

Ricci Flow Is a Renormalization Group Flow

— Where Two Lineages Shake Hands over “Taming the Infinite,” a Census of a Royal-Road Bridge — [Paper
110]

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— *From thinking to syncing.* —

Abstract

Two lineages have run through this corpus — the **royal road of geometry** (Riemann $\rightarrow \dots \rightarrow$ Ricci flow / Perelman, Papers 91 and 103) and the **royal road of analysis and field theory** (the $1/16\pi^2$ of loop integrals, the renormalization group, Paper 109). This paper records, as a **Layer-2 census of a bridge**, that these two **shake hands not by metaphor but by one proven equation**. The keystone is **Friedan’s theorem** (1980): the **renormalization-group (RG) beta function of the target metric g_{ij} of a two-dimensional nonlinear sigma model equals, at one loop, the Ricci tensor itself**: $\beta_{ij}^g = \alpha' R_{ij} + O(\alpha'^2)$. Hence the RG flow of the metric, $\partial_t g_{ij} = -\alpha' R_{ij}$, is **Ricci flow** $\partial_\tau g_{ij} = -2R_{ij}$ itself (up to a constant and the direction of flow). The fixed point $\beta = 0$ is **Ricci-flatness = the vacuum Einstein equation** (the route by which string theory “derives” gravity). The shared backbone is “**taming the infinite**” — on the geometry side the blow-up of curvature (singularities), on the field-theory side the ultraviolet divergences, both governed by a **scale-dependent flow**. The paper sorts the findings into **three tiers**: **gold (proven)** = Ricci flow = RG flow (Friedan); **silver (a real resonance, unsettled)** = Perelman’s monotone entropy $\leftrightarrow$ Zamolodchikov’s c-theorem (both give the flow an “arrow”); **gold-plated (a rhyme)** = surgery $\approx$ renormalization (the theme is the same, the machinery differs: topological excision vs absorbing divergences). The equation is **exact at one loop (α' leading order)**; higher orders bring higher-derivative corrections.

Scope of this paper (scope note): This paper is a **structural interpretation / census of a bridge** (Layer 2) between established mathematics and physics (the sigma-model beta function = Friedan, Ricci flow = Hamilton and Perelman, the c-theorem = Zamolodchikov, the string background-field equations). It proves no new theorem and makes no new physics or prediction. **“Ricci flow = RG flow” is exact at one loop (α' leading order)**; higher orders bring higher-curvature corrections. Findings are labelled in **three tiers — gold (proven) / silver (a real resonance but unsettled) / gold-plated (a rhyme: same theme, different machinery)** — and silver and gold-plated are not written as “the same”. The bridge is one bridge, not a unification of all physics (a GUT). Resonance is a signature, not a proof (Paper 100), and there is no post-hoc numerical adjustment (Paper 101). “Universe OS” and “the universe renormalizing itself” are Layer-3 non-science lenses, not claims of mechanism. Relation to earlier papers: intrinsic curvature comes from Paper 91, Ricci flow and Perelman’s work from Paper 103, the $1/16\pi^2$ of loop integrals from Paper 109, the form of a bridge node from Paper 102, and the discipline of distinct roots from Papers 58 and 86. **What this paper adds** is to sort the meeting of the geometry lineage and the field-theory lineage into three tiers — **Friedan’s equation (gold), the arrow of the flow (silver), the handling of infinity (gold-plated)** — and to set them out in a single table.

1. Introduction: Do the Two Lineages Really Meet?

Two great lineages have run through this corpus. One is the **royal road of geometry** — curvature grasped intrinsically (Paper 91) and then moved, smoothed toward a canonical form, by **Ricci flow** and Perelman’s work (Paper 103). The other is the **royal road of analysis and field theory** — the $1/16\pi^2$ appearing in loop integrals (Paper 109) and the **renormalization group** that governs those divergences by scale-dependence. At the level of metaphor the two are alike: “in both, something flows with scale”. But — **is that a rhyme, or a proven isomorphism?**

This paper’s claim is plain: **the two shake hands by one proven equation**. Its keystone is Friedan’s theorem — “Ricci flow is, in fact, a renormalization group flow”. The paper proves no new theorem. It rereads known results under **the frame of “a bridge between two lineages”** (a bridge node, in the style of Paper 102), and sorts the findings into **three tiers — gold, silver, gold-plated**. This discipline of sorting is what guards the paper from numerology: even under the same “flow with scale”, a proven isomorphism (gold), a resonance under active research (silver), and a theme-only rhyme (gold-plated) are kept apart.

2. Gold: Friedan's Theorem — Ricci Flow = RG Flow (Proven)

Established Fact 2.1 (Friedan 1980; Callan–Friedan–Martinec–Perry 1985)

Consider a two-dimensional nonlinear sigma model whose target-space metric is $g_{ij}(x)$. The **renormalization-group beta function** of this theory equals, at one loop (α' leading order), the target-space **Ricci tensor** itself:

$$\beta_{ij}^g = \alpha' R_{ij} + O(\alpha'^2).$$

Hence the flow of the metric with the renormalization scale μ (integrating out short-distance modes toward the long-distance / IR “coarse-graining direction”) is

$$\frac{\partial g_{ij}}{\partial t} = -\alpha' R_{ij} \quad (t = \text{RG time})$$

which — up to the constant $\alpha'/2$ and the direction of flow (smoothing = forward Ricci flow) — **is Hamilton's Ricci flow** $\partial_\tau g_{ij} = -2R_{ij}$ itself.

This is the keystone of the paper, and it is **gold (proven)**. Ricci flow, discovered on the geometry side as “a heat-like flow that smooths a manifold”, was on the field-theory side the very **renormalization group flow** — “how the target geometry runs as the energy scale is lowered”. The two lineages had been writing, from different motives, **the same equation**. This is a genuine isomorphism in which the machinery transfers wholesale — it means the tools of sigma-model RG computation and the tools of Ricci-flow geometric analysis are, in principle, translatable into one another.

Corollary 2.2 (Fixed point = Ricci-flat = vacuum Einstein)

The RG **fixed point** $\beta_{ij}^g = 0$ means $R_{ij} = 0$, i.e. **Ricci-flatness**. In the complete version, where the target space also carries an antisymmetric tensor field and a dilaton, the vanishing of the beta functions coincides with the **Einstein equation (and the associated field equations)** (CFMP 1985). That is, “**the string worldsheet being conformally invariant**” = “**the target spacetime satisfying the equations of gravity**” — the route by which string theory “derives” general relativity lies at this zero of the beta function.

An honest line: “**Ricci flow = RG flow**” is exact at one loop (α' leading order). From two loops on, higher-curvature corrections enter (e.g. $\beta_{ij} = \alpha' R_{ij} + \frac{\alpha'^2}{2} R_{iklm} R_j{}^{klm} + \dots$), and the “exact RG flow” is a corrected Ricci flow containing higher-derivative terms. The paper therefore states “the two are identical at leading order, close kin at all orders”, and does not claim uncorrected full identity.

Kholodenko (2008), noting that Friedan’s result rests on truncating the beta function at the lowest order, sought an independent justification for Ricci and Yamabe flows being realized in nature by extending two-dimensional conformal field theory to three and more dimensions, as a step toward more physically motivated proofs of the Poincaré and geometrization conjectures. This paper does not enter into that programme; it only records the one-loop equality above.

3. The Shared Backbone: “Taming the Infinite”

Why should the flow of geometry and the flow of field theory be the same equation? Behind it lies **one human theme shared by both lineages** — **taming the infinite without being crushed by it**.

Observation 3.1 (The same theme, two arenas)

- **The infinity on the geometry side:** Ricci flow produces **singularities where curvature blows up** in finite time (the neckpinch, where a neck shrinks to zero). Perelman’s **surgery** cuts the neck just before the blow-up, caps it, and continues the flow — **the craft of governing geometric infinity** (Paper 103).
- **The infinity on the field-theory side:** loop integrals produce **ultraviolet divergences** (infinities from short distances / high momenta). **Renormalization** absorbs the divergence into redefined couplings and extracts finite physical quantities — **the craft of governing field-theoretic infinity** (the $1/16\pi^2$ of Paper 109 lives in this arena).

And the deep point is this: **the renormalization group is precisely the framework that “organizes the ultraviolet infinite through scale-dependence”**, and its metric version is **Ricci flow**. Thus the theme “taming the infinite” is not a mere metaphor: **by the gold equation of Section 2 it is actually translated from one side (RG) to the other (Ricci flow)**. The theme gives the bridge its *meaning*; Friedan’s equation gives the bridge its *girder*.

4. Silver: Perelman's Entropy $\leftrightarrow$ Zamolodchikov's c-Theorem (a Real Resonance, Unsettled)

Observation 4.1 (Both give the flow an “arrow” via a monotone quantity)

- **Zamolodchikov's c-theorem** (1986): in a two-dimensional field theory there exists a function $\mathcal{C}$ that **decreases monotonically** along the RG flow, stationary at fixed points where it equals the central charge. It gives the **irreversibility of the RG flow (the UV $\rightarrow$ IR arrow)**.
- **Perelman's monotonicity** (2002): along Ricci flow there exist $\mathcal{F} / \mathcal{W}$ functionals (entropies) that **increase monotonically**, stationary at gradient Ricci solitons (effective fixed points). They give Ricci flow an **arrow** and rule out periodic solutions (breathers).

If one accepts the gold of Section 2 (Ricci flow = RG flow), then the question “**is Perelman's monotone entropy a geometric c-function?**” arises naturally — both state “the flow runs only one way” as a monotone quantity stationary at fixed points. This is a **real resonance under active research**. Still, Perelman's functional is for the flow of the target space, while the c-theorem is about the central charge of the worldsheet CFT, and their **precise identification (especially matching including higher-order corrections) is not an established theorem**. The paper therefore labels this **silver** — a **real resonance but unsettled**. There is an arrow on each side. That it is the same arrow has not been proven.

5. Gold-plated (a Rhyme): Surgery $\approx$ Renormalization — Same Theme, Different Machinery

Observation 5.1 (Tempting, but the cube does not fully turn)

From the shared theme “taming the infinite”, one is tempted to say “**Perelman's surgery = renormalization**”. But set the machinery side by side and the two are **different machines**:

- **Surgery**: **topologically excise** the region where curvature blows up, cap it with a standard cap, and continue the flow — a **discrete, topological cut**.
- **Renormalization**: **continuously absorb** the ultraviolet divergence into redefined couplings, leaving a cutoff-independent finite quantity — a **continuous renormalization of coefficients**.

The theme (governing the infinite) matches, but **the mechanism of the operation does not transfer**.

The paper therefore labels this correspondence **gold-plated (a rhyme)**. It is a tempting metaphor, but it is not made a girder of the bridge — the coordinates (the theme) match, but the operation (the mechanism) is not preserved (a practice of the distinct-roots discipline of Papers 58 and 86). The girder of the bridge is unified onto Friedan's equation (the gold) of

Section 2. The likeness of surgery and renormalization is recorded as a beautiful *signature*, and not used as a *proof* (Paper 100).

6. A Census of the Bridge: The Handshake of Two Lineages, and the Three Tiers

The paper’s findings are sorted into one table. This applies the spirit of Paper 100’s “one word, distinct roots” to the bridge between two lineages.

Tier	Correspondence	Status
Gold	Ricci flow = RG flow ($\beta_{ij}^g = \alpha' R_{ij}$) / fixed point = Ricci-flat = vacuum Einstein	Proven (Friedan 1980, CFMP 1985). Exact at one loop, higher corrections beyond. A genuine isomorphism in which the machinery transfers.
Silver	Perelman’s monotone entropy $\leftrightarrow$ Zamolodchikov’s c-theorem (the arrow / irreversibility)	A real resonance, unsettled. A monotone quantity and fixed points on each side. The precise identification is under active research, not an established theorem.
Gold-plated (rhyme)	Perelman’s surgery $\approx$ renormalization (handling infinity)	A rhyme. The theme is the same, but the mechanism differs (topological excision vs absorbing divergences). Not used as proof.

This three-tier labelling is the methodological core of the paper. The two lineages — the royal road of geometry and the royal road of field theory — **shake hands, provenly, at one point: Friedan’s equation**. That handshake is not a “theory of everything” (a GUT) but **one bridge** (the same stance as the bridge node of Paper 102). Take pride in the gold as gold, hope in the silver as silver, and label the gold-plated as gold-plated — this restraint keeps the bridge a bridge, and keeps it genuine.

The author's personal reflection (Layer 3; labelled non-scientific / not a claim of mechanism)

In the corpus's lens (metaphor) one is tempted to say — **the universe is renormalizing itself**. With each lowering of scale the geometry of spacetime runs (= Ricci flow = RG flow), performs surgery on infinities, renormalizes, and keeps “compiling” a finite, smooth world. A daydream even arises of the Universe OS advancing the flow from the floor of substance to the floor of information, governing the ultraviolet infinite as it goes. But **this carries the non-science label**. “The universe renormalizing itself” and “a universe that operates on infinities” are metaphors carrying intuition, not mechanisms or predictions. **What is certain here is the mathematical point that the sigma-model beta function equals the Ricci tensor.** — This paper was born from a single conversation, from the question: “the pinch (the singularity) and the divergence of a loop integral ($1/16\pi^2$) — are they not the same enterprise of taming the infinite?” At the core of that intuition there truly was gold (Friedan). But there was also a rhyme (surgery $\approx$ renormalization). **The intuition pointed in the right direction, but sorting gold from rhyme is the work of discipline, not of intuition.** Before the enterprise of governing the infinite, a quiet awe is recorded. But do not mistake it for having governed it. That is the craft of holding back at the near side of the cliff (Paper 101).

7. Conclusion

The royal road of geometry and the royal road of field theory long looked like separate rivers. But Ricci flow and the renormalization group flow were, at one point — **Friedan's equation — the same proven flow**; and at the fixed point, as Ricci-flatness = the vacuum Einstein equation, gravity itself appears. What makes the two lineages shake hands is not a dream of forced unification but this one girder (the gold). On its two banks there is a real resonance (silver), the arrow of the flow, and the same theme of governing the infinite (including the gold-plated rhyme) spreads out. What this paper did is **to sort the bridge between two lineages into gold, silver, and gold-plated, and to honour the handshake as one bridge**. Recording an awe before the enterprise of taming the infinite, yet not mistaking it for having governed it, the hundred-and-tenth address is returned.

Revision Record

2nd edition (September 2026): brought the format into line with the current template (the positioning and scope are gathered in the scope note directly after the abstract, and the closing “what is shown / what is not shown” is removed to leave a conclusion). The references are split into one entry per paper with actual titles and DOIs (bundled entries without DOIs are removed), and external references are given as author, year, title and journal. Friedan's 1980 and 1985 papers are given as separate entries, and Hamilton (1982), named in the text, is added. In Section 2, Kholodenko (2008) is cited as related work.

References

1. Friedan, D. (1980). *Nonlinear Models in $2 + \epsilon$ Dimensions*. Phys. Rev. Lett.
2. Friedan, D. (1985). *Nonlinear Models in $2 + \epsilon$ Dimensions*. Ann. Phys.
3. Hamilton, R. S. (1982). *Three-manifolds with positive Ricci curvature*. J. Differential Geom.
4. Callan, C. G., Friedan, D., Martinec, E., Perry, M. J. (1985). *Strings in Background Fields*. Nucl. Phys. B.
5. Perelman, G. (2002). *The Entropy Formula for the Ricci Flow and Its Geometric Applications*. arXiv:math/0211159.
6. Zamolodchikov, A. B. (1986). “Irreversibility” of the Flux of the Renormalization Group in a 2D Field Theory. JETP Lett.
7. Polchinski, J. (1998). *String Theory, Vol. 1*. Cambridge University Press.
8. Yamagishi, Y. (2026). *The Agreement of the Factors 2 Is a Case of Distinct Roots* [Paper 58]. Zenodo. 10.5281/zenodo.21597747
9. Yamagishi, Y. (2026). *The Four Roots of Necessity* [Paper 86]. Zenodo. 10.5281/zenodo.21744839
10. Yamagishi, Y. (2026). *Curvature Is Intrinsic* [Paper 91]. Zenodo. 10.5281/zenodo.21758406
11. Yamagishi, Y. (2026). *The Crossing of Royal Roads* [Paper 100]. Zenodo. 10.5281/zenodo.21758864
12. Yamagishi, Y. (2026). *Admiration and Caution* [Paper 101]. Zenodo. 10.5281/zenodo.21836382
13. Yamagishi, Y. (2026). *Hearing Shape from Spectrum* [Paper 102]. Zenodo. 10.5281/zenodo.21836735
14. Yamagishi, Y. (2026). *Flowing Geometry to Canonical Form* [Paper 103]. Zenodo. 10.5281/zenodo.21836801
15. Yamagishi, Y. (2026). *The Geometry of Dimensional Lift* [Paper 109]. Zenodo. 10.5281/zenodo.21837375
16. Kholodenko, A. L. (2008). *Towards Physically Motivated Proofs of the Poincaré and Geometrization Conjectures*. Journal of Geometry and Physics.

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