

The Answer to "Why 137" Cannot Take Two of the Four Roots

— Topology yields only integers and dimension never touches a dimensionless quantity — the separator is
whether the gap shrinks under refinement — [Paper 1061]

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— *From thinking to syncing.* —

Abstract

Paper 1057 showed how the reciprocal of the fine-structure constant freezes at low energy, and measured that what freezes it is not a number but a single mass. Its scope note then said that it does not answer why the value is 137, that the two are different questions, and that answering the former leaves the latter standing. This paper adds one thing to what was left standing—not the value, but which shapes an answer could take, tested against the four roots of Paper 86. No new theorem or law is claimed.

First, what "close to 137" measures is separated out in advance—under the ruler that asks whether the leading digits are round it is **0.370359992**, and under the ruler that asks whether the value itself is near an integer it is **0.000262768**; the two differ by a factor of **1409.5**, and only the latter is used (Section 2)。

Second, the choice of units cannot produce it—across four unit systems the value of the speed of light spreads by **1.000×10^4** , while the reciprocal of the fine-structure constant spreads by **0.000×10^6** . The mechanism Paper 1055 found for the speed of light, a roundness made by mixed bases, is unavailable here (Section 3)。

Third, three human choices nevertheless do have an effect—writing the reciprocal brings it **161.6597** times nearer an integer, 137 is prime so it is round in none of the bases from 2 to 20, and among seven dimensionless quantities the reciprocal ranks third, with one of them **10.0211** times nearer an integer (Section 4)。

Fourth, this nearness to an integer is not rare—the probability that a number of the same magnitude lies this near an integer is **0.205970** over 200000 draws and **0.205413** in closed form, about one time in five (Section 5)。

Fifth, and this is one core of the paper. The topological root cannot be taken—topology yields integers, yet the reciprocal stands **1.6419×10^6** times its own uncertainty away from 137. The Dirac condition constrains a product and not one factor, and varying the partner over four values moves the charge with a spread of **13.6000** (Section 6)。

Sixth, the dimensional root cannot be taken either—the subtraction of Paper 1049 removes one unit frame for each independent constant fixed, but the dimension vector of the fine-structure constant is zero, so however many frames are removed it is never touched (Section 7)。

Seventh, the remaining two do not fall. Structure can produce a non-integer real number—from the superstable points of period doubling a constant **4.668954** was obtained that the calculation does not carry as input (difference **0.000248** from the literature value 4.669201609). Yet no rational with denominator up to 100 reaches the precision **1.6×10^{-10}** (the closest is relative **1.061×10^{-6}**), so whether the algebraic root can be taken is undecided (Section 8)。

Keywords: fine-structure constant, roots of necessity, topological invariant, dimensional analysis, Dirac quantization condition, period doubling, numerology.

Scope of this paper (scope note): No new theorem or law is claimed—that topological invariants are integers, that the Dirac quantization condition constrains a product, that dimensionless quantities do not depend on the unit system, and that a universal constant appears in period doubling are all standard. The value is not derived—no attempt is made to produce the number 137 itself. Paper 1056 measured 10769 formulas and found 0 hits at this precision, and that measurement is taken as given. No fifth root is named—that two of the four fall is shown, but the remaining two are left undecided. No new root is given a name and no count is fixed. Unification values are not used—the claim that three couplings meet at one point at high energy follows from an added premise, and is not treated here. No window for the existence of observers is used—the ranges within which atoms or chemistry are said to be possible are not cited, because their sources have not been checked. The period-doubling constant and the fine-structure constant are not called one root—Section 8 shows only that an exit exists by which structure produces a non-integer real number. No mechanism linking the two is measured. It is not claimed that the symmetry and algebraic roots can or cannot be taken—what each would require is written, but no verdict is given. The measured values are not questioned—the reciprocal and its uncertainty are received from outside. Relation to earlier papers: the scope note of Paper 1057 says that it does not answer why the value is 137, that it answers only why the value freezes, and that the two are different questions—this paper takes that seat (no earlier occupant). It does not produce the value, but measures which root an answer could come from. Paper 86 divided the reasons a value must be what it is into four roots—this paper applies those four to a single dimensionless quantity and shows in numbers that two of them fall. Paper 1050 counted that four are not enough and six are not enough either, and its scope note said that no new root would be named and no count fixed—this paper keeps that limit. It closes two of the four and raises no fifth. Paper 1055 showed that numbers separate by whether they move when units are changed—this paper adds, for a number on the unmoving side, that the base does not matter either. Paper 1056 showed that the fine-structure constant does not move with units but moves with energy, and measured that hitting 137 with a formula in the circle constant is governed by the tolerance—this paper receives that measurement and does not pursue the direction of hitting the value. Paper 1058 showed that exactness comes in two kinds, one set by units and one that continuous deformation cannot move—the separator of this paper is taken from there: whether the gap shrinks under refinement. Paper 1049 counted that fixing one independent constant removes exactly one unit frame—this paper uses the fact that the subtraction never touches a dimensionless quantity as the ground on which the dimensional root falls. Paper 320 treated the period-doubling constant—this paper re-derives it from superstable points and cites it as a precedent for structure producing a non-integer real number. Paper 1021 showed that continued-fraction statistics do not distinguish transcendental from algebraic numbers—it is cited here in the form that the algebraic root cannot be judged from the appearance of the number. Paper 101 treated the numerology of Eddington as a caution—Sections 4 and 5 of this paper continue that caution. Paper 300 showed that whether two things share a root is decidable—following that discipline, the period-doubling constant and the fine-structure constant are not called one root. What is added: separating the two different rulers that "close to 137" had been mixing, narrowing to three the places where a human choice has an effect and showing each in numbers, showing that the topological root falls, both by the multiple of the

uncertainty and by the Dirac condition, showing that the dimensional root falls by the dimension vector, and leaving the remaining two undecided while writing what each would require.

1. Setting and question

Paper 1057 measured how the reciprocal of the fine-structure constant freezes at low energy. What freezes it is not a number but a single mass, and it stops because the fall is quadratic rather than logarithmic. Its scope note then said the following.

It does not answer why the value is 137—it answers only why the value freezes. The two are different questions, and answering the former leaves the latter standing.

This paper adds one thing to what was left standing. It does not derive the value. Paper 1056 measured 10769 formulas built from the circle constant and found 0 hits at this precision. That direction is closed.

What is asked instead is which shapes an answer could take. Paper 86 divided the reasons a value must be what it is into four roots—topology, symmetry, algebra, dimension. Those four are applied here to a single dimensionless quantity.

2. What "close to 137" measures

The rulers are separated first, because mixing them makes the argument impossible.

Ruler ① asks whether the leading digits are round. It is the one Paper 1055 used for the speed of light, measuring how near the leading part 2.997925 of 2.997925×10^8 is to the integer 3.

Ruler ② asks whether the value itself is near an integer. That is how near 137.035999177 is to 137.

Ruler	Value for the reciprocal	What it looks at
① leading digits	0.370359992	the leading part 1.370360 is far from 1
② nearness to an integer	0.000262768	the value 137.035999 is near 137

The two differ by a factor of **1409.5**. What is meant by "close to 137" is ②, and only that one is used below.

⚠ While preparing this paper the writer once confused the two—applying the ruler used for the speed of light and producing the irrelevant number 0.370359992. The habit of naming the ruler is needed.

3. The choice of units cannot produce it

Paper 1055 showed that the speed of light is near 3 hundred million because length is decimal and time is sexagesimal.A mixture of bases chosen by people makes that digit string.Can the same mechanism be used here?
It cannot.Four unit systems are applied.

Quantity	Spread across four unit systems
speed of light	1.000 ×10 ⁴
reciprocal of the fine-structure constant	0.000 ×10 ⁰

The speed of light moves.Switching to decimal time alone makes the roundness of its leading digits **197.450715** times worse.The reciprocal does not move, because it is dimensionless.

⇒ The line that a human choice of units produced 137 falls here.

4. Three places where a human choice does have an effect

Units have no effect. Three other choices do.

The first is the habit of writing the reciprocal.

Written as	Value	Nearest integer	Ruler ②
directly (×1000)	7.297353	7	0.042478938
reciprocal	137.035999177	137	0.000262768

Writing the reciprocal brings it **161.6597** times nearer an integer.The face "137" appears only under that convention.

The second is the base—and this one has no effect.Since 137 is prime, in none of the bases from 2 to 20 does it come out with all digits after the leading one equal to 0 (0 hits).And nearness to an integer is itself independent of the base.

⇒ The roundness of the speed of light was made by the base; here the base is irrelevant.The two part company at this point.

The third is which combination is made the protagonist.

Dimensionless quantity	Value	Nearest integer	Ruler ②
reciprocal × twice the circle constant	861.022576584	861	0.000026221
proton-to-electron mass ratio	1836.152673430	1836	0.000083155
reciprocal of the fine-structure constant	137.035999177	137	0.000262768
muon-to-electron mass ratio	206.768283000	207	0.001119406

Among seven, the reciprocal ranks third.The first is **10.0211** times nearer an integer.

⇒ "Close to 137" is not even the best coincidence at hand.Which one is made the protagonist is a human choice.

5. This nearness is not rare

The control that Paper 1056 applied to formulas is applied here.How probable is it that a number of the same magnitude (from 100 to 1000) lies this near an integer?

Method	Probability
200000 random draws	0.205970
closed form	0.205413

The two agree (difference 0.000557).It is about one time in five. Not rare.

6. The topological root cannot be taken

Here begins the core.The four roots of Paper 86 are applied in turn.

What topology yields is an integer.Winding numbers, Chern numbers, the Dirac integer——none of them can take a fractional value.Is the reciprocal an integer?

Quantity	Value
gap from 137 (absolute)	0.035999177
its own uncertainty (absolute)	2.193 $\times 10^{-8}$
the gap is, in units of that uncertainty	1.6419 $\times 10^6$

It is not an integer.It stands 175 ten-thousand times its measurement precision away.A mechanism that yields only integers cannot yield a number that is not one.

One further route that is often raised is also examined.The Dirac quantization condition constrains the product of electric and magnetic charge to an integer multiple of twice the circle constant.Does that fix the electric charge?It does not, because what is constrained is the product.Varying the magnetic partner over four values moves the charge with a spread of **13.6000**.

⇒ The separator lies here.Paper 1058 measured that the gap of a Chern number from an integer shrinks as the lattice is refined (ratios **3.998** and **4.001**).That gap is a discretization error, and the number really is an integer.The gap of the reciprocal does not shrink.Improving the measurement leaves 137.035999177 where it is.

Whether the gap shrinks under refinement, or does not. That is what separates the two.

7. The dimensional root cannot be taken either

Paper 1049 counted that fixing one independent constant removes exactly one unit frame.

Fixed	Frames remaining
nothing	3
the speed of light	2
the speed of light and the quantum of action	1
those two and the gravitational constant	0

What does that subtraction do to the fine-structure constant?Nothing, because its dimension vector is zero.However many frames are removed, a dimensionless quantity is never touched.

Section 6 of Paper 1057 already states this, and it is used here as the ground on which the dimensional root falls.

8. The remaining two do not fall

Topology and dimension have fallen.Symmetry and algebra remain.One assumption is removed first.

Topology fell because it yields only integers, not because structure yields no numbers.There is a case in which structure produced a non-integer real number.

Take period doubling in the logistic map.The parameters at which a cycle of period a power of 2 becomes superstable are found by bisection, after printing the interval in which the sign changes.The first two are 3.236067977 and 3.498561699, reproducing the known values.The ratio of successive gaps comes out as **4.668954**, differing by **0.000248** from the value 4.669201609 held by Paper 320.

The inputs to this calculation are the form of the map and the range of the parameter, nothing else.The constant is not supplied from outside.A real number that is neither an integer (distance from an integer 0.066160) nor a simple fraction came out of structure alone.

⊖ That constant and the fine-structure constant are nevertheless not called one root.What is shown is only that an exit exists; no mechanism linking the two is measured.

What would the algebraic root require?The reciprocal would have to be an algebraic number of low degree.Bounding the denominator, the nearest rational is sought.

Denominator bound	Nearest rational	Relative distance
10	137/1	2.627×10 ⁻⁴
30	3837/28	2.079×10 ⁻⁶
100	11374/83	1.061×10 ⁻⁶

None reaches the measurement precision 1.6×10⁻¹⁰.And Paper 1021 showed that continued-fraction statistics do not distinguish transcendental from algebraic numbers.The appearance of the number decides nothing.

Symmetry meets the same wall.What symmetry gives directly is an integer, such as the dimension of a representation.There are places where a continuous value appears, but those enter the theory as inputs; symmetry is not fixing the value.

The result of applying the four roots to one dimensionless quantity is set out.

Root	Verdict	Ground
topology	cannot be taken	yields only integers; gap is 1.6419 $\times 10^6$ uncertainties
dimension	cannot be taken	dimension vector is zero; the subtraction never touches it
algebra	undecided	structure can yield non-integers; low degree undecided
symmetry	undecided	representation dimensions are integers; continuous values are inputs

Closing

Closing: the answer to why 137 cannot take two of the four roots. **Topology cannot be taken**——topological invariants yield only integers, while the reciprocal of the fine-structure constant sits **1.6419** $\times 10^6$ of its own uncertainties away from 137. **Dimension cannot be taken** either——the dimension vector of this quantity is zero, and the subtraction of Paper 1049 never touches a dimensionless number. It does not move with the units——across four unit systems the spread is **0.000** $\times 10^0$, against **1.000** $\times 10^4$ for the speed of light. What people chose is the notation, not the value——writing it as a reciprocal brings it **161.6597** times closer to an integer, and the combination made the protagonist ranks third of seven, with the 2π multiple **10.0211** times closer. The algebraic root is still alive——structure does produce non-integer reals: the period-doubling constant comes out at **4.668954**, differing from the value of Paper 320 by only **0.000248**. Stated plainly——this paper does not hit the value 137 and names no fifth root. Two roots are closed; the other two are left undecided, with what each would require written out. The separator——whether the gap shrinks under refinement. A shrinking gap is the coarseness of the calculation and the number really is an integer; a gap that does not shrink means the number really is not one.

Revision Record

2nd edition (September 2026): The fine-structure constant was changed from the CODATA 2018 recommended value to the CODATA 2022 recommended value (its inverse from **137.035999084** to **137.035999177**, the relative uncertainty from 1.5×10^{-10} to 1.6×10^{-10}). Only the last digits changed, and neither the conclusion that two of the four roots cannot be taken nor any ranking changes: ruler ② went from **0.000262767** to **0.000262768**, the effect of writing the inverse from **161.6601** to **161.6597** times, the closeness of the quantity multiplied by 2π from **10.0214** to **10.0211** times, and the departure from an integer from **1.7513** $\times 10^6$ to **1.6419** $\times 10^6$ times the uncertainty. The rank among the seven dimensionless quantities (rank **3**) is the same.

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