

The Two Noethers — Not Conflating the First Theorem (Conservation) with the Second Theorem (Identities and Constraints) — [Paper 107]

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— *From thinking to syncing.* —

Abstract

The corpus has already used Noether's theorem as "conservation is the shadow of symmetry" (Paper 80). But Emmy Noether's 1918 paper contained **two theorems**, and the corpus had used only the **first**. This paper places the **second theorem** correctly and separates the two as **distinct roots**. **First theorem**: a **global** (rigid, finitely many parameters) continuous symmetry $\rightarrow$ a **conservation law** (energy, momentum, angular momentum — measurable conserved quantities). **Second theorem**: a **local** (parametrized by arbitrary functions of spacetime, infinite-dimensional) gauge symmetry $\rightarrow$ **not a conservation law, but identities / dependencies among the equations of motion** (Noether identities). Examples: in electromagnetism $\partial_\mu \partial_\nu F^{\mu\nu} \equiv 0$; in general relativity the contracted Bianchi identity $\nabla_\mu G^{\mu\nu} \equiv 0$ — these are **identities (holding always, off-shell)**, not new conserved quantities, and mean that **the field equations are not all independent (there is gauge freedom)**. The core: **even for "the same Noether," the first (global $\rightarrow$ conservation) and the second (local $\rightarrow$ identities/constraints) are different theorems = distinct roots**. This separation answers a puzzle the corpus met — **why in gauge theory the "charge" becomes a boundary term** (Paper 96): the Noether current of a local symmetry is identically conserved in the bulk, and the true charge appears as a **surface integral at infinity**. The second theorem is also why the 10 equations of $G_{\mu\nu}$ hide 4 identities (general covariance = 4 arbitrary functions), and the general backbone of Paper 72 (constraints, unimodular). **Scope**: a Layer-2 structural interpretation of established mathematical physics (Noether's two theorems); no new mathematics and no new prediction. The Universe OS is a Layer-3 non-science lens.

Positioning of this paper (discipline): This paper is a **structural interpretation** of established mathematical physics (Noether's first and second theorems, the Bianchi identity, constraints in gauge theory). It contains no new mathematics and no new prediction. The point is to **separate as distinct roots Noether's first theorem (global symmetry $\rightarrow$ conservation, Paper 80) and second theorem (local gauge symmetry $\rightarrow$ identities/constraints)**, and to explain by the second theorem **why gauge "charge" becomes a boundary term** (Paper 96). "Universe OS" is a Layer-3 non-science lens, not a claim of mechanism.

1. Introduction: The Same Noether, Two Theorems

In Paper 80, the corpus placed Noether's theorem as "to every continuous symmetry corresponds a conserved quantity" — time translation → energy, space translation → momentum, rotation → angular momentum. But this was **half the story**. What Emmy Noether showed in her 1918 monument *Invariante Variationsprobleme* was in fact **two theorems**. The corpus had used only the **first**.

This paper places the **second theorem** correctly in the corpus. And, most importantly, it separates the two as **distinct roots** — for "the same Noether," the first and the second are **different theorems, in both input (global or local) and output (conservation or identity)**. This distinction gives correct names to several subtleties the corpus touched in gauge (Paper 96) and constraints (Paper 72). It builds no new theorem.

2. The First Theorem: Global Symmetry → Conservation (the Core of Paper 80)

Standard fact (Noether's first theorem)

If the action is invariant under a continuous symmetry with **finitely many constant parameters** (a **global, rigid** symmetry), then to each parameter corresponds a **conserved current** $\partial_\mu j^\mu = 0$ and a **conserved charge** $Q = \int j^0 d^3x$. Time translation → energy, space translation → momentum, rotation → angular momentum, global $U(1)$ phase → electric charge. This is the content of "conservation is the shadow of symmetry" treated in Paper 80.

The output of the first theorem is a **measurable conserved quantity**. Energy, momentum, angular momentum — all measurable. A rigid symmetry (shifting everywhere by the same amount) generates such **genuine conservation laws**. This far, the corpus has already walked.

3. The Second Theorem: Local Gauge Symmetry → Identities and Constraints

Standard fact (Noether's second theorem)

If the action is invariant under a continuous symmetry parametrized by **arbitrary functions of spacetime** (a **local, gauge** symmetry, infinite-dimensional), the output is not a conservation law but **identities / dependencies among the equations of motion** (the Euler–Lagrange equations) — the **Noether identities**.

- **Electromagnetism ($U(1)$ gauge)**: from the antisymmetry of $F^{\mu\nu}$, $\partial_\mu \partial_\nu F^{\mu\nu} \equiv 0$ — this holds **identically** and forces $\partial_\nu J^\nu = 0$.

- **General relativity (diffeomorphism = general covariance)**: the contracted **Bianchi identity** $\nabla_\mu G^{\mu\nu} \equiv 0$ — also holds identically and forces $\nabla_\mu T^{\mu\nu} = 0$.

In general, a local symmetry with r arbitrary functions produces r **identities** among the EL equations.

The output of the second theorem is **different in kind** from the first. It is not a new conserved quantity but an **identity (holding always, before solving the equations)**, and it means "**the field equations are not all independent**" — that is, there is **gauge freedom (redundant degrees of freedom)**. For example, Einstein's equations number 10, but with 4 Bianchi identities only 6 are effectively independent; the remaining 4 "gaps" correspond to the general covariance of freely choosing coordinates (4 arbitrary functions).

4. Distinct Roots: Conservation vs Identity (Why Gauge "Charge" Is a Boundary Term)

Observation 4.1 (the first and second are different theorems = distinct roots)

Under the single phrase "Noether's theorem" stand **two distinct roots**.

First theorem: input = **global** symmetry (rigid, finite-dimensional) / output = **conservation law** (measurable E, p, L, Q).

Second theorem: input = **local** symmetry (gauge, infinite-dimensional) / output = **identities / constraints** (dependence among equations = gauge freedom).

Since both input and output differ, they **must not be conflated**. In particular, jumping to "a local symmetry also has a Noether current giving a conserved charge" is a mistake — the Noether current of a local symmetry is **identically conserved (trivial in the bulk)**, and the meaningful **charge appears as a surface integral at infinity**

$Q = \oint_\infty (\dots) dS$ (a boundary term).

This distinct root answers a puzzle of the corpus. Why do "local symmetry forces an interaction" (Paper 96, Weyl's gauge) and "global symmetry generates conservation" (Paper 80) play **different roles**? — the second theorem is the answer.

Global $\rightarrow$ conservation (first) and local $\rightarrow$ interaction/identity (second) are, from the start, different theorems, hence different roles. The often-confusing fact that "energy and charge in a gauge theory are measured at the boundary" is the fingerprint of the second theorem. That "energy conservation" is subtle in general relativity is exactly this — in a diffeomorphism-invariant theory, energy becomes a **quasi-local, boundary** quantity (the domain of the second theorem).

5. Connection to the Corpus

The second theorem re-tightens several threads of the corpus. **(a) The first theorem = Paper 80**: this paper is its **distinct-root twin**. "The shadow of symmetry" (80) is the first theorem; the second is "the margin (freedom) of symmetry." **(b) Gauge (Paper 96)**: why "local $\rightarrow$ interaction" (96) and "global $\rightarrow$ conservation" (80) are different roles = the second theorem; also why gauge charge is a boundary term. **(c) Constraints / unimodular (Paper 72)**: the second theorem's consequence — general covariance $\rightarrow$ Bianchi identity $\rightarrow$ constraints on Einstein's equations — is the general backbone of Paper 72 (Λ as an integration constant in UG). **(d) The symmetry root of necessity (Paper 86)**: 86 named "symmetry" as one root of necessity; this paper refines it into **two distinct sub-roots — global (conservation) and local (identity)**. **(e) Cartan's forms (Paper 92)**: the Bianchi identity $dF = 0$ (and $d^2 = 0$) shows that identities are naturally written in the language of differential forms (the second theorem is read on 92's alphabet). **(f) The lineage of symmetry (Paper 85)**: it makes explicit the **two phases** of Noether in the Galois $\rightarrow$ Lie $\rightarrow$ Noether lineage (85).

The Author's Personal Note (Layer 3, non-science label / not a claim of mechanism)

In the corpus's lens (metaphor), the Universe OS seems to have two kinds of "symmetry" — a **global symmetry**, unchanged when the whole world is shifted uniformly (from which a "ledger" of conserved quantities is born), and a **local symmetry**, free to choose the calibration at each point (from which not conserved quantities but "the margin of description = gauge freedom" is born). The former looks like the OS's **accounting**, the latter like the OS's **freedom to re-choose coordinates**. But **this carries the non-science label**. What is certain here is only the mathematical point that Noether has two theorems, and global $\rightarrow$ conservation and local $\rightarrow$ identity are different theorems. And a respect not to forget — Emmy Noether, amid the double wall of being a woman and Jewish in early-twentieth-century Germany, championed by Hilbert and Klein, established these two theorems and fled to the United States in 1933. To this master in the corpus's lineage (Paper 85), by separating the two theorems correctly, quiet respect is recorded.

6. Scope (What Is Shown, and What Is Not)

What is shown (descriptive, structural):

- Noether's first theorem (global symmetry $\rightarrow$ conservation, the core of Paper 80) and second theorem (local gauge symmetry $\rightarrow$ identities/constraints = the Bianchi identity) (§§2–3).

- The first and second are different theorems = distinct roots (input: global/local, output: conservation/identity) (§4).
- Why gauge "charge" becomes a boundary term, why "energy conservation" is subtle in GR = the fingerprint of the second theorem (§4); connections to 80/96/72/86 (§5).

What is not shown (restraint):

- **Do not conflate the first and second theorems.** Do not naively expect a conserved charge from a local symmetry (identically conserved in the bulk; charge is a boundary term).
- **Do not mistake an identity (second) for a conservation law (first).** An identity is a manifestation of gauge freedom, not a measurable conserved quantity.
- **No new mathematics or prediction.** Noether's two theorems are standard. Hence Layer 2 (structural interpretation). The OS is Layer-3 non-science.

7. Conclusion

Noether's theorem was not one but **two**. The first theorem the corpus used in Paper 80 (global symmetry $\rightarrow$ conservation) generates measurable conserved quantities. But the second theorem (local gauge symmetry $\rightarrow$ identities/constraints) generates not conserved quantities but identities among the equations — the gauge freedom. Both $\partial_\mu \partial_\nu F^{\mu\nu} \equiv 0$ and $\nabla_\mu G^{\mu\nu} \equiv 0$ are faces of the second theorem.

For "the same Noether," the first and second are **different theorems = distinct roots**. This separation gives correct names to subtleties the corpus met — why gauge charge is a boundary term, why energy in GR is quasi-local. What this paper did is not a new claim but to sort the corpus's symmetry threads (80/85/86/96/72) correctly by the two distinct roots, first and second. To Emmy Noether, one of the greatest mathematicians of the twentieth century, who left two theorems, with that correct separation, the hundred-and-seventh address is returned.

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