A Point of Spacetime Becomes a Sphere in the Space of Light Rays

— The space of rays always has odd dimension, and two such spheres meet only when the points are null-separated — the separator is whether the displacement is along a null direction or not — [Paper 1066]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
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— From thinking to syncing. —

Abstract

Instead of laying out the points of spacetime, one may lay out the light rays themselves as points.This paper counts the dimension of that space, measures what a point of spacetime becomes there, and finds when two such objects meet.No new theorem and no new physical law is claimed. First, the space of rays has one dimension more than spacetime——a starting point and a direction supply six parameters, but sliding along the ray leaves the ray unchanged, so one is spare and the rank is 5. Varying the dimension from 3 to 6, the spare was always 1. Second, that dimension is necessarily odd——an antisymmetric matrix of odd size is degenerate, and the determinants at sizes 3, 5 and 7 reached at most 1.703720×10^{-15}, 4.513467×10^{-14} and 7.741156×10^{-13}, whereas at even sizes the smallest was 1.793350×10^{2}. Third, a single point of spacetime becomes a sphere there——the rays through one point form a two-dimensional family, all directions are unit vectors (largest departure 3.331×10^{-16}), and the solid angle they cover is 12.566370614359. Fourth, and this is the core. Two such spheres meet only when the points are null-separated——among 200000 randomly drawn pairs of points not one shared a ray, while all 20000 pairs constructed to be null-separated shared exactly one. Fifth, they ought not to meet at all——placing two two-dimensional families inside five dimensions gives an intersection of dimension -1. Even so there is only one condition: the rank of the gradient of the interval was 1 on all 200 trials. Stated plainly——what can be placed on the space of rays is not entered; only what cannot is stated with numbers. The measurement is made in flat spacetime alone and is not extended to curved spacetime. **Keywords**: space of light rays, null geodesic, solid angle, causal structure, symplectic structure, dimension counting, separator.

Scope of this paper (scope note): **No new theorem and no new physical law is claimed——the space of light rays, its dimension, the exchange of points for spheres, and the degeneracy of antisymmetric forms in odd dimensions are all standard.No theory is built——spacetime is not reconstructed from rays taken as primitive. What is measured is the dimension and the condition.No complex coordinates are introduced——the tools that such a standpoint properly uses are not touched.Apparent shape and aberration are not entered——how a moving body looks, and how the arrangement of directions changes with speed, lie outside this paper.What can be placed is not entered——only the impossibility of a non-degenerate antisymmetric form is stated with numbers; what may be placed instead is not written.Curved spacetime is not entered——the measurement is made in flat spacetime alone.The two are not called one root——the 4π of solid angle covering one point's sky and the number 5 for the dimension of the ray space come from different places, and no mechanism by which moving one moves the other has been measured.Relation to earlier papers**: **Paper 901 counted the apparent shape of a moving sphere and showed that aberration preserves the total solid angle of the sky while rearranging it, and that circles map to circles——this paper does not enter that side of the sky, and measures instead the space in which rays themselves are the points.Paper 76 placed a covariant symplectic structure on phase space——what is shown here is the single fact that the space of rays cannot take the same form.Paper 111 measured how far light determines geometry along two roads and wrote that both stop at conformal structure——this paper places one instance of that same side, seen by counting rays.Paper 1041 showed that reduced to order alone the fraction of related pairs still gives the dimension——what is used here is not order but whether a ray is shared.Paper 67 read the 4π as a solid angle and showed it to be twice the circle constant times the Euler characteristic of the sphere——one point's sky covers exactly that 4π.Paper 300 showed that whether two things share a root is decidable——following that discipline, the 4π of solid angle and the dimension 5 are not called one root.**What is added is **counting the dimension of the ray space in four dimensions and confirming that the spare is always one**, **showing with numbers that the necessarily odd dimension forbids a non-degenerate antisymmetric form**, **measuring that the rays through one point form a sphere covering that solid angle**, and **showing that the condition for two such spheres to meet is exactly null separation**.

1. Setting and question

To study spacetime one usually lays out its points.But there is another way, in which light itself takes the leading part——each single ray is treated as a point, and the whole collection is laid out as one space.

Where then does a point of spacetime go?Many rays pass through one point, so the point becomes a whole cluster.Measuring what that cluster is, is the first question here.

The second question is when two such clusters meet.To meet is to share a single ray.

Throughout, the speed of light is set to one and only flat spacetime is treated.No extension to curved spacetime is made.

And apparent shape and aberration are not entered.Paper 901 counted how a moving sphere looks and showed that aberration preserves the total solid angle of the sky while rearranging it.What this paper enters is not that side of the sky, but the space in which rays themselves are the points.

2. The space of rays has five dimensions

To fix one ray it suffices to give one point it passes through and one direction of travel.In a spacetime of a given dimension, the point takes that many parameters and the direction two fewer, for a total of twice the dimension minus two.

But sliding the starting point along the ray leaves the same single ray.So one parameter must be spare.The map from the parameters to coordinates that fix the ray itself was therefore differentiated numerically and the rank of the matrix taken.

Spacetime dimension	Parameters supplied	Space of rays	Spare
3	4	3	1
4	6	5	1
5	8	7	1
6	10	9	1

The spare is 1 in every dimension, and its direction is the sliding.The space of rays always has one dimension more than the spacetime.In four dimensions it is 5.

The coordinates used are where the ray crosses time zero, together with its direction.That sliding leaves them fixed, and that other displacements do not, was confirmed before the run.

3. That dimension is necessarily odd

If the spacetime dimension is written as a number, the ray space has twice that minus three.Whatever the dimension, this is odd.And that shuts out one thing that might have been placed there.

An antisymmetric matrix of odd size is always degenerate.Two hundred matrices were drawn at random at each size to confirm it.

Size	Parity	Largest determinant	Largest rank
3	odd	$\mathbf{1.703720}\times10^{-15}$	2
4	even	$\mathbf{1.793350}\times10^{2}$	4
5	odd	$\mathbf{4.513467}\times10^{-14}$	4
6	even	$\mathbf{2.405325}\times10^{3}$	6
7	odd	$\mathbf{7.741156}\times10^{-13}$	6
8	even	$\mathbf{4.358112}\times10^{4}$	8

On the odd side the values stay at the size of the double-precision step and the rank falls one short of the size.On the even side nothing fell below $\mathbf{1.793350}\times10^{2}$ and the rank reached the size.This test can fail.

So no non-degenerate antisymmetric form can be placed on the space of rays.Paper 76 placed a covariant symplectic structure on phase space; the space of rays cannot take the same form.

What may be placed instead is not entered.What this paper states is only what cannot.

4. A point of spacetime becomes a sphere

Fix one point of spacetime and collect every ray through it.Only the direction is free to turn, so the dimension of the collection should be two fewer than that of the spacetime.

Spacetime dimension	Rays through one point	Space of rays
3	1	3
4	2	5
5	3	7
6	4	9

In four dimensions it is two-dimensional, that is, a surface.Every direction is a unit vector, with a largest departure of $\mathbf{3.331}\times10^{-16}$, and the solid angle the collection covers is $\mathbf{12.566370614359}$.

Paper 67 read the 4π as a solid angle and showed it to be twice the circle constant times the Euler characteristic of the sphere.One point’s sky covers exactly that 4π.

So a point of spacetime becomes a sphere in the space of rays——a two-dimensional sphere set inside five dimensions.

5. Two spheres meet only when the points are null-separated

When, then, do two points' spheres meet?To meet is for two points to share one ray.

The place where a ray through a point crosses time zero is the point's spatial position minus its time coordinate times the direction.For two points to share one ray they must give the same direction and the same place.From that follows the condition that the spatial separation equals the time separation times the direction.Since the direction has unit length, this says the spatial separation and the time separation are equal.

Separation of the two points	Rays shared	Interval
Null (joinable by light)	1	**0.000000**
Timelike (slower than light)	0	**3.750000**
Spacelike (faster than light)	0	**−21.000000**
The same point	the whole sky	**0.000000**

This is the core of the paper.Among 200000 pairs of points drawn at random, not one shared a ray.Among 20000 pairs constructed to be null-separated, all 20000 shared exactly one.

And the number shared takes only three values.Ordinarily none, when null-separated exactly one, and for the same point the whole sky.That two points can be joined by light and that their spheres meet turned out to be one and the same thing.

6. They ought not to meet at all

The dimensions are worth counting here.Placing two two-dimensional objects inside five dimensions gives an intersection whose dimension is two plus two minus five, which is negative.Negative means that in general they do not meet.

What is counted	Dimension
Space of rays	5
One point’s sky	2
Sum of two skies	4
General intersection	-1
Space of pairs of points	8
Null-separated configurations	7

Even so, there is only one condition for meeting.Taking the interval as the condition and measuring the rank of its gradient 200 times, both the smallest and the largest were 1.Of the eight dimensions of configurations, seven meet.

Paper 111 measured how far light determines geometry and wrote that both roads stop at conformal structure.What this paper places is one form of that same side, seen by counting rays.

7. The separator

The separator is whether the displacement is along a null direction or not.Two null-separated points were built and one of them moved.

How it was moved	Rays shared	Interval
Slid along the same null direction	1	0.000000
Moved along time	0	1.250000
Moved along space	0	-1.250000
Moved transversally	0	-0.250000

Of the four, the intersection survives only for the slide along the null direction.In the other three the shared ray disappears.

A second handle can be named.Whether a dimension is even or odd decides what form can be placed on it.The space of rays is always odd, so it always falls on the side where nothing can be placed.

8. Relation to earlier papers

Paper 901 counted the apparent shape of a moving sphere and showed that aberration preserves the total solid angle of the sky while rearranging it, and that circles map to circles.This paper does not enter that side of the sky and measures only the space in which rays themselves are the points.The light is the same; what is laid out differs.

Paper 76 placed a covariant symplectic structure on phase space.What this paper shows is the single fact that the space of rays cannot take the same form.The reason lies only in the parity of the dimension and has nothing to do with the physical content.

Paper 111 measured how far light determines geometry along two roads.This paper places one instance of that side, seen by counting rays.

Paper 1041 showed that reduced to order alone the fraction of related pairs still gives the dimension.What is used here is not order but whether a ray is shared.Both look at geometry from the causal side, but what is taken in hand differs.

The 4π of Paper 67 appears here as the solid angle covering one point's sky.It comes from a different place than the dimension 5 of the space of rays, so following the discipline of Paper 300 the two are not called one root.

Closing

Closing: a point of spacetime becomes a sphere in the space of light rays.The space of rays has one dimension more than spacetime——of six parameters one is spare through sliding, and the rank is 5. Varying the dimension from 3 to 6, the spare was always 1.That dimension is necessarily odd——the determinants of antisymmetric matrices at sizes 3, 5 and 7 reached at most 1.703720×10^{-15}, 4.513467×10^{-14} and 7.741156×10^{-13}, while at even sizes the smallest was 1.793350×10^{2}.One point becomes a sphere——the family is two-dimensional, the largest departure of the directions from unit length is 3.331×10^{-16}, and the solid angle covered is 12.566370614359.Two spheres meet only when the points are null-separated——among 200000 random pairs none shared a ray, and among 20000 constructed pairs all 20000 shared exactly one.They ought not to meet at all——the intersection has dimension -1, yet there is only one condition, the rank of the gradient being 1 on all 200 trials.Stated plainly——what can be placed on the space of rays is not entered, only what cannot. The measurement is in flat spacetime alone, with no complex coordinates.The separator——whether the displacement is along a null direction or not. Along a null direction the shared ray survives; in the other three it disappears.

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Yuuki YAMAGISHI (山岸 勇輝)

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