

Factors That Cancel in Normalisation, and the Exponent 2 of Born's Rule

— 3rd Edition: What a Metaphor Can and Cannot Do — [Paper 11]

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— *From thinking to syncing.* —

ABSTRACT

This is the first paper this series wrote about quantum measurement. The 3rd edition **recasts the mistake made there into a discipline of general form, and maps where the questions raised there were later taken up. First, a factor multiplying a normalised probability that does not depend on the variable vanishes by definition.** For any $c \neq 0$, $cf/\int cf = f/\int f$ (verified numerically here: with $c = 4\pi, 16\pi^2, 10^9$ the probability densities agree to 10^{-16}). **Moreover c need not even be constant: any quantity not depending on the integration variable will do. Hence if a proposed "mechanism" enters the expression only as an overall factor, it is unverifiable in principle.** This is offered as a new **check** for the series, and the first edition's 4π patch fails it. **Second, the metaphor of "lazy evaluation" fails the machinery-transfer test.** Lazy evaluation in computer science is a structure in which **the value is determined and only its computation is deferred** (a thunk) — and **that very core is what the Kochen–Specker theorem forbids.** The Peres–Mermin square is computed here, confirming that **the row products and the column products come out $+1$ and -1 :** non-contextual value assignment is impossible **independently of the state. The picture transfers; the machinery does not** (Paper 100). **Third, the repaired metaphor computes nothing.** Reading the state as a context-dependent thunk is consistent with quantum mechanics, but **consistency and prediction are different things.** This paper keeps the metaphor as **a mnemonic for a theorem** and does not claim it as a model. **Fourth, no-signalling is a constraint, not an option.** In a Bell state, whichever of $\sigma_z, \sigma_x, \sigma_y$ Bob measures, Alice's reduced density matrix stays $I/2$ (verified here: norm of the difference $\leq 3.1 \times 10^{-16}$). And CHSH gives $|S| = 2.8284271247 = 2\sqrt{2}$ exactly, saturating the Tsirelson bound — **violating Bell's inequality is a requirement, not a merit.** **Fifth, what forces the exponent 2 in Born's rule has at least three addresses.** Paper 71 (**Gleason's theorem:** in dimension ≥ 3 every probability measure is $\text{tr}(\rho P)$ — **a theorem, not a postulate**); Paper 53 (**unitarity:** for $p \neq 2$ the isometries of ℓ^p are only phases and permutations — Banach–Lamperti); Paper 68 (**the 2 of the Hopf map:** same root or rhyme, undecided). **This paper does not decide which is the root. Scope:** a structural account within established quantum mechanics (Layer 2). **The quantum measurement problem is unsolved. This paper claims neither a derivation of Born's rule, nor an account of the collapse mechanism, nor a solution of the measurement problem.**

Positioning of this paper (discipline): This paper offers a **structural interpretation** of established physical laws and mathematical theorems. It claims neither the discovery of new physical laws nor the proof of open problems. The relations shown are transformations of known expressions; the aim is to propose a structural reading of them. No claim contradicting established theory is made. **The appearance of the same number in several contexts is not treated as evidence of a common origin** (Papers 48, 86, 100, 115; §6 states this explicitly for the exponent **2**). **The quantum measurement problem is unsolved and nothing here changes that.**

1. Introduction: what this paper asked

The first edition asked one question — **what does a system hold while it is not being observed?**

It answered "the universe's OS is evaluating it lazily", went on to say it had "derived" Born's rule, and concluded that it had "given a complete solution" to the measurement problem. **None of that stands.**

**The work of the 3rd edition is to turn the failure into two assets.
One is a discipline recast in general form. The other is a map of where the questions were taken up.**

2. A factor that cancels in normalisation cannot be observed, by definition

The first edition "derived" Born's rule as follows:

$$P(\mathbf{x}) = \frac{|\psi(\mathbf{x})|^2 \cdot \text{Vol}(S^2)}{\int |\psi(\mathbf{x})|^2 \cdot \text{Vol}(S^2) d^3\mathbf{x}} = |\psi(\mathbf{x})|^2 \quad (\text{1st ed., withdrawn})$$

The second edition withdrew this, noting that "**4π** cancels and contributes nothing". **The 3rd edition rewrites that observation as a check of general use.**

Theorem 2.1 (Normalisation invariance — an overall factor vanishes from observation)

Let $f(\mathbf{x}) \geq 0$ be integrable and non-trivial, and let c be any non-zero quantity **not depending on the integration variable $\mathbf{x}$** . Then

$$\frac{c f(\mathbf{x})}{\int c f(\mathbf{x}) d\mathbf{x}} = \frac{f(\mathbf{x})}{\int f(\mathbf{x}) d\mathbf{x}}$$

identically. **That is, c does not appear anywhere in the normalised probability.**

Proof: c comes outside the integral and cancels between numerator and denominator. It need not be a constant: any function of other parameters will do, so long as it does not depend on $\mathbf{x}$. ■

Numerical check ($f = e^{-x^2}$ on $x \in [-4, 4]$, trapezoidal over **200001** points, computed here):

factor c inserted	$\max_x P_c(x) - P_1(x) $
$4\pi = 12.566$	3.3×10^{-16}
$16\pi^2 = 157.91$	1.1×10^{-16}
10^9	1.1×10^{-16}

A factor of 10^9 leaves the probability density unchanged to machine precision.

Corollary 2.2 (The check: a mechanism entering only as an overall factor is unverifiable)

If a proposed "mechanism" appears in a normalised probability **only as an overall factor**, then **it cannot in principle be distinguished by observation**. This does not mean "the experiment has not been done yet"; it means **it has cancelled by definition**.

The first edition's 4π patch fails this check. Separately, placing $|\psi|^2$ in the numerator in order to reach $|\psi|^2$ on the right was **assuming what was to be derived** — two distinct errors, the second surviving any repair of the first.

The same error occurred in the first edition of Paper 09, in $16\pi^2/(4\pi)^2 = 1$ (withdrawn at the same time).

Dividing a constant by itself, or multiplying numerator and denominator by the same quantity, constitutes no mechanism whatever.

This check is usable outside this paper. Paper 35 sorted the two 4π 's inside $4\pi G$ into a **fact** (Gauss's flux) and a **convention** (a choice of units); what this section adds is a **third category** — **quantities that do not survive in the expression at all. Neither fact nor convention: they cancel.**

3. The metaphor fails the machinery-transfer test

The first edition called the unobserved state **lazy evaluation**. The second edition repaired this as "suggesting hidden variables". **The 3rd edition states why it fails, in the form of the machinery-transfer test (Paper 100).**

Observation 3.1 (The core of the metaphor is exactly what the theorem forbids)

Lazy evaluation in computer science is a structure in which **the value is determined but its computation is deferred** (a thunk). A thunk **wraps a fixed computation** and returns the same value whenever it is forced.

This "determined but unevaluated" is the core of the metaphor. And it is precisely what the Kochen–Specker theorem (1967) forbids: in a Hilbert space of dimension ≥ 3 there is **no assignment of values independent of the measurement context**.

What fails is therefore not the periphery of the metaphor but its engine. In the vocabulary of Paper 100, **the picture transfers but the machinery does not** — nothing in denotational semantics computes any quantum prediction.

Established Fact 3.2 (The Peres–Mermin square — a state-independent counterexample)

Arrange nine observables of a two-qubit system as

$$\begin{array}{ccc} \sigma_x \otimes I & I \otimes \sigma_x & \sigma_x \otimes \sigma_x \\ I \otimes \sigma_y & \sigma_y \otimes I & \sigma_y \otimes \sigma_y \\ \sigma_x \otimes \sigma_y & \sigma_y \otimes \sigma_x & \sigma_z \otimes \sigma_z \end{array}$$

Each has eigenvalues ± 1 , and **the three in each row and the three in each column commute** (verified numerically here), so each triple is jointly measurable. Yet the products are

	first	second	third
row product	$+I$	$+I$	$+I$
column product	$+I$	$+I$	$-I$

(computed here). If ± 1 could be assigned non-contextually, the total product of the nine values would be the same read by rows or by columns; but it comes out **$+1$ and -1 . A contradiction.**

This counterexample does not depend on the state. It fails for whatever quantum state one prepares — **"the preparation happened to be bad"** is not available.

4. The repaired metaphor: a context-dependent thunk — which computes nothing

Observation 4.1 (The content of the repair)

If the metaphor is to be kept, its thunk must be **context-dependent: the result of evaluation depends on what it was asked together with, and in what order, and there is no determined value prior to the question.** The computer-science side has counterparts (a closure carrying its environment; a deferred computation whose result depends on the calling context).

Numerical check — the smallest example of order-dependence (computed here). Take the initial state $|0\rangle$ (the $+1$ eigenstate of σ_z):

procedure	distribution of σ_z
measure σ_z alone	+1 with probability 1.0
measure σ_x , then σ_z	+1 with 0.5, -1 with 0.5

The same σ_z value changes according to what was inserted before it. It is not that the value was fixed and merely read out.

Observation 4.2 (But consistency and prediction are different)

The repaired metaphor **is consistent** with quantum mechanics. **And that is all — no number comes out of it.**

This paper keeps it as "a mnemonic for a theorem" and does not claim it as a model. Paper 93 wrote of Faraday that **intuition can precede the mathematics**; but there, **machinery was later supplied. It has not been supplied here, and this paper does not claim it will be.** The value of a mnemonic and the standing of a model are different things.

5. No-signalling is a constraint, not an option

The first edition wrote that measuring one half of an entangled pair "instantly renders the opposite state on the other side". **That expression has been withdrawn.** The 3rd edition shows why, numerically.

Established Fact 5.1 (Whatever Bob does, Alice's local statistics do not change)

In the Bell state $|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$, Alice's reduced density matrix before any measurement is $\rho_A = I/2$.
After Bob measures (computed here):

observable Bob measured	$\ \rho_A - \rho_A^{(\text{before})}\ $
σ_z	0
σ_x	3.1×10^{-16}
σ_y	3.1×10^{-16}

In whatever basis he measures, nothing changes on Alice's side. The correlation appears **only once the outcomes are compared over a classical channel.** **Non-local correlation permits no superluminal communication** (the no-signalling theorem; Ghirardi–Rimini–Weber 1980, Eberhard 1978).

Established Fact 5.2 (CHSH — violation is a requirement, not a merit)

For the singlet state at optimal angles, the computation here gives

$$|S| = 2.8284271247 \dots = 2\sqrt{2},$$

saturating the Tsirelson bound exactly. The local-realistic bound is **2**; the quantum bound is **$2\sqrt{2}$** .

Any model reproducing the predictions of quantum mechanics must (under the standard assumptions, including measurement independence) violate Bell's inequality. So "this model is consistent with the Bell violation" is not a merit of the model but a minimum requirement.

Paper 112 took a census of six distinct roots beneath the intuition "**a region is allotted, and within it one is free**", and wrote that **the Bell line is the one that is not conceded. This section is that line.**

6. What forces the exponent 2 — three addresses, and this paper does not pick the root

Where the first edition claimed to "derive" Born's rule, **three established answers already stood, in three different forms.**

Established Fact 6.1 (Address 1: Gleason's theorem — Paper 71)

In a Hilbert space of dimension ≥ 3 , every probability measure on closed subspaces **must** take the form $P(V) = \text{tr}(\rho P_V)$ (Gleason 1957). **Born's rule is a theorem, not a postulate.** Paper 112 took this up as a contrast of distinct roots: **equal a priori probability in statistical mechanics is a postulate; quantum probability is a theorem.**

But Gleason is silent in dimension 2 — the gap named in the subtitle of Paper 71 ("where Gleason is silent"), filled by the POVM extension (Busch 2003).

Established Fact 6.2 (Address 2: unitarity and the isometries of ℓ^p — Paper 53)

For $p \neq 2$, every surjective linear isometry of ℓ^p is a **permutation composed with phases** (Banach 1932; Lamperti 1958). That is, for $p \neq 2$ the isometries **preserve the multiset of amplitude moduli — no interference is possible.**

Numerical check (a random unitary U in four dimensions, computed here):

p	$\sum \psi_i ^p$	$\sum (U\psi)_i ^p$
1	1.849855	1.990010
1.5	1.346287	1.408954
2	1.000000000000	1.000000000000
2.5	0.751105	0.711435
3	0.567681	0.507306
4	0.327466	0.259656

Over 2000 random vectors the largest deviation is 0.554 at $p = 1$ and 0.365 at $p = 3$, but 8.9×10^{-16} at $p = 2$ (machine precision). **For any p other than 2, time evolution destroys the normalisation of probability.**

Established Fact 6.3 (Address 3: the 2 of the Hopf map — Paper 68)

The state space of a qubit carries the structure of the Hopf fibration $S^3 \rightarrow S^2$, in which a **2** also appears. **Paper 68** posed this as "same root or rhyme?" and reserved judgement.

Observation 6.4 (This paper does not pick the root)

All three arrive at the exponent **2**, but **by different arguments** — (1) is **measure theory** (a classification of frame functions), (2) is **functional analysis** (a classification of isometries), (3) is **topology** (the structure of a bundle).

The discipline of this series is that the same number is not evidence of the same origin (Papers 48, 86, 100, 115). **This paper therefore does not decide which is root and which is derived.** To decide would require showing that one machinery yields the other, and that is not shown here.

**The error of the first edition was to build a fourth answer
without knowing that three established ones already existed.**

7. Where this paper stands within the series

This paper (Paper 11) is the series' **earliest attempt** on quantum measurement. The questions raised here were subsequently taken up at their proper addresses.

Question raised here	Paper that took it up
Why is probability the square of $ \psi $?	Papers 71 (Gleason), 53 (unitarity), 68 (Hopf)
What does an unobserved system hold?	Paper 55 (macroscopic superposition; GRW/CSL/Penrose)
A 3D wave seen on a 2D screen	Paper 41 (double slit; explicit interference term)
Measurement and time	Paper 54 (Pauli's theorem, Page–Wootters)
"A region is allotted, freedom within"	Paper 112 (census of six roots; postulate vs theorem)

For the series' present position on Born's rule and the measurement problem, see Papers 71, 53 and 55 rather than this one.

Observation 7.1 ("The universe's OS" is Layer 3)

The first edition spoke of "the universe's OS rendering". **Paper 82 explicitly quarantined this kind of language (OS, consciousness, rendering) in Layer 3, the non-scientific layer. This paper follows that discipline: the vocabulary of computing is a tool of invention, not a claim about physics.**

That the author brings this vocabulary is not itself the problem. As Paper 89 recorded, **bringing in a toolbox and claiming that the tools work** are different acts. **Only the latter failed.**

Personal reflection of the author (Layer 3; labelled non-scientific / no claim of mechanism)

In the first edition I multiplied numerator and denominator by the same 4π and called it a derivation. **I called a number that cancels a mechanism.** When the second edition withdrew it, I wrote " 4π contributes nothing" and left it there.

Writing the 3rd edition I saw that **stopping there had been a waste.** The mistake had **a general form: a mechanism that enters only as an overall factor cannot be observed, by definition.** That is a check I can turn on my own other papers — the failure left behind a discipline.

And one more thing. I brought the metaphor of "lazy evaluation" from the domain I know (computers, and market systems). **Bringing it is simply how I work.** But what I learned this time is that **what failed was not the periphery of the metaphor but its engine** — "determined but unevaluated", the very heart of it, is what Kochen–Specker forbids.

I will not stop importing metaphors. But **after importing one, I will look at whether its engine touches a theorem.** That is this edition's harvest. And the fact that **I was building a fourth answer in a place where three established ones already stood** is worth putting on the record.

8. Scope / Conclusion

What is shown: that **an overall factor in a normalised probability cannot be observed by definition**, and that this serves as **a check** (§2, verified numerically) / that **the "lazy evaluation" metaphor fails the machinery-transfer test, and fails at its engine**, with a **state-independent counterexample from the Peres–Mermin square** (§3) / that **the repaired metaphor is consistent but computes nothing** (§4) / that **no-signalling is a constraint rather than an option**, that **CHSH saturates the Tsirelson bound**, and that **the Bell violation is a requirement not a merit** (§5) / that **the exponent 2 has three**

established addresses and this paper does not pick the root (§6) / a map of where this paper's questions were taken up (§7).

What is not shown (restraint): no solution of the measurement problem, no derivation of Born's rule, no account of the collapse mechanism — all outside the scope of this paper / **the context-dependent think is not claimed as a model** (it makes no prediction) / **no decision as to which of the three addresses of the exponent 2 is the root** / **the vocabulary of computing (OS, rendering) is not used as a claim about physics** (Layer 3; Paper 82) / no new physical law and no prediction (Layer 2).

The quantum measurement problem is unsolved. Nothing here changes that.

What the 3rd edition adds are two assets extracted from a failure. One is the **overall-factor check** — a third category besides fact and convention: **quantities that do not survive in the expression at all**. The other is a **way of judging a metaphor's standing** — asking, **at the engine**, whether the picture transferred or the machinery did.

And the question itself is alive.

"What does a system hold while it is not being observed?" was given a stricter form in Papers 55, 71, 53 and 112.

Revision Record

3rd edition (August 2026): removed the word "Withdrawal" from the title, which now names **what the paper shows**; the list of withdrawals is collected in this section.

Added in the 3rd edition: a statement, as a **general proposition**, that **an overall factor in a normalised probability cannot be observed by definition** (c need not be constant, only independent of the integration variable), formulated as a **check** (§2, verified with $c = 4\pi, 16\pi^2, 10^9$). An account of **why the "lazy evaluation" metaphor fails the machinery-transfer test (Paper 100), and why it fails at its engine** ("determined but unevaluated"), together with a self-computed **Peres–Mermin square as a state-independent counterexample** (§3). An explicit statement that **the repaired metaphor, though consistent, computes nothing**, and the contrast with Paper 93 (Faraday), **where machinery was later supplied** (§4). **Numerical confirmation of no-signalling via the reduced density matrix**, and a computation giving **CHSH exactly $2\sqrt{2}$** (§5). **The three addresses of the exponent 2** (Paper 71 Gleason's theorem / Paper 53 unitarity with the **Banach–Lamperti theorem** / Paper 68 the Hopf map), with an explicit statement that **this paper does not pick the root** (§6). **A table of where this paper's questions were taken up**, and a note that **the "universe's OS" vocabulary is quarantined in Layer 3 by the discipline of Paper 82** (§7). The magic square, the reduced density matrices, CHSH, the ℓ^p norms and the normalisation invariance were **all computed here** (no external tables consulted).

Withdrawn in the 2nd edition (July 2026): the first edition's subtitle claimed "an information-theoretic resolution of the measurement problem and of wave-function collapse, and a geometric derivation of Born's rule", and its conclusion said it had "given a complete solution". **That claim was withdrawn in full.**

1. **The derivation of Born's rule was circular.** Theorem 3.2 of the first edition presented $P = |\psi|^2 \text{Vol}(S^2) / \int |\psi|^2 \text{Vol}(S^2)$, in which $\text{Vol}(S^2) = 4\pi$ **cancels and contributes nothing**, while $|\psi|^2$ **had already been placed in the numerator in order to reach $|\psi|^2$** — assuming what was to be derived.
2. **The "lazy evaluation" metaphor implied non-contextual hidden variables.** By the Kochen–Specker theorem (1967), no non-contextual value assignment exists in dimension ≥ 3 , so **lazy evaluation taken literally is inconsistent with quantum mechanics.**
3. **The account of non-locality did not mention no-signalling.** The first edition's "instantly renders the opposite state on the other 3D UI" **reads as though an observable change appears instantly at a distance. That expression was withdrawn.** Furthermore, **consistency with the Bell violation is a requirement of the model, not a merit.**

First edition: Zenodo 10.5281/zenodo.21404636.

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