

The Geometry Where the Invariant Lives — Minkowski's Spacetime, and Not Conflating the "Geometries of Relativity" — [Paper 108]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

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— *From thinking to syncing.* —

Abstract

To close Chapter 2 (connecting royal-road nodes, Papers 101–108), the corpus places one of the headwaters of the geometric mainstream, Hermann Minkowski. In 1908 he **translated Einstein's special relativity into "the geometry of spacetime"** — "space by itself, and time by itself, are doomed to fade into mere shadows, and only a kind of union of the two preserves an independent reality." Its core is the **invariant interval** $ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$. Time and length change when the observer changes, but this **interval alone is common to everyone (invariant)**. The Lorentz transformation is the **(hyperbolic) rotation of spacetime** preserving this interval, and $ds^2 = 0$ carves the **light cone = the causal structure**. The core of this paper is the **separation of distinct roots (Paper 58)**: the corpus has several "geometries of relativity," easily conflated. **(a) Minkowski's flat spacetime ds^2** (this paper — the geometry of the invariant interval, flat, Lorentzian signature) / **(b) the hyperbolic geometry of velocity space $K = -1/c^2$** (Paper 60 — the space of velocities, negative curvature) / **(c) proper time $\tau = \int ds/c$** (Paper 82 — the invariant accumulated along a worldline). These share "invariant / relativity" but are **distinct roots**. Minkowski is **the geometry of the interval itself (the flat limit)**. Moreover, where Ignatowski (Paper 83) answered "why the Lorentz group" by **group theory**, Minkowski gives the **geometry (the invariant) that group preserves — 83 (the group root) and 108 (the geometry root) are complementary but distinct roots**. And Minkowski's flat spacetime is the **zero-curvature special case** of Riemann's intrinsic curvature (Paper 91) = the flat limit of general relativity, whose algebra is Clifford's spacetime algebra (Paper 104). **Scope**: a Layer-2 structural interpretation of established relativity; no new mathematics and no new prediction (SR is not rewritten). The Universe OS is a Layer-3 non-science lens.

Positioning of this paper (discipline): This paper is a **structural interpretation** of established special relativity (Minkowski spacetime, the invariant interval, the Lorentz transformation as hyperbolic rotation, the light cone). It contains no new mathematics and no new prediction, and **does not rewrite or re-derive SR**. The point is to **separate as distinct roots** the corpus's "geometries of relativity" — Minkowski's flat interval / the hyperbolic geometry of velocity space (60) / proper time (82) — and to place Ignatowski (83, the group root) and Minkowski (108, the geometry root) as complementary distinct roots. (This paper partly overlaps in theme with Papers 82/83/60 — as the finale of Chapter 2, it makes explicit their "geometric core of the invariant.") "Universe OS" is a Layer-3 non-science lens, not a claim of mechanism.

1. Introduction: The Man Who Made Relativity into "Geometry" (the Finale of Chapter 2)

Chapter 2 (Papers 101–108) has applied the corpus's jigs to the royal-road nodes of the lineage map, cutting new roads — Eddington, Connes–Kac, Perelman, Clifford, Donaldson, Wiener, Noether. To close it, the corpus places one of the headwaters of the geometric mainstream, Hermann Minkowski (1864–1909). Once Einstein's teacher, in his 1908 lecture "Space and Time" he translated special relativity into **a single geometry called spacetime**.

The corpus has touched "invariant" and "relativity" many times — the curvature of velocity space (60), the Renko chart = proper time (82), why a maximum speed exists (83). But it had never squarely placed their **geometric core = the invariant interval**. This paper places it, and at the same time sorts the "geometries of relativity" as **distinct roots**. It builds no new theorem.

2. Minkowski's Spacetime: Invariant Interval, Hyperbolic Rotation, Light Cone

Standard fact (Minkowski spacetime, 1908)

Minkowski unified time and space into one 4-dimensional **spacetime** and placed upon it the **invariant interval**:

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2.$$

When the observer (inertial frame) changes, dt and dx change, but ds^2 takes the same value in every inertial frame (invariant). The **Lorentz transformation** between inertial frames is the transformation preserving this ds^2 = a **hyperbolic rotation** in the (t, x) plane of spacetime (a speed v corresponds to rapidity φ , $\tanh \varphi = v/c$). The surface $ds^2 = 0$ is the **light cone**, and the distinction timelike ($ds^2 > 0$) / spacelike ($ds^2 < 0$) / null ($= 0$) is the very structure of causality.

This transformed the view of relativity. Where Einstein showed "time dilation and length contraction" as **phenomena**, Minkowski showed that "they are merely **shadows** of one invariant spacetime geometry, seen from a different angle by

each observer." Time and length change with the observer — but **the interval ds^2 is common to all**. Beneath the relative lies an absolute geometry.

3. What Is Absolute: Not Space, Not Time, but the Interval

The meaning of Minkowski's famous declaration — "space by itself, and time by itself, are doomed to fade into mere shadows, and only a kind of union of the two preserves an independent reality" — lies here. **What is absolute is not space, not time, but the spacetime interval ds^2** . Newton placed absolute space and absolute time separately (cf. the Leibniz–Clarke debate, Paper 97), but in relativity **those two become "shadows" whose apportionment changes with the observer**, and only the **interval woven from both remains invariant**.

This is the **geometric core** of the corpus's "invariant." The corpus has often asked "what is invariant, what is a convention" (Paper 95: the value of c is a convention, the finiteness of c is a fact). Minkowski gives, on the "fact" side, a **concrete invariant — the interval**. Relativity is not "everything is relative" but **"the interval is absolute, and time and space are relative as its shadows."**

4. Distinct Roots: The Three Geometries of Relativity, and Complementarity with the Group

Observation 4.1 (the "geometry of relativity" is not one)

The corpus has several "geometries of relativity," sharing words (relativity, invariant, curvature) but of **distinct roots**.

(a) **Minkowski's flat spacetime** (this paper): the stage is **spacetime (t, x, y, z)** , the geometry is **flat** with Lorentzian signature, the protagonist is the **invariant interval ds^2** .

(b) **The hyperbolic geometry of velocity space** (Paper 60): the stage is the space of **velocities**, the geometry is **negatively curved $K = -1/c^2$** (hyperbolic), the protagonist is the composition of velocities and **E** read from curvature.

(c) **Proper time** (Paper 82): the protagonist is the invariant **accumulated along** a worldline, **$\tau = \int ds/c$** (the core of event-based time).

The three share "invariant / relativity" but are **distinct roots** — different stage (spacetime / velocity space), different curvature (flat / negative), different quantity (interval / composition / proper time). Minkowski is **the geometry of the interval itself (the flat limit)**.

Observation 4.2 (the group root 83 and the geometry root 108 are complementary)

Paper 83 (Ignatowski) answered "why the Lorentz group" by **group theory** — from the relativity principle + homogeneity/isotropy + group structure, the transformations are limited to the Galilei type or the Lorentz type. Minkowski gives the **geometry (the invariant interval) that the Lorentz group preserves**. **83 = the group root (what is the transformation group)** and **108 = the geometry root (what invariant that group preserves)** are **complementary but distinct roots**. Not "group first or geometry first," but two faces of the same relativity (the transformation side and the invariant side).

This separation befits the finale of Chapter 2. Under the single word "relativity" stand **several distinct roots** — the interval of flat spacetime (108), the hyperbolic geometry of velocity space (60), proper time (82), the Lorentz group (83). All are different faces of the same relativity and must not be mixed — the relativity version of the corpus's "one word, distinct roots" (Paper 100).

5. Connection to the Corpus

Minkowski tightens several threads of the corpus. **(a) Riemann (Paper 91) = the flat limit**: Minkowski's flat spacetime is the **zero-curvature** special case of Riemann's intrinsic curvature. The **tangent space at each point (the local flat limit)** of general relativity (curved spacetime) is Minkowski spacetime; 91 (intrinsic curvature) and 108 (flat interval) are distinct instances joined by "curved / flat." **(b) Clifford (Paper 104) = the algebra of spacetime**: the Clifford algebra of Minkowski space (the spacetime algebra $\text{Cl}(1, 3)$, gamma matrices) gives the algebraic skeleton of this geometry (108's geometry rides on 104's alphabet). **(c) Ignatowski (Paper 83)**: the group root (83) and the geometry root (108) are complementary distinct roots. **(d) Velocity space / proper time (Papers 60/82)**: distinct roots among the three geometries of relativity. **(e) Convention and fact (Paper 95)**: it gives the "fact" of c 's finiteness a concrete invariant, the interval. **(f) Cartan (Paper 92): ds^2** is a metric (a symmetric tensor), of a different layer from the exterior algebra (92, metric-free) — it belongs to the metric-holding Clifford side (104) (a concrete instance of the 92/104 distinct root).

The Author's Personal Note (Layer 3, non-science label / not a claim of mechanism)

In the corpus's lens (metaphor), for the Universe OS the "real data" is neither time nor space but one invariant, the spacetime interval ds^2 , and each observer's time and length feel like the **rendering (shadow)** of that invariant projected onto their own screen. Minkowski's "space and time fade into shadows" sounds like an echo of a design principle that **separates the OS's internal representation (the invariant) from its display (each observer's coordinates)**. But **this carries the non-science label**. What is certain here is only the physical point that the absolute quantity of relativity is neither space nor time but the interval ds^2 . And the corpus's discipline — the "geometry of relativity" is not one, but several distinct roots stand side by side: the interval (108), velocity space (60), proper time (82), the group (83). Finding an absolute invariant beneath the relative — that stance shares the same blood as the corpus's core of separating "what is a convention, what is a fact" (95). To the headwater master who made relativity into geometry, quiet respect is recorded.

6. Scope (What Is Shown, and What Is Not)

What is shown (descriptive, structural):

- Minkowski's spacetime, the invariant interval ds^2 , the Lorentz transformation as hyperbolic rotation, the light cone (§2); what is absolute is not space or time but the interval (§3).
- The distinct roots of the three geometries of relativity (Minkowski's flat interval / the hyperbolic geometry of velocity space 60 / proper time 82) (§4).
- The group root (Ignatowski 83) and the geometry root (108) are complementary but distinct (§4); connections to the flat limit of Riemann 91 and the spacetime algebra of Clifford 104 (§5).

What is not shown (restraint):

- **SR is not rewritten or re-derived.** This is a structural interpretation of Minkowski's standard geometric formulation.
- **Do not conflate the geometries of relativity.** The interval (108), velocity space (60), proper time (82), and the group (83) are distinct roots.
- **No new mathematics or prediction.** Minkowski spacetime is standard. Hence Layer 2 (structural interpretation). The OS is Layer-3 non-science.

7. Conclusion (the Finale of Chapter 2)

Minkowski made relativity into "the geometry of spacetime." What is absolute is neither space nor time but the **invariant interval ds^2** woven from both — time dilation and length contraction are "shadows" of this single invariant. The Lorentz

transformation is the hyperbolic rotation preserving the interval; the light cone is the skeleton of causality. This is the geometric core of the "invariant" the corpus has touched again and again.

And the "geometry of relativity" is not one — Minkowski's flat interval (108), the hyperbolic geometry of velocity space (60), proper time (82), and the Lorentz group (83) are distinct roots sharing a word. What this paper did is not a new claim but to place the geometric core of the invariant and to sort correctly the several geometries of relativity as distinct roots. With this, the **eight papers of Chapter 2 (connecting royal-road nodes, 101–108)** are complete — from Eddington's caution, through Connes–Kac's bridge, Perelman's dynamical uniqueness, Clifford's algebra, Donaldson's four dimensions, Wiener's signal, Noether's two theorems, to Minkowski's interval. Each is a road cut by applying the corpus's jigs to a royal-road node and grafting it by the discipline of distinct roots. To the headwater master who made relativity into geometry, as the finale of Chapter 2, the hundred-and-eighth address is returned.

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