What Sets the Quantum Ceiling Is Not Dimension but the Number of Measurements

— Adding settings moves the ceiling toward the algebraic one while the margin over the classical one thins — the separator is whether one moves the size of the space or the number of measurements — [Paper 1062]

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— *From thinking to syncing.* —

Abstract

Paper 160 showed that one inequality carries three ceilings, and divided them: 2 is combinatorial, 2√2 is geometric, 4 is arithmetic.Its scope note then said that it does not answer why nature stops at 2√2.This paper does not answer that either——it answers a different question, namely what sets that ceiling.No new theorem or law is claimed. First, enlarging the space does not move the ceiling——optimising state and measurements alternately from two levels up to five, all four reach $\mathbf{2.828427124746}$, with a spread of $\mathbf{2.220}\times10^{-15}$. Second, and this is the core. Increasing the number of measurements does move it——with n settings on each side, the quantum ceiling divided by the algebraic one is $\mathbf{0.707106781187}$ at $n = 2$ and $\mathbf{0.999969158}$ at $n = 200$. Third, two ratios then move in opposite directions——the algebraic ceiling is approached, while the margin over the classical one falls from $\mathbf{1.414213562373}$ to $\mathbf{1.004994128}$. The algebraic side moves by $+\mathbf{0.292862376}$ and the classical side by $-\mathbf{0.409219434}$. Fourth, the state moves it too——an unentangled state stops at exactly $\mathbf{2.000000000}$, and only a maximally entangled one reaches $\mathbf{2.828427125}$. Fifth, units do not move it——giving directions by components and changing the unit of length four ways leaves a spread of $\mathbf{0.000}\times10^{0}$, while the same changes move the dimensionful speed of light by a relative $\mathbf{1.000000}\times10^{9}$. Stated plainly——why nature stops at this ceiling is not answered. Any link to 4π was measured and rejected: the nearest candidate differs by a relative $\mathbf{0.113773075}$. **Keywords**: Bell inequality, nonlocal correlation, upper bound, entanglement, dimension, measurement settings, separator.

Scope of this paper (scope note): No new theorem or law is claimed——Bell's inequality, its chained form with more settings, the no-signalling condition, and the procedure of alternately optimising state and measurements are all standard.Why nature stops at that ceiling is not answered——that is an open research question. What is written here is what moves the ceiling and what does not.The ceiling is not derived from a principle——work deriving the quantum ceiling from principles such as information causality or local orthogonality is not entered into. Only numbers computed here are written.A search maximum is not called an upper bound——alternating optimisation evaluates from below. What can be said is that the value is reached and that no configuration exceeding it was found.No link to 4π is drawn——Section 7 compares them and measures that none is close.The period-doubling constant is not said to share a root——it is written only that two examples exist in which structure fixes a non-integer real number.Experiments are not discussed——how far loophole-free experiments have confirmed this ceiling lies outside the scope.Relation to earlier papers: Paper 160 took a census of three ceilings over one inequality, divided them as combinatorial, geometric and arithmetic, and said in its scope note that it does not answer why nature stops at 2√2 ——this paper does not answer that either. It answers what sets the ceiling, which is a different question.Paper 227 measured that the value of a PR box reaches 4 with signalling 0——that algebraic ceiling is cited here as the destination reached as settings are added.Paper 11 computed that an entangled pair gives exactly 2√2——that is taken as the starting point, and dimension, number of settings and state are then moved.Paper 164 measured the polytope built from local assignments and the distance to its face——the classical ceiling here comes from enumerating the same assignments exhaustively.Paper 68 showed that a qubit is the Hopf map itself and that an angle on the sphere is twice the angle between states——that the optimal configuration of Section 6 is 90 degrees on the sphere and 45 degrees between states is that doubling.Paper 1055 showed that numbers separate by whether they move when units are changed——this ceiling is on the unmoving side, and Section 7 measures that with a control.Paper 1061 wrote that an exit exists by which structure produces a non-integer real number——this ceiling is a second example, and one that can be written exactly in closed form.Paper 300 showed that whether two things share a root is decidable——following that discipline, the two examples are not called one root. What is added: **applying dimension, number of settings and state to one ceiling and separating what moves it from what does not, confirming exhaustively that the quantum-to-algebraic ratio matches a cosine of the number of settings, showing in numbers that two ratios move in opposite directions, and measuring and rejecting any link to 4π.**

1. Setting and question

Paper 160 took a census of three ceilings over one inequality.Local hidden variables give 2, quantum theory gives 2√2, and imposing no-signalling alone gives 4.It assigned the three to separate origins: 2 is combinatorial, 2√2 is geometric, 4 is arithmetic.

Its scope note reads as follows.It does not answer why nature stops at 2√2——that is an open research question, and what it writes is only the addresses of the three ceilings.

This paper does not answer that question either.It asks a different one: what moves that ceiling.Once an address is known, the next thing that can be measured is what the place responds to.

The conventions are fixed once.Two parties each hold two settings, and every outcome is $+1$ or -1.Four averaged correlations are combined and the magnitude is taken.Enumerating all 16 local assignments gives a maximum of exactly $\mathbf{2.000000000000}$.An entangled pair with angles 0 and 90 degrees against 45 and -45 degrees gives $\mathbf{2.828427124746}$.

The sign convention is settled at the outset.Taking the fourth angle mistakenly as 135 degrees in the same expression gives $\mathbf{0.000000000000}$, which is $\mathbf{2.828427125}$ away from the ceiling.This control comes first because every computation below rests on that convention.

2. Enlarging the space

The first thing moved is the size of the space in which the state lives.The same quantity is computed not only for a pair of two-level systems but for three-, four- and five-level pairs.The measurements are found by alternating optimisation: fix the observables and take the best state, fix the state and take the best observables, and repeat.

Levels on one side	Maximum reached	Difference from the ceiling
2	$\mathbf{2.828427124746}$	$\mathbf{0.000}\times10^{0}$
3	$\mathbf{2.828427124746}$	$\mathbf{2.220}\times10^{-15}$
4	$\mathbf{2.828427124746}$	$\mathbf{8.882}\times10^{-16}$
5	$\mathbf{2.828427124746}$	$\mathbf{1.776}\times10^{-15}$

The spread across the four is $\mathbf{2.220}\times10^{-15}$.That is the scale of the double-precision step, and does not count as movement.All four reach the ceiling as well——a larger space is not penalised either.

One caution is recorded here.Alternating optimisation evaluates from below.What can be said is that the value is reached and that no configuration exceeding it was found; the upper bound itself has not been proved.Where Paper 160 confirmed non-excess by random search, this paper looks at the same place from the side of reaching it.

3. Increasing the number of measurements

The next thing moved is the number of settings each party holds.Each of the two parties holds n settings, and neighbouring pairs are joined in a ring of $2n$ terms.The case $n = 2$ is the form used above.

Each of the three ceilings is produced separately.The classical one comes from enumerating all ± 1 assignments——up to $n = 8$ the values run 2, 4, 6, 8, 10, 12, 14, matching $2n - 2$.The algebraic one is the number of terms itself, $2n$ .The quantum one comes from taking the angles of an entangled pair at equal spacing and adding the terms.

Settings	Classical	Quantum	Algebraic
2	**2.000000000**	**2.828427125**	**4.000000000**
3	**4.000000000**	**5.196152423**	**6.000000000**
4	**6.000000000**	**7.391036260**	**8.000000000**
6	**10.000000000**	**11.591109915**	**12.000000000**
8	**14.000000000**	**15.692564486**	**16.000000000**
12	**22.000000000**	**23.794676673**	**24.000000000**
20	**38.000000000**	**39.876693349**	**40.000000000**
50	**98.000000000**	**99.950656037**	**100.000000000**
200	**398.000000000**	**399.987663058**	**400.000000000**

All three move with n.The three that Paper 160 divided as combinatorial, geometric and arithmetic all respond to the number of settings——what differs is only how they move.

The quantum ceiling divided by the algebraic one sits cleanly on a cosine of the number of settings.Comparing the value obtained by adding terms with the value of the closed form, the discrepancy across all nine values of n is at most 1.110×10^{-16}.

Settings	Quantum over algebraic (added)	Closed form
2	**0.707106781187**	**0.707106781187**
3	**0.866025403784**	**0.866025403784**
6	**0.965925826289**	**0.965925826289**
20	**0.996917333733**	**0.996917333733**
200	**0.999969157645**	**0.999969157645**

So 2√2 is not a number belonging to quantum theory.It is the value for two parties holding two settings each.Change the counting and the same quantum theory carries a different ceiling.

4. Two ratios moving in opposite directions

The same table is read again through two ratios.One is the quantum value divided by the algebraic one, the other the quantum value divided by the classical one.

Settings	Quantum over algebraic	Quantum over classical
2	**0.707106781187**	**1.414213562373**
4	**0.923879532511**	**1.231839376682**
8	**0.980785280403**	**1.120897463318**
20	**0.996917333733**	**1.049386667088**
50	**0.999506560366**	**1.019904653434**
200	**0.999969157645**	**1.004994128286**

The left column rises and the right column falls.The algebraic side moves by $+\mathbf{0.292862376}$ and the classical side by $-\mathbf{0.409219434}$, opposite in sign.

Distances show the same thing.The gap to the algebraic ceiling is $\mathbf{1.171572875}$ at $n = 2$ and $\mathbf{0.012336942}$ at $n = 200$, a ratio of $\mathbf{94.964608512}$.The excess over the classical ceiling is $\mathbf{0.828427125}$ at $n = 2$ and $\mathbf{1.987663058}$ at $n = 200$, a ratio of $\mathbf{2.399321556}$.

The absolute excess grows while the proportional advantage disappears.The more settings are added, the closer quantum theory comes to the ceiling of a world constrained only by no-signalling, and the thinner its margin over the classical one becomes in proportion.The value 2√2 is the value at the single point where those two are still widely apart.

5. Moving the state

The third thing moved is the state.The degree of entanglement is written with one angle, and the measurements are optimised for each angle.At 0 degrees there is no entanglement, and 45 degrees is maximal.

Entanglement angle (deg)	Maximum reached	Ratio to the ceiling	Independent formula
0	**2.000000000**	**0.707106781**	**2.000000000**
10	**2.113743389**	**0.747321142**	**2.113743389**
20	**2.377541513**	**0.840587863**	**2.377541513**
30	**2.645751311**	**0.935414347**	**2.645751311**
45	**2.828427125**	**1.000000000**	**2.828427125**

An unentangled state stops at exactly 2.000000000——the same place as the classical ceiling.Only a maximally entangled state reaches the ceiling.The optimised values agree with a separately stated closed form to at most 8.882 ×10⁻¹⁶.That is a result and at the same time a check that the method is not broken.

6. Moving the angle

Paper 160 showed the origin of this ceiling in numbers, as the geometry of an inner product.The same place is confirmed here independently, by another route.A family of four directions is built from a single angle *t*, and *t* is stepped finely.

Stepping through 200000 points, the maximum is 2.828427124746 and the angle is exactly 45.000000000 degrees.The values of this family agree with a closed form containing a triple angle, with a discrepancy of 0.000×10⁰ at five points.

At that configuration the two directions held by one observer make 90.000000000 degrees on the sphere.Seen between states, that is half of it, 45.000000000 degrees.Paper 68 showed that a qubit is the Hopf map itself and that an angle on the sphere is twice the angle between states——what appears here is that doubling.

7. What does not move

Two things that do not move are recorded last.

The first is the choice of units.Directions are given by components, the unit of length is changed four ways so that the components are rescaled, and the angle is recovered from the normalised inner product.All four give 2.828427124746, with a spread of 0.000×10⁰.As a control, the dimensionful speed of light recomputed under the same four changes has a relative spread of 1.000000×10⁹.On the separator of Paper 1055, this ceiling sits on the unmoving side.

The second is 4π.The solid-angle number at the centre of this body of work is compared with the ceiling directly.

Compared number	Value	Relative difference
4π	**12.566370614359**	**3.442882938**
4π over the square root of 2	**8.885765876317**	**2.141592654**
4π over the ceiling	**4.442882938158**	**0.570796327**
4π over 9	**1.396263401595**	**0.506346340**
π over the square root of 2	**2.221441469079**	**0.214601837**
The square root of 2π	**2.506628274631**	**0.113773075**

Even the nearest differs by a relative 0.113773075.This ceiling is not of the 4π family.That a number sits at the centre of a body of work does not draw dimensionless constants toward it——that was measured, and found not to hold.

8. Relation to earlier papers

Paper 1061 applied four roots to one dimensionless constant and showed that two of them fall.It then wrote that the algebraic root is still alive, because structure does produce non-integer real numbers and an exit exists.

The ceiling of this paper is a second example of that exit.It is a stronger one than the first——it can be written exactly in closed form, it is fixed by structure alone, and it lies within reach of experiment.

Following the discipline of Paper 300, however, the two are not called one root.All they share is the single point that structure can fix a non-integer real number, and no mechanism by which moving one moves the other has been measured.

And this paper has not, to the end, answered the question Paper 160 declined.Why nature stops at this ceiling remains unwritten.What could be written is what the ceiling responds to, and what it does not.

Closing

Closing: what sets the quantum ceiling is not dimension but the number of measurements.The size of the space has no effect——from two levels to five, all four reach 2.828427124746, with a spread of only 2.220×10^{-15}.The number of settings does have an effect——the quantum ceiling divided by the algebraic one runs from 0.707106781187 to 0.999969157645, agreeing with a closed form to within 1.110×10^{-16} at all nine values.Two

ratios then move in opposite directions——the algebraic side rises by +0.292862376 and the classical side falls by −0.409219434. The gap ratio is 94.964608512 and the excess ratio 2.399321556.The state has an effect too——an unentangled state stops at 2.000000000, and only a maximally entangled one reaches the ceiling.Units have none——four changes leave a spread of 0.000×10⁰, while the control speed of light moves by 1.000000×10⁹.Stated plainly——why nature stops at this ceiling is not answered. There is no link to 4π either——the nearest candidate differs by a relative 0.113773075. Alternating optimisation evaluates from below and is not a proof of the bound.The separator——whether one moves the size of the space or the number of measurements. Moving the former leaves the ceiling untouched; moving the latter moves the ceiling itself.

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