What Decides the Sign of the Running Is a Single Thing: Whether the Group Is Abelian

— Electromagnetism freezes at low energy while the strong force diverges — the separator is whether the adjoint Casimir is zero or positive — [Paper 1059]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
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— From thinking to syncing. —

Abstract

Paper 1057 measured why the electromagnetic coupling freezes at low energy, and stated that only three leptons were treated.This paper measures what changes when quarks and gluons are put in.No new theorem or law is claimed.

First, and this is the core. What reverses the sign is a single term——the one-loop coefficient is a difference of two terms, and the first carries the adjoint Casimir, which is exactly zero for an abelian group. Changing that value from 0 to 3 without altering a single particle count turns the coefficient from -4.000000 into $+7.000000$(Sections 2 and 3).

Second, the threshold where the sign changes is a number of quark species——it is 11N/2, which for SU(3) is exactly 16.5. With 16 species the coefficient is $+0.333333$, with 17 it is -0.333333, and nature's 6 lies below(Section 3).

Third, over the same span of energy the two couplings move quite differently, and in opposite directions——the electromagnetic coupling moves from an inverse of 137.035999177 to 128.939785, a ratio of only 1.062790660, while the strong coupling differs by 2.859843602 between 1 GeV and the Z boson mass. And electromagnetism grows at the high end while the strong force weakens there(Section 5).

Fourth, the way each ends at low energy is also opposite——electromagnetism stops because the lightest charged mass sets a floor, whereas the inverse strong coupling falls as 2.960791, 2.115022, 0.996975, 0.373672 towards zero and the formula itself ceases to apply. One has its value stop, the other has its formula break(Section 6).

Fifth, the effect of heavy particles dropping out is present in both, but what remains afterwards is opposite——each species that drops out raises the coefficient by exactly 0.666666667, so the running gets faster and the strong coupling at 1 GeV rises from 0.290729 to 0.499221(Section 7).

Sixth, this is confirmed to be the same formula written in Paper 05——the two ways of writing it agree for species 3 through 6, and the one-loop value of 88 MeV that Paper 05 reported is reproduced independently here as 88.3451 MeV. That value is Paper 05's result, not a new one(Section 4).

Keywords: asymptotic freedom, beta function, adjoint Casimir, running of couplings, decoupling, non-abelian gauge theory.

Scope of this paper (scope note): **No new theorem or law is claimed——the form of the one-loop coefficient of the running, asymptotic freedom, screening and antiscreening, and the decoupling of heavy fields at low energy are all standard.Confinement is not derived——a coupling growing large and quarks being unextractable are separate claims. The latter requires lattice computation and is not entered here.Chiral symmetry breaking is not treated——the origin of hadron masses is not the subject.Nothing beyond one loop is treated——as Paper 29 noted, these coefficients do receive higher corrections. Every number here is a one-loop approximation.No new value for the strong interaction is produced——the scale at which the formula diverges was given species by species in Paper 05. This paper only reproduces it independently and checks the agreement.It is not claimed that the strong force fails to freeze——it is not that it fails to end, but that it ends differently.No lattice result is borrowed——only the one-loop formula and two inputs taken from the literature are used.Relation to earlier papers**: **the scope note of Paper 1057 says that hadrons and bound states are not included and that only three leptons are treated——this paper takes that seat (no earlier occupant). Putting quarks and gluons in reverses the sign.Paper 05 treated dimensional transmutation, wrote the one-loop coefficient in terms of N and the number of species, and gave the divergence scale species by species——this paper cites that formula and those values, and adds the side of the sign.Paper 29 wrote that these coefficients receive higher corrections——the one loop used here sits inside that caution.Paper 1056 showed that the fine-structure constant does not move with the units but moves with energy——here that motion is placed beside the motion of another coupling.Paper 1058 showed that the word exact does two different jobs——neither of these couplings is exact in either sense.Paper 110 showed that Ricci flow is a renormalization group flow——there geometry is what runs, here it is the coupling.Paper 300 showed that whether two things share a root is decidable——following that discipline, the two runnings are not called one root.**What is added: **showing, without changing any particle count, that the sign is reversed by the Casimir term alone**, **confirming on both sides that the threshold is a species count of 11N/2**, **placing the two couplings side by side over the same span and showing in numbers that the directions are opposite**, **showing that the low-energy endings split into stopping and breaking**, **and writing that the decoupling effect is present in both while what remains is opposite**.

1. Setting and question

Setting: **Paper 1057 measured why the electromagnetic coupling stops running at low energy.What stops it is the mass of the lightest charged particle.Its scope note stated that only three leptons were treated.**

Question: **what happens when quarks and gluons are put in?The strong coupling is known to weaken at high energy, the opposite of electromagnetism.Where does that difference of sign come from?**

Only that one point is measured here.Neither confinement nor hadron masses are treated——only where the sign is decided.

2. The one-loop coefficient is a difference of two terms

The one-loop coefficient of the beta function can be written as follows.

$$\beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_F n_f$$

Here C_A is the adjoint Casimir, equal to N for SU(N) and exactly 0 for an abelian group.n_f counts the species of charged matter.A positive β_0 means weakening at high energy, a negative one means strengthening.

Group	Casimir	Species	Coefficient	At high energy
U(1) (electromagnetism)	0.0	3.0	−4.000000	grows
SU(2)	2.0	6.0	3.333333	weakens
SU(3) (strong force)	3.0	6.0	7.000000	weakens
SU(3) (5 species)	3.0	5.0	7.666667	weakens
SU(3) (3 species)	3.0	3.0	9.000000	weakens

For the abelian group it is -4.000000, for SU(3) it is $+7.000000$.Control: setting the species count to 0 still does not make the abelian coefficient positive——it stays at 0, because the first term was never there.

3. What reverses the sign is the Casimir term alone

Which of the two terms is responsible is settled by moving them.Keeping the particle counts fixed, the Casimir alone is changed from 0 to 3.

At 6 species: from -4.000000 to $+7.000000$, the sign turning over.With the electromagnetic particle count (3 species) the same thing happens, from -4.000000 to $+7.000000$.

⇒ what decides the sign is not the number of particles but the Casimir.And the Casimir is zero only when the group is abelian.

Can the sign then be restored by adding particles?Solving for the threshold gives $n_f = 11N/2$.

Group	Threshold species	Species in nature
SU(2)	11.0	—
SU(3)	16.5	6
SU(4)	22.0	—

Both sides of the threshold were checked.At 16 species the coefficient is $+0.333333$, at 16.5 exactly 0, at 17 it is -0.333333.Nature's 6 lies below the threshold, and that is why the strong force weakens at high energy.

4. It is the same formula that Paper 05 wrote

Paper 05 treated dimensional transmutation, wrote the one-loop coefficient as $(11N - 2n_f)/3$, and gave the divergence scale species by species——246 MeV for 3 species, 153 for 4, and 88 for 5.

Whether that writing and the present one are the same formula was checked.For species 3 through 6 the difference is within 10^{-12}.

The divergence scale was also computed independently.One loop at 5 species gives 88.3451 MeV——in agreement with the 88 MeV that Paper 05 reported.As a check, inserting that value into the formula makes the inverse exactly 0 .

⇒ this value is Paper 05's result, not a new one produced here.What is cited is the formula and the value; what is added is the side of the sign.As Paper 29 wrote, these coefficients receive higher corrections——every number here is a one-loop approximation.

5. Over the same span, both the size and the direction differ

The two couplings are placed side by side over the same span of energy.

Q (GeV)	Strong coupling (one loop)	
91.1876	0.118100000	Z boson mass
10.0000	0.173298875	
5.0000	0.203061843	
2.0000	0.262704297	
1.0000	0.337747529	

Between 1 GeV and the Z boson mass the strong coupling differs by 2.859843602.Paper 1056 gave the electromagnetic side, where the inverse moves from 137.035999177 to 128.939785——a ratio of 1.062790660 in the coupling itself, a relative motion of 0.059080929.

The ratio of the ratios is 2.6909.And the directions are opposite——electromagnetism grows at the high end, the strong force weakens there.

6. The way each ends at low energy is also opposite

Paper 1057 measured that the electromagnetic screening falls as Q squared at low energy and stops.What stops it is the mass of the lightest charged particle.The same place is examined on the strong side.

Q (GeV)	Strong coupling	Its inverse
1.000	0.337748	2.960791
0.500	0.472808	2.115022
0.200	1.003034	0.996975
0.120	2.676147	0.373672

The inverse falls monotonically towards zero as 2.960791, 2.115022, 0.996975, 0.373672.Where it reaches zero the coupling diverges and the formula ceases to apply.

⇒ electromagnetism has its value stop; the strong force has its formula break.Both can be described as running ending at low energy, but the endings are opposite.Paper 300 showed that whether two things share a root is decidable——these two are the same one-loop formula with opposite signs, yet as endings they are different things.

What can be said here is only that the one-loop formula ceases to apply.That quarks cannot be extracted is a separate claim, and it is not derived here.

7. Decoupling is present in both, but what remains is opposite

As the energy is lowered, heavy quarks cease to act one after another.The coefficient changes accordingly.

Species	Coefficient	Strong coupling at 1 GeV
6	7.000000	0.290729
5	7.666667	0.337748
4	8.333333	0.402908
3	9.000000	0.499221

Each species that drops out raises the coefficient by exactly 0.666666667.The larger it gets the faster the running, and the coupling at 1 GeV rises from 0.290729 to 0.499221.

Paper 1057 measured the same thing for electromagnetism——there is no charged particle lighter than the electron, so the screening stops there.The mechanism is shared.

But the outcome is opposite.In electromagnetism, when the partners are gone the running stops.In the strong force, when the partners are gone the running speeds up——because the first term remains.

8. The separator: abelian or non-abelian

	Casimir	At low energy	At high energy
Electromagnetism (abelian)	0.0	freezes	grows slowly
Strong force (non-abelian)	3.0	diverges	weakens

The difference between the two reduces to the first column.With a Casimir of zero only screening is left; with a positive one antiscreening wins.And the Casimir is zero only when the group is abelian.

Paper 1058 showed that the word exact does two different jobs——neither of these couplings is either.They cannot be fixed by definition, they are not integers, and they move.Paper 110 showed that Ricci flow is a renormalization group flow——there geometry is what runs, here it is the coupling.

Closing

Closing: what decides the sign of the running is a single thing, whether the group is abelian.What reverses the sign is the Casimir term alone——changing that value from 0 to 3 without altering a single particle count turns the coefficient from -4.000000 into $+7.000000$.The threshold is a number of quark species——11N/2, exactly 16.5 for SU(3). At 16 species it is $+0.333333$, at 17 it is -0.333333, and nature's 6 lies below.Over the same span both the size and the direction differ——the electromagnetic ratio is only 1.062790660 (a relative motion of 0.059080929) while the strong coupling differs by 2.859843602, and it weakens at the high end.The low-energy endings are opposite too——electromagnetism has a mass set a floor and its value stops; the strong coupling has its inverse fall to 0.373672 towards zero and its formula break.Decoupling is present in both——each species dropping out raises the coefficient by 0.666666667 and the coupling at 1 GeV rises from 0.290729 to 0.499221. The mechanism is shared, what remains is opposite.It is the same formula Paper 05 wrote——agreeing in four cases, and the 88 MeV Paper 05 reported is reproduced as 88.3451 MeV. That value is Paper 05's result; what is added here is the side of the sign.Stated plainly——everything here is a one-loop approximation, and neither confinement nor hadron masses are derived. All that can be said is where the sign is decided.The separator——whether the adjoint Casimir is zero or positive. Zero leaves only screening and freezes at low energy; positive lets antiscreening win and diverges at low energy.

Revision Record

2nd edition (September 2026): The fine-structure constant on the electromagnetic side was changed from the CODATA 2018 recommended value to the CODATA 2022 recommended value (its inverse from 137.035999084 to 137.035999177, the same value as in the second edition of Paper 1056). Only the last digit of the electromagnetic coupling ratio changed (from 1.062790659 to 1.062790660 times); the conclusion that the sign of the running is fixed by whether the group is commutative, and the other numbers, are unchanged.

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Yuuki YAMAGISHI (山岸 勇輝)

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