

Whether a Point Lies Inside Comes Out as an Integer When the Swept Solid Angle Is Divided by 4π

— A single ray distinguishes the shell, while the swept solid angle distinguishes the interior and even gives the direction of time — the separator is whether one counts intersections or swept solid angle — [Paper 1067]

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— *From thinking to syncing.* —

Abstract

In the space where light rays are laid out as points, a point of spacetime becomes a sphere, and two spheres meet exactly when the points can be joined by light. But meeting treats the case of lying inside the light cone and the case of lying outside it alike, calling both "not meeting". This paper places one way of counting that recovers the distinction. No new theorem and no new physical law is claimed. First, meeting sees only the shell—the pair with interval **3.750000** and the pair with interval **−21.000000** have opposite signs, yet both share zero rays. Second, and this is the core. Dividing the solid angle swept by the difference direction by 4π yields an integer—for the pairs lying inside the magnitudes are **1.000014668** and **1.000012830**, and for those lying outside they are **0.000000823** and **0.000000645**. Third, the departure from the integer is a discretization error—halving the step shrinks it fourfold each time, with measured ratios between **3.9995** and **4.0041**. Fourth, its sign gives the direction of time—all three pairs agreed with the reversed sign of the time separation. Fifth, the 4π here plays the opposite role to the one it plays in earlier papers—in the cancellation of density against volume element the value **12.566370614** stands in the numerator, whereas here it stands in the denominator, dividing the swept solid angle **12.566554932** into an integer. Sixth, the shortfall for meeting does not depend on the dimension—from three to eight dimensions the shortfall was always one, while the exponent of the cancellation runs from 1 to 6. Stated plainly—no theorem is proved. The measurement is numerical, in flat spacetime, and says no more than how fast it converges. **Keywords:** space of light rays, solid angle, degree of a map, light cone, causal structure, convergence of quadrature, separator.

Scope of this paper (scope note): No new theorem and no new physical law is claimed—the space of light rays, the exchange of points for spheres, the degree of a map, and the fact that the degree is written as an integral of solid angle are all standard. No theorem is proved—the measurement is numerical in flat spacetime, and says no more than how it converges as the step is refined. Curved spacetime is not entered—the measurement is made in flat spacetime alone. No structure is placed on the space of rays—what is counted is only an integer obtained from two spheres; nothing is loaded onto the space itself. The two roles are not called one root—the 4π that stands in the numerator as the result of a cancellation and the 4π that stands in the denominator to produce an integer come from different places, and no mechanism by which moving one moves the other has been measured. It is not claimed that three exhaust them—a third kind, independent of dimension, is merely set beside the two an earlier paper separated. Apparent shape and aberration are not entered—how the arrangement of directions changes with speed lies outside this paper. Spacetime is not rebuilt from order—the road that takes causal order alone as input is not taken. Relation to earlier papers: Paper 1066 counted the dimension of the space of rays and showed that two spheres meet only when the points are null-separated. Its scope note declared "no theory is built—what is measured is the dimension and the condition"—this paper sits in the seat of that scope note and goes one step past the condition, separating the two cases to which the condition returned the same answer (no prior occupant). Paper 01 proved that for a spherically symmetric power-law density there is exactly one exponent for which the increase per shell becomes independent of the radius—what this paper sets beside it is the contrast that this exponent moves with the dimension while the shortfall does not. Paper 02 showed that the volume element and the density cancel so that the mass inside the unit ball equals the total solid angle—here that same number stands on the dividing side. Paper 67 read the 4π as a solid angle and showed it to be twice the circle constant times the Euler characteristic of the sphere—what this paper divides by is exactly that solid angle. Paper 1052 showed that "subtracting one" comes in two kinds, one counting holes and the other counting dimensions—what this paper places is a third, which is neither. Paper 300 showed that whether two things share a root is decidable—following that discipline, the two roles of 4π are not called one root. What is added is showing with numbers that meeting does not distinguish the inside of the light cone from the outside, measuring that dividing the swept solid angle yields an integer that does distinguish them, confirming by ratios that the departure from the integer shrinks with the square of the step, and confirming that its sign gives the direction of time.

1. Setting and question

Paper 1066 measured the space in which light rays themselves are laid out as points. There a point of spacetime becomes a sphere, and when, and only when, two points can be joined by light, their two spheres share a single ray.

But that way of measuring misses one thing. Two points lying inside a light cone and two points lying outside it both come out as "zero rays shared". Counting intersections alone cannot tell inside from outside.

Separating the inside of a sphere from its outside, however, is something earlier papers already handled elsewhere. Papers 01 and 02 counted the quantity belonging to a single shell and the quantity belonging to the

interior separately.

This paper joins the two. The question is one: what must be counted to recover the distinction that intersection dropped?

Throughout, the speed of light is set to one and only flat spacetime is treated. Apparent shape and aberration are not entered.

2. Meeting sees only the shell

First, what is being lost is confirmed. Using the measurement of Paper 1066 unchanged, the origin was paired with three points.

Separation of the pair	Rays shared	Interval
Null (on the shell)	1	0.000000
Timelike (inside)	0	3.750000
Spacelike (outside)	0	−21.000000

The signs of the intervals, **3.750000** and **−21.000000**, are opposite. Even so both share zero rays, and the answers coincide.

What meeting sees is only whether one lies on the shell. Inside or outside it does not see.

3. Dividing the swept solid angle separates inside from outside

The two spheres sit at displaced positions on the slice of time zero. Choosing one direction fixes the difference between the places where the two rays cross that slice. That difference is the spatial separation minus the time separation times the direction.

Turning the direction through all directions moves this difference as well. So the solid angle swept by the direction of the difference was divided by the total solid angle.

Separation	Time	Space	Step 40	Step 80	Step 160
Timelike (inside)	2.0	0.500000	−1.000234691	−1.000058671	−1.000014668
Timelike (oblique)	3.0	1.486607	−1.000205244	−1.000051318	−1.000012830
Timelike (reversed)	−2.0	0.500000	1.000234691	1.000058671	1.000014668
Spacelike (outside)	2.0	5.000000	−0.000013181	−0.000003292	−0.000000823
Spacelike (oblique)	1.0	2.596151	−0.000010335	−0.000002581	−0.000000645

For the three pairs lying inside, the magnitude approaches one. For the two lying outside it approaches zero. The two that intersection called zero alike have separated here.

The reason is plain. The difference traces a sphere whose radius is the time separation and whose centre is the spatial separation. If the origin lies inside that sphere the difference direction turns through all directions once; if outside, it does not. And "the origin lies inside" means that the time separation exceeds the spatial one, that is, that the pair lies inside the light cone.

4. The departure is a discretization error

The values do not sit neatly at one or zero. So what happens as the step is refined was measured.

Separation	Step 40	Step 80	Step 160	Ratio	Ratio
Timelike (inside)	2.347 $\times 10^{-4}$	5.867 $\times 10^{-5}$	1.467 $\times 10^{-5}$	4.0002	4.0000
Timelike (oblique)	2.052 $\times 10^{-4}$	5.132 $\times 10^{-5}$	1.283 $\times 10^{-5}$	3.9995	3.9999
Timelike (reversed)	2.347 $\times 10^{-4}$	5.867 $\times 10^{-5}$	1.467 $\times 10^{-5}$	4.0002	4.0000
Spacelike (outside)	1.318 $\times 10^{-5}$	3.292 $\times 10^{-6}$	8.229 $\times 10^{-7}$	4.0033	4.0008
Spacelike (oblique)	1.034 $\times 10^{-5}$	2.581 $\times 10^{-6}$	6.451 $\times 10^{-7}$	4.0041	4.0010

Each halving of the step shrinks the departure from the integer fourfold. The measured ratios lie between **3.9995** and **4.0041**, and all ten fell in that range.

This is the shape of an error proportional to the square of the step in a numerical integral. The departure from the integer is the coarseness of the measurement, not a wobble in the value itself.

When this paper was begun the author set a tolerance before measuring and three tests failed.They failed not because the values were wrong but because the estimate was too fine.Replacing the tolerance on the value itself by a measurement of how it approaches removed the failures.

5. The sign gives the direction of time

The integer that comes out carries a sign.For the three pairs lying inside, that sign was compared with the time separation.

Separation	Time	Value obtained
Timelike (inside)	2.0	−1.000014668
Timelike (oblique)	3.0	−1.000012830
Timelike (reversed)	−2.0	1.000014668

All three agreed with the reversed sign of the time separation.So not only whether one lies inside, but which of the two comes first, is obtained.

Paper 1041 showed that reduced to order alone the fraction of related pairs still gives the dimension, but what this paper takes as input is not order.Order comes out of a quantity swept by turning a direction through a full turn.

6. The 4π here plays the opposite role

The total solid angle used as the divisor is the number Paper 67 read as twice the circle constant times the Euler characteristic of the sphere.The same number appears in Papers 01 and 02, but stands in a different place.

Paper 02 showed that the volume element and the density cancel so that the mass inside the unit ball equals the total solid angle.The values extended to a general exponent are laid out.

Exponent	Mass inside	Sheets
0.0	4.188790205	0.333333333
1.0	6.283185307	0.500000000
2.0	12.566370614	1.000000000
2.5	25.132741229	2.000000000

Here the total solid angle stands in the numerator, since it is what the cancellation produces.

Here it is the other way round.Dividing the swept solid angle by the total gives an integer.

Separation	Swept solid angle	After dividing
Timelike (inside)	12.566554932	1.000014668
Timelike (oblique)	12.566531840	1.000012830
Spacelike (outside)	0.000010341	0.000000823
Spacelike (oblique)	0.000008107	0.000000645

The swept solid angle for the pair lying inside is **12.566554932**, approaching the divisor **12.566370614**.The same number stands in the numerator as the result of a cancellation in one place, and in the denominator to produce an integer in the other.

These two are not called one root.One comes from the balance of density against volume element, the other from all directions amounting to one full turn.No mechanism by which moving one moves the other has been measured.

7. The shortfall is the same in every dimension

Finally the dimensions are counted.By Paper 1066 the space of rays has twice the spacetime dimension minus three, and one point’s sky has two fewer than that.The shortfall on placing two skies is laid out.

Spacetime dimension	Space of rays	One point’s sky	Shortfall
3	3	1	−1
4	5	2	−1
5	7	3	−1
6	9	4	−1
7	11	5	−1
8	13	6	−1

The shortfall is the same single one in every dimension. By contrast the exponent producing the cancellation in Paper 01 is the spatial dimension minus one, running 1, 2, 3, 4, 5, 6 from two to seven dimensions.

Paper 1052 showed that "subtracting one" comes in two kinds, one counting holes and the other counting dimensions. The shortfall here is neither—a third kind, responding to neither topology nor dimension. It is not claimed, however, that three exhaust them.

8. The separator

The separator is whether one counts intersections or swept solid angle.

Counting intersections separates only the shell. The pair with interval **3.750000** and the pair with interval **−21.000000** both answer zero. Counting swept solid angle separates inside from outside, giving **1.000014668** against **0.000000823**. And its sign gives the direction of time as well.

A second handle can be named. Whether the total solid angle stands in the numerator or the denominator changes what comes out. In the numerator it becomes a mass; in the denominator, an integer.

9. Relation to earlier papers

Paper 1066 counted the dimension of the space of rays and showed that two spheres meet only when the points are null-separated. Its scope note declared "no theory is built—what is measured is the dimension and the condition", and this paper sits in the seat of that scope note. It goes one step past the condition and separates the two cases to which the condition returned the same answer. There is no prior occupant of that seat.

Paper 01 proved that there is exactly one exponent for which the increase per shell becomes independent of the radius. This paper set that exponent beside the shortfall and compared how each moves.

Paper 02 showed that the volume element and the density cancel so that the mass inside equals the total solid angle. Here that same number stands on the dividing side.

Paper 67 read the 4π as a solid angle. What this paper divides by is exactly that solid angle.

Beside the two kinds of "subtracting one" in Paper 1052 stands a third, independent of dimension. And following the discipline of Paper 300, the two roles of 4π are not called one root.

Closing

Closing: whether a point lies inside comes out as an integer when the swept solid angle is divided by 4π . Meeting sees only the shell—the pair with interval **3.750000** and the pair with interval **−21.000000** have opposite signs, yet both share zero rays. Dividing the swept solid angle yields an integer—inside it is **1.000014668** and

1.000012830, outside **0.000000823** and **0.000000645**. The departure is a discretization error——each halving of the step shrinks it fourfold, with measured ratios between **3.9995** and **4.0041**. The sign gives the direction of time——all three pairs agreed with the reversed sign of the time separation. The role of 4π is the opposite of the one it plays in earlier papers——in the cancellation **12.566370614** stands in the numerator, while here it divides the swept solid angle **12.566554932**. The shortfall does not depend on the dimension——from three to eight dimensions it was always one, while the exponent of the cancellation runs from 1 to 6. Stated plainly——no theorem is proved; the measurement is numerical in flat spacetime and says no more than how fast it converges. No structure is placed on the space of rays. The separator——whether one counts intersections or swept solid angle. The first separates only the shell; the second separates the interior and gives the direction of time.

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