.

- (a) The road of definition: α is dimensionless, so its value does not move when the scale is chosen (Section 4). A number that does not move has no destination to be sent to.
- (b) The road of topology: α is not an integer. With $1/\alpha = 137.035999178$, the departure from the nearest integer is **0.035999178**.

That this departure is not a near-integer is checked against numerical error. The departure of the Chern number shrank with the mesh, whereas the departure of α is 1.6419×10^6 times its own measurement uncertainty of 2.193

$\times 10^{-8}$ and does not shrink with finer computation.

Quantity	Dimension	Exact?	Source of exactness
h/e^2	yes	exact	definition of the units (2019)
$h/2e$	yes	exact	definition of the units (2019)
Chern number	none (integer)	exact	topology (unmoved by deformation)
μ_0	yes	measured	ceased to be exact in 2019
$1/\alpha$	none (not an integer)	measured	neither road is open
proton-to-electron mass ratio	none (not an integer)	measured	the same

The last row is a control. The mass ratio 1836.15267343 is also dimensionless and not an integer, so it cannot be fixed by definition, yet it does not run. Paper 1056 showed that α does not move with the units but moves with energy—so being unable to be exact and running are not the same thing.

The proposal that arrived said that topology is doing the freezing. What was measured is the reverse: what froze R_K was the decision of 2019, and what topology freezes is the integer. Even so it was a good question—it is what made visible that two exactnesses share one formula.

Closing

Closing: there are two kinds of exactness, one set by the units and one unmoved by continuous deformation. A quantity with a dimension can be made exact by definition—the von Klitzing constant is 25812.807459 ohms and the flux quantum $2.067833848 \times 10^{-15}$ weber, both from defined values alone. The 2019 revision was a trade—the permeability of vacuum is recovered from the fine-structure constant to a relative 5.718×10^{-12} , moving by 1.000×10^{-6} when that constant moves by a relative 1×10^{-6} , and the departure from $4\pi \times 10^{-7}$, which moved with the revision of the recommended values from 5.444×10^{-10} (CODATA 2018) to 1.320×10^{-10} (CODATA 2022, 0.825 times the uncertainty), is the price. A dimensionless quantity cannot be made exact by definition—across five scales the spread of the inverse fine-structure constant is 0.000×10^0 against a control spread of 1.000×10^{14} . Topological exactness is not the fixing of a value—the Chern number comes out as -0.999891028 and the like, but the departure is numerical error shrinking from 4.358×10^{-4} to 2.724×10^{-5} by a factor of 15.996, while continuous motion of the parameter leaves the integer unmoved (spread 1.882×10^{-3}). The measured quantum Hall resistance is the product of the two—the list divided by integers that Paper 514 produced is reread as a product of exactness by definition and exactness by topology. The fine-structure constant can be neither—dimensionless, so not definable,

and with a departure from an integer of **0.035999178** that is **1.6419**×10⁶ times its own uncertainty, so not protected. Stated plainly—the proposal that arrived had the order reversed, but it was the proposal that made visible that two exactnesses share one formula. The separator—whether the quantity has a dimension, is an integer, or is neither. With a dimension it can be fixed by definition, as an integer by topology, and a quantity that is neither can take no road at all.

Revision Record

2nd edition (September 2026): The fine-structure constant was changed from the CODATA 2018 recommended value **7.2973525693** × 10^{−3} to the CODATA 2022 recommended value **7.2973525643** × 10^{−3} (its computed inverse is **137.035999178**; that it differs in the last digit from the recommended inverse **137.035999177** is due only to the rounding of the recommended value; the relative uncertainty is **1.6** × 10^{−10}). With this, the relative difference in recovering the permeability of vacuum from the fine-structure constant became **5.718** × 10^{−12}, and the departure from the nearest integer became **1.6419** × 10⁶ times its own uncertainty (the first edition had **1.7513** × 10⁶ times). For the departure from **4π** × 10^{−7} the first edition quoted the number of Paper 1057 (with CODATA 2018, **5.444** × 10^{−10}, or **3.629** times the uncertainty); with CODATA 2022 it is **1.320** × 10^{−10}, or **0.825** times the uncertainty, inside the range, and its sign reverses. The reading in Section 3 and in the closing was therefore changed from “the size of the departure is the price” to “since the definition was given up, the price is that the value moves each time the recommended values are revised”, and the two values are set side by side. The numbers and conclusions of the other sections are unchanged.

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