# In a System with Only One Scale Factor, the Ratio Is Determined but the Position Is Not

— The hierarchy is spaced by a fixed ratio, but where it begins is not fixed by the equation — the separator is whether the ratio moves with a substitution rule or with a coefficient — [Paper 1060]

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*— From thinking to syncing. —*

**Abstract**

**Paper 1053 showed that weakening the requirement of scale invariance to a single ratio loses the uniqueness of the exponent, and that what becomes unique instead is the ratio.Its scope note, however, said that it does not claim log-periodic factors are seen in real systems, leaving the physical side open.This paper places one example there.No new theorem or law is claimed. First, the scale factor of a physical system with only one of them was actually obtained ——bisecting the transcendental equation that holds for three equal masses in the zero-range limit, after printing the values to find where the sign changes, gives 1.006237825103, with a residual of 4.441×10⁻¹⁶. The ratio of lengths is 22.694382595 and the ratio of energies is its square, 515.035001**(Section 2). **Second, and this is the core. Only the ratio is determined; the absolute position of the hierarchy is not——changing the phase moves the rungs by as much as 7.300000000, while the spread of neighbouring ratios is 0.000×10⁰. Fixing the position requires one more input from outside this equation**(Section 3). **Third, periodicity holds only inside the logarithm ——multiplying the length by the square of the ratio returns the value exactly (difference 0.000×10⁰), while the first power departs by 2.000000000. There is no symmetry under continuous dilation, only under one factor**(Section 4). **Fourth, what sets the ratio is the coefficient of that equation——moving the coefficient from 4.000000 to 6.000000 makes the length ratio fall monotonically through 33.006464, 22.694383, 19.129043 and 13.814519**(Section 5). **Fifth, this ratio and the ratio of Paper 1053 do not share a root——after reproducing independently the 2.618033989 that 1053 obtained from a substitution matrix, changing that matrix moves only the tiling ratio, to 1.618033989, leaving the three-body ratio untouched; changing the coefficient moves only the three-body ratio, to 13.814519, leaving the tiling ratio untouched. Each is blind in the direction the other sees**(Section 6). **Sixth, stated plainly. The rungs run out quickly——the ratio is large, so even a span of a million in length admits only 4.425045 rungs. An infinite hierarchy belongs to the ideal limit**(Section 7). **Keywords**: discrete scale invariance, log-periodicity, three-body bound states, transcendental equation, Perron-Frobenius, hierarchy.

**Scope of this paper (scope note)**: **No new theorem or law is claimed——that discrete scale invariance permits a log-periodic factor, that a three-body hierarchy is spaced by a fixed ratio, and that the largest eigenvalue of a non-negative matrix is unique are all known.The transcendental equation is not derived here——its form is borrowed from the literature as it stands. What this paper does is solve it, obtain the ratio, and measure what moves that ratio.No experimental value is used——how many rungs have actually been seen is not treated.The unequal-mass case is not entered——the equation is different there, so only equal masses are examined.It is not claimed that the hierarchy is infinite——that belongs to the ideal limit, and as counted in Section 7 the rungs run out quickly over any real span.The binding mechanism is not derived——why three bodies bind at all is not the subject; only the structure of the ratio is examined.Paper 1053 is not called mistaken——it stated itself that it is confined to a mathematical fact, and this paper only places one example outside that.Relation to earlier papers**: **the scope note of Paper 1053 says that it does not claim log-periodic factors are seen in real systems, and that what it shows is only the mathematical fact that a discrete requirement fails to narrow the solutions——this paper takes that seat (no earlier occupant). Where the claim was confined to mathematics, one physical system with only a single scale factor is placed.Paper 1053 showed that on the discrete side what becomes unique is the ratio rather than the exponent——this paper confirms that the ratio plays the same role in a physical system, and adds what fails to be determined.Paper 117 divided into four the ways in which requiring scale invariance fixes an exponent——all four presuppose continuous dilation, and the present system lies outside that premise.Paper 269 showed that what a crystal forbids is not the symmetry but the periodicity——the golden ratio appearing here is the same number as there, in a different role.Paper 1052 showed that two similar forms are separated by what each responds to——the same test is applied here, and the separation is between a substitution rule and a coefficient.Paper 300 showed that whether two things share a root is decidable——following that criterion, the two ratios are not called one root.**What is added: **obtaining, for a physical system with only one scale factor, that factor by actually solving the equation, showing in numbers, by changing the phase, that only the ratio is determined and the position is not, confirming in both directions that periodicity holds only inside the logarithm, showing by varying the coefficient that it is the coefficient that sets the ratio, and confirming with four cases that the ratio of Paper 1053 and the present ratio are blind in opposite directions.**

## 1. Setting and question

**Setting**: **Paper 1053 showed that weakening the requirement of scale invariance to a single ratio makes the exponent of a power law non-unique.What becomes unique instead is the ratio.**

**Its scope note, however, made a reservation——it does not claim that log-periodic factors are seen in real systems, and what it shows is only the mathematical fact that a discrete requirement fails to narrow the solutions.**

**Question**: **in a physical system that has only one scale factor, what is determined and what is not?This paper places one such system there and counts.**

## 2. Solving the transcendental equation

**Consider three bodies of equal mass in the limit of zero interaction range.The equation that holds there is written in the literature in the following form.**

$$s\cosh\left(\frac{\pi s}{2}\right)=\frac{8}{\sqrt{3}}\sinh\left(\frac{\pi s}{6}\right)$$

**This form is not derived here.It is borrowed from the literature as it stands, and the work begins from solving it.**

**The values were first printed to find where the sign changes.At $s=1.00$ the difference is $-0.021248326$, at $s=1.20$ it is $+0.945905724$.Bisecting that interval gives $\mathbf{1.006237825103}$.Inserting it back leaves a residual of $\mathbf{4.441}\times10^{-16}$.**

**Control: $s=0$ also satisfies the equation, but it is the trivial solution and lies outside the interval searched.**

**Two ratios follow from this solution.**

| Quantity | Expression | Value |
|---|---|---|
| Exponent | solution of the equation above | 1.006237825103 |
| Ratio of lengths | exp(π/s) | 22.694382595 |
| Ratio of energies | exp(2π/s) | 515.035001 |

**The ratio of energies is exactly the square of the ratio of lengths (relative $\mathbf{0.000}\times10^{0}$).**

## 3. Only the ratio is determined, not the position

**The position of the rungs is set by a phase inside the logarithm.That phase does not come out of the equation above.Three phases were tried and the rungs listed.**

| Phase | Rungs (lengths, first four) |
|---|---|
| 1.000 | 1.0000／22.6944／515.0350／11688.4014 |
| 2.000 | 2.0000／45.3888／1030.0700／23376.8027 |
| 7.300 | 7.3000／165.6690／3759.7555／85325.3300 |

**The positions move by as much as $\mathbf{7.300000000}$.Yet the neighbouring ratios are identical in all nine cases, with a spread of $\mathbf{0.000}\times10^{0}$.**

**⇒ what the equation fixes is the ratio, not the position.Fixing the position requires one input from outside this equation.Paper 1053 showed that on the discrete side what is unique is the ratio——here the other side is counted, namely what fails to become unique.**

## 4. Periodicity holds only inside the logarithm

**Writing the content of the phase as a function gives something that oscillates in the logarithm of the length.The length was multiplied repeatedly to see whether the value returns.**

| Length | Value of the phase |
|---|---|
| 1.000000 | 1.000000000000 |
| 515.035001 | 1.000000000000 |
| 265261.052652 | 1.000000000000 |
| 136618726.619734 | 1.000000000000 |

**Multiplying by the square of the ratio returns the value exactly (difference $0.000\times10^{0}$).Control: multiplying by the first power does not return it, departing by $2.000000000$.**

**⇒ there is no symmetry under continuous dilation, only under one factor.Paper 117 divided into four the ways in which requiring scale invariance fixes an exponent——all four presuppose continuous dilation, and this system lies outside that premise.**

## 5. What sets the ratio is the coefficient of the equation

**Where, then, does this ratio come from?The coefficient of the equation was varied to see how the ratio moves.**

| Coefficient | Exponent | Ratio of lengths |
|---|---|---|
| 4.000000 | 0.898444130 | 33.006464 |
| 4.618802 | 1.006237825 | 22.694383 |
| 5.000000 | 1.064510839 | 19.129043 |
| 6.000000 | 1.196468970 | 13.814519 |

**The larger the coefficient, the smaller the ratio, with a spread of 19.191945.The second row, 4.618802, is the coefficient of the equation in the literature.**

**⇒ what sets the ratio is the coefficient of this equation.That single line is what the next section needs.**

## 6. This ratio and the ratio of Paper 1053 do not share a root

**Paper 1053 obtained the ratio of tile counts from the eigenvalue of a substitution matrix——2.618033989 .Computing it independently here gives 2.618033989, in agreement.**

**Both ratios come from systems that have only one scale factor.Do they share a root?The criterion of Paper 300 is to see what moves each.**

| What is moved | Tiling ratio | Three-body ratio |
|---|---|---|
| (original values) | 2.618033989 | 22.694383 |
| the substitution matrix | 1.618033989 | 22.694383 |
| the coefficient of the equation | 2.618033989 | 13.814519 |

**Changing the substitution matrix moves only the tiling ratio, to 1.618033989, and the three-body ratio does not move at all.Changing the coefficient moves only the three-body ratio, to 13.814519, and the tiling ratio does not move at all.**

**⇒ each is blind in the direction the other sees.By the criterion of Paper 300 they do not share a root.Paper 1052 showed that two similar forms are separated by what each responds to——the same test applies here, and the separation is between a substitution rule and a coefficient.Paper 269 showed that what a crystal forbids is not the symmetry but the periodicity, and the golden ratio appeared there——the same number appears here in the different role of a ratio of tile counts.**

## 7. The rungs run out quickly

**One thing should be stated plainly at the end.Because the ratio is large, the hierarchy runs out quickly.**

| Span in length | Rungs that fit |
|---|---|
| 100 | 1.475015 |
| 1000 | 2.212523 |
| 1000000 | 4.425045 |

**Even a span of a million in length admits only 4.425045 rungs.An infinite hierarchy can be spoken of only in the ideal limit; once a span is fixed, the number of rungs is finite and determined.**

**How many have been seen experimentally is not treated here.What is counted is only the arithmetic that a large ratio leaves little room.**

## 8. The separator: what is determined and what is not

| | Determined | Not determined |
|---|---|---|
| Continuous dilation | the exponent | (the exponent is unique) |
| One factor (mathematical side) | the ratio | the exponent |
| One factor (present system) | the ratio | the absolute position |

**The third row is what this paper adds.The equation fixes the ratio but not the position——that requires another input.**

**Closing**

**Closing: in a system with only one scale factor, the ratio is determined and the position is not.The factor comes out of the equation——borrowing the form from the literature and solving it gives an exponent of 1.006237825103 (residual $4.441\times10^{-16}$), a ratio of lengths of 22.694382595 and a ratio of energies equal to its square, 515.035001 .But the position is not determined——changing the phase moves the rungs by as much as 7.300000000 while the spread of neighbouring ratios stays at $0.000\times10^{0}$.Symmetry holds only under one factor——multiplying by the square of the ratio returns the value with a difference of $0.000\times10^{0}$, while the first power departs by 2.000000000 .What sets the ratio is the coefficient of the equation——varying it moves the length ratio from 33.006464 to 13.814519, a spread of 19.191945.It does not share a root with the ratio of Paper 1053——changing the substitution matrix moves only the tiling ratio, to 1.618033989; changing the coefficient moves only the three-body ratio, to 13.814519. Each is blind where the other sees.Stated plainly——the rungs run out quickly: a span of a**

**million admits only 4.425045. An infinite hierarchy belongs to the ideal limit, and neither the derivation of the equation nor any experimental value is treated here.The separator——whether the ratio moves with a substitution rule or with the coefficient of an equation. Moving the former leaves the latter's ratio untouched, and moving the latter leaves the former's untouched.**

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