

The Allotted Region and the Freedom Within

— A Census of Six Distinct Roots Sharing One Rhyme: Prohibition, Resolution, Postulate, Theorem, Exclusion,
Capacity — [Paper 112]

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— *From thinking to syncing.* —

ABSTRACT

"A region is given, and within it one is free." This intuition recurs across physics with the same rhyme — in the flight of gas molecules, in the electron's cloud, in the shells of chemistry, in the reach of light. This paper calls the roll of **six structures** that share the rhyme, as a Layer-2 **distinct-roots census**, and shows that **in all six the word "freedom" means something different**. **(A) Causal**: the light cone and causal diamond. Inside one can influence; outside one **cannot** — a **prohibition** (Papers 111, 66, 108). **(B) Phase space**: the Planck cell $h^3 = (2\pi\hbar)^3$. The number of states is V/h^3 , and inside a cell nothing is **distinguishable** — a **limit of resolution** (Papers 77, 88). **(C) Statistics**: equal a priori probability / Jaynes's maximum entropy. The system **spreads uniformly** over the allowed region — but this is a **postulate**, not a theorem (Paper 98). **(D) Quantum**: the measure on Hilbert space is not postulated but **fixed uniquely by a theorem** (Gleason 1957; Paper 71). **Here lies the decisive distinction from (C)**: one is a **postulate**, the other a **theorem**. **(E) Chemistry**: Pauli exclusion. Seats are **filled** one at a time, shell capacity $2n^2$ (2, 8, 18, 32) — not freedom but **exclusion** (Papers 68, 104). **(F) Holographic**: $S \leq A/4\ell_P^2$. What sets the ceiling on the amount that can be packed in is **not the volume but the area** — breaking the naive picture "region = volume" itself (Papers 73, 88, 65). **An honest line**: reading the quantum cloud as "it really is somewhere, we simply cannot see where" is excluded, as a local hidden-variable picture, by **Bell's theorem (1964) and experiment (from 1982 onward; the loophole-free tests of 2015)**. This paper therefore does not adopt the reading "a cloud of ignorance." **Scope**: a structural interpretation of established results (Layer 2). It yields no new mathematics and no prediction. **The six are not unified** — sharing a rhyme is a signature, not a proof (Paper 100). The picture of a designer allotting modules is quarantined **at Layer 3 with a non-scientific label**.

Positioning of this paper (discipline): This paper offers a **structural interpretation and distinct-roots census** of established results (causal structure; Liouville's theorem and the counting of states in phase space; equal a priori probability and Jaynes's maximum entropy; Gleason's theorem; Pauli exclusion; the holographic bound; Bell's theorem and its experimental tests). It proves no new theorem and claims no new physics or prediction. **The six are not unified under one principle** — what they share is a **rhyme** (a region, and freedom within), **not machinery**. The main point is precisely that **the word "freedom" carries six different meanings** (prohibition / resolution / postulate / theorem / exclusion / capacity). **The quantum cloud is not read as "ignorance"** — local hidden variables are excluded by Bell's theorem and experiment (this line is experimental fact, and is not conceded). **The statistical postulate of equal weights and the quantum measure (Gleason's theorem) are not identified.** "Resonance is a signature, not a proof" (Paper 100); "no post-hoc numerical adjustment" (Paper 101). **A designer, module allotment, and the Universe OS are Layer-3 non-scientific lenses**, not claims about mechanism.

1. Introduction: One Intuition, Six Addresses

This paper begins from a plain intuition the author has long held: **"a region is given, and within it one may behave freely."** The ball light can reach in unit time, $\frac{4}{3}\pi(ct)^3$, looks like that; so does the flight of gas molecules, the whereabouts of an electron, the shells of chemistry. As if **a frame had been issued** to each stage.

This intuition **really does recur throughout physics**. But — and this is the core of the paper — **each time it recurs, both the making of the "region" and the meaning of "freedom" are different.**

The same rhyme. Not the same machinery.

What this paper does is not unification but a **roll call**. It sets six addresses side by side and writes out, for each, what fixes the region and what "freedom" means. This applies the discipline this series has used repeatedly ("one word, distinct roots"; Papers 86, 100) to **a single picture: region and freedom**. The contrast between (C) and (D) in particular — **the same "measure on a region," yet one a postulate and the other a theorem** — is the sharpest point in the paper.

2. Address A: Causal — "One Cannot Influence the Outside" (Prohibition)

Established Fact 2.1 (Causal structure and the light cone)

The range that an event p can influence is confined to the interior of the **future light cone** of p . For two points $p \prec q$, the **causal diamond** $J^+(p) \cap J^-(q)$ is the set of all events that can occur after p and before q . In flat spacetime, the volume of the ball light can reach in time t is

$$V(t) = \frac{4}{3}\pi(ct)^3,$$

and no signal whatever can be sent outside that ball.

Here "freedom" is unambiguous: **inside one can exert influence, outside one cannot**. This is a **prohibition**, not ignorance and not resolution. And as Paper 111 showed, **the structure of this cone fixes nine of the ten components of the spacetime metric (the conformal class)**. The intuition that "a region has been given" is, at least here, touching **the very core of geometry**.

An honest line. The number $\frac{4}{3}\pi c^3$ is an artefact of units (the value of c is an SI defining constant; Paper 95). What survives is not the number but the **structure**: "the causal reach is a ball, and its volume scales as $(ct)^3$." This paper keeps that distinction.

3. Address B: Phase Space — "Inside, Nothing Is Distinguishable" (Resolution)

Established Fact 3.1 (Liouville's theorem and the Planck cell)

The time evolution of a Hamiltonian system **preserves phase-space volume** (Liouville; Paper 77). In quantum theory, moreover, the number of states contained in a phase-space volume V is counted as

$$N = \frac{V}{h^3} = \frac{V}{(2\pi\hbar)^3}, \quad d^3x d^3p / (2\pi\hbar)^3.$$

That is, phase space is divided into **cells of size h** , and **one cell corresponds to one state**.

Here "freedom" is **a different thing** from A: **inside a cell, no further distinction can be made**. This is not a prohibition on influence but a **limit of resolution**. Rather than "free inside," it is more accurate to say **"there is no structure inside."** This is probably the address closest in shape to the author's intuition — and what stands out is that **the size of the ration, h^3 , is quantitatively fixed**.

(In this series, the $\hbar^{-3}$ appearing in the Stefan–Boltzmann constant in Paper 43 came precisely from this density of states, $1/(2\pi\hbar)^3$.)

4. Address C: Statistics — "Spread Uniformly over the Allowed Region" (Postulate)

Established Fact 4.1 (Equal a priori probability and maximum entropy)

For an isolated system (with E , V , N specified), statistical mechanics **posits that all accessible microstates are equally probable** (the **postulate of equal a priori probability**, the basic assumption of the microcanonical ensemble). Boltzmann's entropy counts the number of states W inside that region: $S = k \log W$ (Paper 98). Jaynes (1957) recast this as a **principle of inference: under given constraints, the least-biased distribution is the one that maximises entropy**.

The flight of gas molecules (the Maxwell–Boltzmann distribution) is a resident of this address. Here "freedom" means a **uniform measure over the allowed region**.

And the decisive point: equal a priori probability is a postulate, not a theorem derived from dynamics. Whether real systems truly explore the region uniformly (ergodicity) is in general an open and hard question. On Jaynes's reading it is **not even a claim about nature, but a rule of inference under incomplete information. This paper records that weakness of status openly** — because the next section stands exactly opposite to it.

5. Address D: Quantum — The Measure Is Not a Postulate but a Theorem

Established Fact 5.1 (Gleason's theorem, 1957)

When the dimension of the Hilbert space is at least three, any probability measure defined on the lattice of projections **must be given by a density operator** — that is, **the Born rule is not a postulate but follows uniquely** (Paper 71).

Corollary 5.2 (The core of this paper: the same "measure on a region," different status)

In statistical mechanics the measure on the allowed region is **postulated** (equal weights; §4). In quantum mechanics the measure on the state space is **fixed uniquely by a theorem** (Gleason). **Although the picture is the same — a region is given and a probability rides on it — one is an assumption laid down, the other a proven uniqueness.** These are **distinct roots** and must not be bundled together by the similarity of the rhyme.

An honest line (the point this paper concedes least). One is tempted to read the electron's "cloud" as **"it really is at some definite point, and we simply cannot see it."** That is a natural reading, and the picture most people — the author included — form first. But **this naive picture is excluded by experiment.**

Established Fact 5.3 (Bell's theorem and its experimental tests)

Bell (1964) showed that any attempt to reproduce quantum correlations by **local hidden variables** must satisfy a certain inequality. Experiments from Aspect (1982) onward, and in particular **several "loophole-free" experiments in 2015**, confirm that this inequality is **violated** (Nobel Prize in Physics, 2022). That is, **the local picture "the particle really is at a definite place and we merely do not know it" does not hold.**

This paper therefore does **not** read the cloud as **"a cloud of ignorance."** What can accurately be said here is only this — **boundary conditions fix a region, and a measure rides on that region** — and **not** that the measure represents **ignorance about a hidden position**. This line is not a matter of interpretive taste but **experimental fact**, and the paper does not concede it.

6. Address E: Chemistry — The Seats Get Filled (Exclusion)

Established Fact 6.1 (Pauli exclusion and shell capacity)

Identical fermions cannot occupy the same quantum state (Pauli 1925). The number of electrons a shell of principal quantum number n can hold is

$$2n^2 \quad (n = 1, 2, 3, 4 \Rightarrow 2, 8, 18, 32),$$

with subshell capacities $s=2, p=6, d=10, f=14$. The lengths of the periods in the periodic table follow from this **rationing of seats**.

Here "freedom" **inverts**. Seats are indeed allotted — this is the address where the author's intuition lands most directly — but **one does not roam freely inside; the seats are filled one at a time**. What governs is not freedom but **exclusion**. And that exclusion comes from **spin statistics** (the antisymmetry of fermions; the "two" of spin in Paper 68, the spinors of Paper 104), so its **origin is altogether different** from A or B.

This is why chemistry looks so "modular." The structure of the periodic table is **not a geometric allotment of regions but a consequence of counting quantum states together with antisymmetry**.

7. Address F: Holographic — The Ceiling Is Set by Area, Not Volume

Established Fact 7.1 (The holographic / covariant entropy bound)

The amount of information (entropy) that can be packed into a region is bounded above by the **area of its boundary**:

$$S \leq \frac{A}{4\ell_P^2}$$

(Bekenstein–Hawking $S = A/4$, Paper 73; Bousso's covariant version, 1999).

This address delivers **a blow that decisively separates it from the other five**. The naive intuition expects "the larger the region, the more fits inside" — proportionality to **volume**. But the holographic bound says the ceiling is set by **area**. **The naive picture "region = amount of content" itself breaks down here**.

As Paper 88 distinguished, **the entropy of coarse-graining (addresses B and C) and the entropy of area (address F) are distinct roots**, and the only thing linking them is this holographic bound. And **why it is area remains unsolved** (Paper 88; admire).

8. Census: The Six "Freedoms" All Mean Different Things

Address	What fixes the region	What "freedom" means	Status
A Causal	The speed of light and causal structure (the cone)	Cannot influence the outside	Prohibition (geometry; 111)
B Phase space	The Planck cell h^3	Cannot distinguish inside	Resolution (77, 88)
C Statistics	Constraints (E, V, N) and the accessible set	Spreads uniformly over the region	Postulate (equal weights; 98)
D Quantum	Boundary conditions and Hilbert space	The measure is uniquely fixed	Theorem (Gleason; 71)
E Chemistry	The counting of quantum states	The seats are filled one by one	Exclusion (Pauli; 68, 104)
F Holographic	The area of the boundary	A ceiling on what can be packed in	Capacity (73, 88; unsolved)

Observation 8.1 (Six reasons not to unify)

The six draw the same picture (region and freedom), but **the machinery does not transfer**. Prohibition (A) cannot be translated into resolution (B); a postulate (C) differs in status from a theorem (D). Exclusion (E) is in fact the **opposite** of freedom, and capacity (F) breaks the very premise that "region = volume." **What transfers is the picture, not the apparatus** — this is the verdict of the machinery-transfer test of this series (Papers 58, 86, 100).

The **contrast between C and D** in particular is where this paper puts most of its weight. Statistical mechanics' equal-weight rule is **an assumption laid down**, and whether real systems satisfy it is in general open (the ergodic problem). The quantum measure, by contrast, is **unique by Gleason's theorem**. To intuition both look like **"a probability riding on a region," yet one has the weakest status (a postulate) and the other the strongest (a uniqueness theorem)**. Bundle them under one word and that gap disappears.

The author's personal reflection (Layer 3; labelled non-scientific / not a claim about mechanism)

I have vaguely thought this: **if there were something like a designer, perhaps they issue each module a frame — "here is your range; do as you like within it."** As ink spreads on paper, scattering freely but only inside the region given. And perhaps when physicists of quantum mechanics look at that, they call it a "cloud," or "probability."

What writing this paper taught me is that **this intuition is, over a surprisingly wide range, "on target."** The frames really are there — in the light cone, in the cell of size $\hbar^3$, in the capacity of a shell, in the ceiling set by area. **The picture of rationing genuinely exists all over physics.**

But at the same time, **seen precisely, the frames were made in six different ways.** And in exactly one place there was a **barrier the intuition cannot pass through as it stands:** reading the electron's cloud as "it really is somewhere, we just cannot see it" has been **closed off by Bell experiments.** That is not a matter of preference; it is a door that experiment has shut.

I do not receive this as a denial of the intuition. **Rather, I feel that a map has been drawn of how far the intuition is right and where it turns into something else.** Whether a designer exists is not a question this paper asks (that belongs to Layer 3 and cannot be handled in the language of science). **What is certain is only the six established results cited in the body.** Before the picture of rationing I record a quiet awe — but **I do not reduce the six to a single will.** That is the practice of stopping short of the cliff (Paper 101).

9. Scope / Conclusion

What is shown (descriptive; a distinct-roots census): that the picture "a region is given, and within it one is free" appears at six addresses — causal (A), phase space (B), statistics (C), quantum (D), chemistry (E), holographic (F); that **"freedom" means something different in all six** — prohibition, resolution, postulate, theorem, exclusion, capacity (§8); in particular that **the statistical postulate of equal weights and the quantum measure (Gleason's theorem) are distinct roots differing in status** (§5); and that the holographic bound breaks the naive premise "region = volume" itself (§7).

What is not shown (restraint): the six are not unified under one principle (a rhyme is a signature, not a proof; Paper 100); **the quantum cloud is not read as "a cloud of ignorance"** (local hidden variables are excluded by Bell's theorem and experiment; this line is not conceded); **it is not claimed that equal weights can be derived from dynamics** (the ergodic problem is open); **why the ceiling is area is not solved** (holography remains open; Paper 88; admire); the **number $\frac{4}{3}\pi c^3$** is not treated as a physical quantity (a convention of units; Paper 95); no post-hoc numerical adjustment (101); no new mathematics or prediction (Layer 2); **the designer and module allotment are quarantined as Layer-3 non-scientific lenses.**

Starting from one plain intuition — **a frame is issued, and within it one is free** — we visited six addresses. The conclusion is neither a denial nor an endorsement of the intuition. **The frames really do exist, all over.** In the reach of light, in the cell of phase space, in the shells of the electron, in the area of a horizon. **But the apparatus that makes each frame was different in all six cases.** One frame forbids influence; one permits no distinction; one is laid down as a postulate; one is fixed uniquely as a theorem; one imposes not freedom but exclusion; and one sets its ceiling by area rather than volume. **The picture transfers. The apparatus does not.** What this paper has done is not to make a new claim, but **to set side by side the six realities that one intuition touches, and to give each "freedom" its proper name.** And it returns address number 112.

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