

How Far Does Light Determine Geometry?

— Two Roads That Stop at Conformal Structure (Differential Geometry and Order Theory), and the Distinct
Roots of the Residual "Scale" — [Paper 111]

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— *From thinking to syncing.* —

ABSTRACT

This series has treated the speed of light c repeatedly (Papers 63, 66, 83, 95, 108). Yet one question was **never asked directly: given light alone, how much of spacetime geometry is determined?** This paper answers with three established theorems, as a Layer-2 structural interpretation. **(i) Ehlers–Pirani–Schild (EPS, 1972): light rays determine a conformal structure** (the null cones, i.e. the metric up to a conformal factor), **freely falling particles determine a projective structure** (the "straightness" of geodesics), and their compatibility fixes a **Weyl structure** uniquely. Reaching a Lorentzian metric requires **one further step: the absence of the second-clock effect**. **(ii) Zeeman (1964):** the bijections of Minkowski spacetime (dimension ≥ 3) that **preserve the causal order** are generated by orthochronous Lorentz transformations, translations, and dilatations — **with neither continuity nor linearity assumed**. **(iii) Hawking–King–McCarthy (1976) / Malament (1977):** in general (distinguishing) spacetimes as well, **the causal structure determines the conformal structure, the topology, and the differentiable structure**. Starting from entirely different mathematics (differential geometry / order theory), the three **hit the same wall: light and causality determine the metric up to a conformal factor, and stop there**. Of the ten independent components of a four-dimensional Lorentzian metric, light determines **the nine of the conformal class**; the remaining one — the scale — it does not. **The core of this paper is the separation of that residual "scale" into two distinct roots carried by one word:** what remains in EPS is **a function on spacetime** (the local conformal factor, removed by clocks — the absence of the second-clock effect; Paper 96), while what remains in Zeeman is **a global constant** (the dilatation, a choice of unit — a convention; Paper 95, SI 2019). **A function and a constant must not be bundled under the single word "scale"**. The paper also takes a roll call of **three distinct roots** leading to the Lorentz group: **group axioms** (Ignatowski, Paper 83) / **the geometry of the invariant interval** (Minkowski, Paper 108) / **causal order** (Zeeman, this paper). **Scope:** a structural interpretation of established theorems (Layer 2). It yields no new mathematics and no prediction. **It does not claim that spacetime can be built from light** — what it records is rather **the limit of light**; EPS requires particles. Layer-3 lenses are quarantined with a non-scientific label.

Positioning of this paper (discipline): This paper offers a **structural interpretation and distinct-roots census** of established theorems (EPS 1972, Zeeman 1964, Hawking–King–McCarthy 1976, Malament 1977). It proves no new theorem and claims no new physics or prediction. **It does not claim that spacetime can be constructed from light.** On the contrary, what it records is **the one component light does not determine**. EPS requires **freely falling particles as an indispensable second input**, and its final step (Weyl structure $\rightarrow$ Lorentzian metric) requires an additional axiom (the status of that step has continued to be examined in later literature; this paper adopts only the uncontested point that *a further step is needed*). **The residual "scale" is not bundled under one word:** the EPS residue is **a function** (the local conformal factor), the Zeeman residue is **a constant** (the global dilatation) — distinct roots. "Resonance is a signature, not a proof" (Paper 100); "no post-hoc numerical adjustment" (Paper 101). The Layer-3 lens (reading light as building material) is **quarantined from the body with a non-scientific label** and is not offered as a mechanism.

1. Introduction: A Question Never Asked Directly

This series has handled $\mathfrak{c}$ many times: as the curvature of velocity space (Paper 63), as the distinct roots of local $\mathfrak{c}$ and coordinate $\mathfrak{c}$ (Paper 66), as what follows from group axioms without assuming light (Paper 83), as the line between convention and fact (Paper 95), as the geometry of the invariant interval (Paper 108). But **one question was never asked head-on:**

Given light alone, how much of spacetime geometry is determined?

This question **already has established answers** — and two of them, from entirely different mathematics. One is the road of **differential geometry** (EPS 1972): axiomatic general relativity, reconstructing geometry from light rays and freely falling particles. The other is the road of **order theory** (Zeeman 1964), which begins from nothing but the relation "which event can send a signal to which." The former treats curved spacetime; the latter assumes not even continuity, in flat spacetime. They share almost no tools.

And yet **the two hit the same wall**. This paper records that wall — and, this being its core, **distinguishes the "scale" that survives beyond the wall as two different things in the two roads**. The same single word names **a function on spacetime** in one case and **a single constant** in the other. This separation is the practice, in the case of $\mathfrak{c}$, of a discipline this series has applied repeatedly ("one word, distinct roots": Papers 86, 100).

2. Road A: EPS — Light Gives Conformal Structure, Particles Give Projective Structure

There is an axiomatic road that does not *begin* from the metric $g_{\mu\nu}$, but **assembles geometry from observable material: light rays and freely falling particles**. This is the 1972 work of Ehlers, Pirani, and Schild.

Established Fact 2.1 (Ehlers–Pirani–Schild 1972)

Take events, light rays, and freely falling particles as the material of spacetime. Then:

- **Light rays** determine a **conformal structure** $\mathcal{C}$ — the null cone at each point, i.e. **the metric up to a positive conformal factor**, $g \sim \Omega^2 g$. Angles and causal cones are fixed; **lengths are not**.
- **Freely falling particles** determine a **projective structure** $\mathcal{P}$ — the family of preferred paths, i.e. **the connection up to reparametrisation**. "Straightness" is fixed; **the affine parameter (the stride) is not**.
- When the two are **compatible** (particle worldlines lie inside the null cones, and the conformal and projective structures agree), a **Weyl structure** is fixed **uniquely** — a torsion-free connection compatible with the conformal structure, permitting non-metricity $\nabla g = A \otimes g$ through a one-form A .
- Passing from the Weyl structure to a **Lorentzian metric** requires **one further step**: that A vanish (or be exact) — physically, **the absence of the second-clock effect**.

Let us count. A four-dimensional Lorentzian metric is symmetric 4×4 , with **ten independent components**. The conformal class (the quotient by a positive factor) has **nine**. That is:

Light determines nine of the ten components.

The remaining one (the conformal factor — the ruler) it does not determine.

This single line measures **both the reach of light and its limit**. Light carries almost all of the geometry. It does not carry the ruler.

Corollary 2.2 (Direct link to Paper 96: a failure returns as an axiom)

The **absence of the second-clock effect**, which EPS requires at the end, is **precisely the objection Einstein raised against Weyl's theory in 1918** (Paper 96): if scale is transported by a gauge field, two clocks that travel different paths disagree on reunion, in conflict with the uniformity of atomic spectra. In Paper 96 this was recorded as **the failure of Weyl's 1918 theory**. In EPS the same point appears **with its sign reversed** — not as an epitaph, but as **the final axiom actively imposed in order to reach metric geometry**. One and the same fact serves as a **headstone in one place and a cornerstone in the other**.

An honest line. The status of the argument for the EPS final step (Weyl structure $\rightarrow$ Lorentzian metric) has continued to be examined — whether an additional axiom is genuinely required, and in what minimal form (see Trautman's editorial note to the 2012 reprint, among others). This paper does not enter that discussion and adopts only the uncontested part: **light and particles alone reach the Weyl structure, and a further step is required to arrive at a metric**.

3. Road B: Zeeman — The Lorentz Group from Causal Order Alone

There is an entirely different entrance: discard geometry and differentiability, and retain **only the order relation "which event can send a signal to which."**

Established Fact 3.1 (Zeeman 1964)

Equip Minkowski spacetime M (of dimension ≥ 3 , i.e. excluding $1 + 1$) with the timelike order $x < y$. Define a **causal automorphism** as a bijection $f : M \rightarrow M$ with $x < y \iff f(x) < f(y)$. Then the group of causal automorphisms is **generated by the following three**:

orthochronous Lorentz transformations $\times$ translations $\times$ dilatations ($x \mapsto \lambda x$, $\lambda > 0$).

The force of this theorem lies in the poverty of its assumptions. Continuity, differentiability, and linearity are **not assumed** — they are **conclusions**. From the single condition of preserving the order, even the linearity of the Lorentz transformations follows. The original title, "Causality Implies the Lorentz Group," is not an overstatement.

Observation 3.2 (One line for the census of dimensions)

Zeeman's theorem requires **dimension ≥ 3** . It fails in $1 + 1$ dimensions, where the causal automorphism group is far larger (it admits an independent increasing function on each of the two null coordinates) and is not narrowed to the Lorentz group. This adds **one line from order theory** to the **census of dimensions** in Papers 64, 69, 105: $1 + 1$ is "too loose" for causal structure to determine geometry. **This is not a proof of "why 3+1"** (the discipline of Paper 69). It adds one independent entry to the census, nothing more.

And what deserves attention is **what remained**. The order **cannot exclude dilatation**: stretching all of M by λ changes nothing about which event precedes which. **Causal order, too, does not determine the ruler.**

4. Confluence in Curved Spacetime: Hawking–King–McCarthy / Malament

Zeeman's is a theorem about flat spacetime. What of curved spacetime? Here a theorem joins the two roads **in a proven form**.

Established Fact 4.1 (Hawking–King–McCarthy 1976; Malament 1977)

Between spacetimes satisfying suitable causality conditions (past- and future-distinguishing), a **bijection preserving the timelike order** is a **smooth conformal isometry**. That is, **the causal structure determines the topology, the differentiable structure, and the metric up to a conformal factor**.

This is decisive. **In curved spacetime as well, what the causal structure reaches is the conformal structure, and there it stops.** Zeeman's flat-spacetime result turns out to be a special face of this general theorem.

**The road of differential geometry (EPS) and the road of order theory (Zeeman → Malament)
hit the same wall: conformal structure.**

This is not a coincidence. Both are different faces of the same geometric fact — "null cone = conformal structure" — and reduce to the single point that changing the conformal factor does not move the null cones. **The conclusion transfers; the machinery does not.** EPS is local, differential-geometric, and works in curved spacetime; Zeeman is global, order-theoretic, and assumes no continuity in flat spacetime. **They hit the same wall with different tools, independently.** This is what this paper calls **distinct roots** (Papers 86, 100).

5. The Core: The Residual "Scale" Is Two Distinct Roots Under One Word

Here is the heart of the paper. Both roads **leave a scale behind**. Naively one wants to say "the same thing remained." But **seen precisely, the surviving freedoms are different objects**.

Observation 5.1 (Do not bundle a function and a constant under one word)

- **What remains in EPS is a function on spacetime:** the conformal factor $\Omega(x)$ can be chosen independently at each point — a **local** degree of freedom. It is a genuine geometric freedom, and it is removed by **clocks** — more precisely, by the physical requirement of the absence of the second-clock effect (Paper 96).
- **What remains in Zeeman is a global constant:** the dilatation $x \mapsto \lambda x$ stretches **the whole spacetime uniformly** — a **single number**. Because the stage is fixed as flat, the local function never arises; what remains is the choice of unit, that is, a **convention** (Paper 95; SI 2019).

A function is not a constant. The former is filled by **physics** (the behaviour of clocks), the latter by **convention** (an agreement on units). The single sentence "light does not determine the scale" names **two entirely different things**.

This separation meshes exactly with the existing line this series draws around c . Paper 95 stated that **the value of c is a convention (SI 1983, 2019), while the finiteness of c is a fact**. This paper adds one line: **what belongs to convention is Zeeman's dilatation (a global constant); the EPS conformal factor (a local function) is settled not by convention but by physics**. The two things called by the one word "scale" are placed, for the first time, on different shelves.

	Road A: EPS (1972)	Road B: Zeeman (1964) / Malament (1977)
Input	Light rays + freely falling particles (light alone is not enough)	Causal order only (no continuity assumed)
Mathematics	Differential geometry (conformal, projective, Weyl connection)	Order theory and group theory (automorphisms of an ordered set)
Stage	General (curved) spacetime; local	Flat Minkowski ($\text{dim} \geq 3$) / general spacetime (Malament)
Reach	Conformal + projective $\rightarrow$ Weyl $\rightarrow$ (+ axiom) metric	Lorentz $\times$ translations $\times$ dilatations / conformal isometry
Wall	Conformal structure (9 of the metric's 10 components)	Conformal structure (up to conformal isometry)
Residual freedom	A function: the local conformal factor $\Omega(x)$	A constant: the global dilatation λ
Who fills it	Physics: clocks / absence of the second-clock effect (Paper 96)	Convention: the choice of unit (Paper 95; SI 2019)

6. Census: Three Distinct Roots to the Lorentz Group

Paper 108 took a roll call of Ignatowski (the group root) and Minkowski (the geometry root) as **distinct roots**. Zeeman, in this paper, adds **a third**.

Root	Starting point	Status
Group root (Paper 83)	Relativity principle + isotropy + group structure. Light is not assumed	Ignatowski 1910. The existence of an invariant speed is forced; its value is settled by experiment
Geometry root (Paper 108)	The invariant interval $ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$	Minkowski 1908. Lorentz transformation = hyperbolic rotation. What is absolute is the interval
Order root (this paper, 111)	Causal order $x < y$ alone. Neither continuity nor linearity assumed	Zeeman 1964. Linearity is a conclusion. A dilatation remains

The three reach the same group, but they are not the same road. The group root needs no light; the geometry root posits the interval; the order root looks only at the cones. **Agreement is a signature, not a proof** (Paper 100). What this roll call shows is not that "the Lorentz group is the one truth," but **the descriptive fact that three independent ways of asking converge on the same place**. And **only the order root lets the dilatation slip through** — that difference, too, is evidence that the roots are distinct.

The author's personal reflection (Layer 3; labelled non-scientific / not a claim about mechanism)

I have long held, privately, **a lens that reads light as "building material"** (Layer 3, non-scientific). This paper did **two things at once** for that lens. One is **encouragement**: light was carrying almost all of the geometry, in a countable rather than a metaphorical sense. Nine of ten components. A single structure — the null cone — fixes angles, causality, topology, and differentiable structure. **This is far larger than I had imagined**. The other is **a precise brake**: light still **does not carry the ruler**. **EPS required freely falling particles**. Light alone does not complete spacetime.

I do not receive this brake as a denial. **Rather, I feel the lens has acquired an outline for the first time**. "Light makes everything" cannot be said. But **"light makes the cone, and clocks put in the ruler"** at least makes clear what is *not* being said. And I think **a dream whose limits are known carries much farther than a vague one**. — This is a personal reflection with a non-scientific label; it is neither a mechanism nor a prediction. What is certain is only the four theorems cited in the body. Before the fact that light arriving from an infinite distance had, by its null cones alone, written down nine tenths of the shape of geometry, I record a quiet awe. But **I do not attribute the remaining tenth to light**. That is the practice of stopping short of the cliff (Paper 101).

7. Scope / Conclusion

What is shown (descriptive; a distinct-roots census): that light rays determine a conformal structure and freely falling particles a projective structure, whose compatibility fixes a Weyl structure uniquely (EPS, §2); that of the ten components of a four-dimensional metric, light determines the nine of the conformal class (§2); that the final EPS axiom is **the same point inverted** as the failure of Weyl's 1918 theory in Paper 96 (§2); that the Lorentz group, translations, and dilatations follow from causal order alone, with no continuity assumed (Zeeman, §3); that in curved spacetime as well the causal structure reaches the conformal structure (HKM / Malament, §4); that **the residual "scale" is a function in EPS and a constant in Zeeman — distinct roots** — the former filled by physics (clocks, Paper 96), the latter by convention (units, Paper 95) (§5); and a roll call of three distinct roots to the Lorentz group (83 / 108 / 111, §6).

What is not shown (restraint): it is not claimed that spacetime can be constructed from light (EPS requires particles as an indispensable second input); **the status of the argument for the EPS final step is not entered into** (only the uncontested point that a further step is required); **"why 3+1 dimensions" is not proved** (Zeeman's dimension condition is one line for the census, not a derivation; Paper 69); **the agreement of the three roots is not read as unification** (agreement is a signature, not a proof; Paper 100); **the two residual scales are not bundled under one word** (a function and a constant are distinct roots); no post-hoc numerical adjustment (101); no new mathematics or prediction (Layer 2); Layer-3 lenses quarantined with a non-scientific label.

This series has treated c as the curvature of velocity space, as the distinct roots of the local and the coordinate, as the boundary between convention and fact, and as the geometry of the invariant interval. What this paper adds is **one question and its honest answer: how far does light determine geometry? Answer: as far as the conformal structure. There it stops.** Two roads — differential geometry and order theory — hit that wall independently, sharing almost no tools. But what survives beyond the wall must not be bundled under one word. **What remains in EPS is a function on spacetime; what remains in Zeeman is a single constant.** The former is filled by clocks, the latter agreed upon by human beings. Light gives the cone, clocks give the ruler, convention gives the unit — **the three come from different roots.** What this paper has done is not to make a new claim, but **to set four established theorems about c side by side and to count, honestly and in components, the reach of light and its limit.** And it returns address number 111.

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