

The Fourth Clock

— The Time of Ricci Flow and Renormalisation-Group Flow Is Neither dt nor $d\tau$ nor dS — [Paper 119]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

August 2026

— *From thinking to syncing.* —

ABSTRACT

Paper 118 showed that the single word “time” covers **three logical types**: the coordinate dt , the functional of a curve $d\tau$, and the functional of a measure dS . **This paper finds that the roll-call was incomplete — and finds the gap inside this series' own papers.** The “time” appearing in Ricci flow $\partial_t g_{ij} = -2R_{ij}$ (Paper 103) and in renormalisation-group flow (Paper 110) is **none of the three: it is a fourth type. First, the dimension settles it.** Solving Ricci flow on a round n -sphere gives $r^2(t) = r_0^2 - 2(n-1)t$ (solved here; for $n=2$, $r_0=1$ the extinction time is $t^* = 1/2$). Matching dimensions on both sides forces $[t] = L^2$ — **a squared length, not a second.** Since dt and $d\tau$ both carry dimension T , and **two quantities of different dimension cannot be the same quantity** (the measuring stick of Paper 116), the identification is closed. The RG parameter $\log \mu$ is **dimensionless**, a third dimension again. **Second, the domain differs.** $d\tau$ lives on curves **inside** a manifold, whereas the Ricci parameter lives on **a path inside the space of metrics $\text{Met}(M)$** — and since **the very instrument of measurement is being deformed**, it cannot be a length measured by that instrument. **Third, the root of the arrow differs.** Ricci flow is parabolic, so running it backwards makes every Fourier mode blow up (factor $e^{+k^2|t|}$: at $k=20$, $|t|=1$ this is $\sim 10^{173}$; verified numerically here) — **the equation itself is irreversible.** Hamiltonian flow, by contrast, is **reversible**, and the arrow of dS comes from **coarse-graining**, a matter of the observer's resolution (Papers 77, 78). **The same word “irreversible” covers two different roots. Fourth, and this is the core: inside the fourth type, the split between dt and dS recurs.** The parameter t is a **relabelable label** ($\partial_s g = -2\lambda \text{Ric}$ traces the same trajectory on a different scale), while **what actually carries a direction is a monotone** — Perelman's $\mathcal{W}$ -entropy (non-decreasing) and Zamolodchikov's c -function (non-increasing in two dimensions). **So the fourth type is not one new clock but a new pair (label, monotone), structurally parallel to (dt, dS) but living in a different space. Fifth, a tension to record honestly:** by Friedan the one-loop beta function of the two-dimensional sigma model equals **Ric**, so **the two flows are the same equation** (Paper 110). **Yet there are two accounts of why it cannot be reversed** — on the Ricci side **parabolicity** (an analytic obstruction), on the RG side **integrating out modes** (discarded information). **Matching equations do not license merging the accounts** (the discipline by which Paper 110 warned that “surgery $\approx$ renormalisation is a rhyme”). **Self-audit:** Paper 103 already braked, writing that Perelman's entropy **is a control device for the flow and not the arrow of time of the universe. But it did not give that quantity a type.** This paper fills that blank. **Scope:** a structural interpretation and roll-call of established standard facts (Layer 2). No new mathematics, no prediction. **Neither the Poincaré conjecture nor geometrisation is touched, and $\mathcal{W}$ is not identified with dS .**

Position of this paper (discipline): every ingredient is a **known standard result** — Hamilton's Ricci flow, its explicit solution on spheres, the backward ill-posedness of parabolic equations, the monotonicity of Perelman's $\mathcal{W}$ -entropy, Zamolodchikov's c -theorem, Friedan's beta function. **No new theorem is proved.** The contribution is **the addition of a type and the description of its internal structure:** (i) showing by **dimension and domain** that the flow parameter is **none of $dt/d\tau/dS$** ; (ii) showing that the root of its arrow is **parabolicity, not coarse-graining**; (iii) stating that **the (dt, dS) split recurs inside the fourth type**; (iv) recording, without merging them, that **one equation carries two accounts of irreversibility**; and (v) filling the type-name that Paper 103 left blank. **The most important restraint: $\mathcal{W}$ is not identified with thermodynamic entropy** (Papers 88, 103). **The Ricci parameter is not called physical time. Nothing is claimed about the Poincaré or geometrisation conjectures, in whole or in part. The two accounts of irreversibility are not fused** (Paper 110). “A resonance is a signature, not a proof” (Paper 100). Layer-3 lenses are quarantined with a non-scientific label.

1. Introduction: The Roll-Call Was One Short

Paper 118 showed that the phrase “time passes” covers **three logical types: the coordinate dt** (a label with no domain), **the functional of a curve $d\tau$** , and **the functional of a measure dS** . The three differ not in value but **in type**, and so there was never an arena in which to compare them.

This paper reports that the roll-call was incomplete. And the omission was found not in someone else's work but **inside this series' own papers.**

**The “time” in Ricci flow (Paper 103) and in renormalisation-group flow (Paper 110)
is neither dt , nor $d\tau$, nor dS .**

Paper 103 had already applied one brake to this quantity: one must not leap to the conclusion that “Perelman also treated the arrow of time with entropy”, because it is **a control device for the flow, not the arrow of time of the universe. That judgement is correct. But Paper 103 did not say what type of quantity it then is.** This paper fills that blank.

2. The Dimension Settles It

Established Fact 2.1 (Ricci flow and its explicit solution on spheres)

Ricci flow on a Riemannian manifold is defined by

$$\frac{\partial}{\partial t} g_{ij} = -2R_{ij}$$

(Hamilton 1982). On a round n -sphere of radius r one has $\text{Ric} = \frac{n-1}{r^2}g$, so writing $g = r^2 g_{\text{unit}}$ gives $\frac{d}{dt}(r^2) = -2(n-1)$, that is

$r^2(t) = r_0^2 - 2(n-1)t$

and the sphere collapses to a point at the finite time $t^* = \frac{r_0^2}{2(n-1)}$ (for $n=2, r_0=1$: $t^*=1/2$; solved here).

Observation 2.1 ($[t] = L^2$: not a second)

Match dimensions in the boxed relation. The left-hand side is L^2 , and the coefficient on the right is dimensionless, so $[t] = L^2$.

The “time” of Ricci flow is a squared length. The structure is that of the heat equation $\partial_t u = \kappa \Delta u$, where $[\kappa] = L^2/T$ and hence $[\kappa t] = L^2$; Ricci flow corresponds to $\kappa = 1$, so the parameter itself carries the L^2 .

Parameter	Dimension	Domain
dt (coordinate time)	T	a coordinate on spacetime
$d\tau$ (proper time)	T	worldlines in a Lorentzian manifold
dS (entropy)	k_B	probability measures on phase space
t (Ricci flow)	L^2	a path in the space of metrics $\text{Met}(M)$
$\log \mu$ (RG)	dimensionless	a path in the space of couplings

Two quantities of different dimension cannot be the same quantity.
The measuring stick Paper 116 used on the exponent of c works here unchanged.

3. The Domain Differs — the Instrument Itself Is Moving

The dimension alone closes the question, but **the domain shows why**.

Observation 3.1 (Inside a manifold, or through the space of manifolds)

$d\tau$ lives on **curves inside** a manifold: name one worldline and a value follows. **The Ricci parameter is not advancing inside the manifold at all**. What advances is **a single path inside the space of metrics $\text{Met}(M)$** , and t is the parameter along that path.

The decisive point is this: under Ricci flow **the metric g is itself changing**. That is, **the very instrument that measures length is being deformed**. Since the instrument is moving, this cannot be a length measured by the instrument.

$d\tau$ advances inside a manifold. The fourth clock advances through the space of manifolds.

4. The Root of the Arrow Differs — Parabolicity, Not Coarse-Graining

The fourth type has **a direction**. But the source of that direction **differs** from the source of the arrow of dS .

Established Fact 4.1 (Parabolic equations cannot be run backwards)

In Fourier space the heat equation gives $u_k(t) = u_k(0)e^{-k^2 t}$. **Forwards ($t > 0$) every mode decays; backwards ($t < 0$) every mode blows up**. At $|t| = 1$, verified numerically here,

$$k = 1 : e^1 \approx 2.7, \quad k = 5 : e^{25} \approx 7.2 \times 10^{10}, \quad k = 20 : e^{400} \approx 5.2 \times 10^{173},$$

so the backward problem is **ill-posed**. Ricci flow is likewise **parabolic** (under the DeTurck trick) and carries the same obstruction.

Observation 4.1 (The two irreversibilities are distinct roots)

The irreversibility of Ricci flow is built into the equation — an analytic consequence of parabolicity, with no observer involved.

Hamiltonian flow, by contrast, is reversible (Liouville's theorem; Paper 77). The microscopic law carries no arrow, and the arrow of dS comes from coarse-graining, a matter of the observer's resolution (Papers 77, 78).

Hence the single word “irreversible” covers two distinct roots — an analytic root (parabolicity) and a resolution root (coarse-graining). They must not be mixed.

5. Inside the Fourth Type, the (dt, dS) Split Recurs

Here is the core of this paper. The fourth type is not one new clock.

The parameter is a label

Observation 5.1 (t can be relabelled)

Rewriting Ricci flow as $\frac{\partial}{\partial s} g_{ij} = -2\lambda R_{ij}$ with $\lambda > 0$, the substitution $s = t/\lambda$ traces the same trajectory in $\text{Met}(M)$. What is geometrically invariant is the trajectory, not the ticks marked along it.

So t has the same standing as dt : it is a label.

What carries a direction is a monotone

Established Fact 5.1 (Monotones along the flows)

Perelman's $\mathcal{W}$ -entropy is **non-decreasing** along Ricci flow (Perelman 2002). In two-dimensional field theory, Zamolodchikov's c -function is **non-increasing** along renormalisation-group flow (the c -theorem, 1986).

Observation 5.2 (The monotone occupies the position of dS)

What actually gives a flow its direction is not the parameter but the monotone. Were $\mathcal{W}$ or c not monotone, the flow would have no “which way”. This is structurally the same position that dS occupied.

	Label (relabelable)	Monotone with a direction
Spacetime / phase space	dt	dS (from coarse-graining)
$\text{Met}(M)$ / theory space	$t, \log \mu$	$\mathcal{W}, c$ (from parabolicity, mode integration)

The fourth type is not one clock but a new pair,
structurally parallel to (dt, dS) but living in a different space.

Brake: this parallel must not be read as sameness. $\mathcal{W}$ is not thermodynamic entropy — it is a functional on triples of metric, function and scale, not a functional of a probability measure on phase space. **The line Paper 88 drew between two entropies is drawn once more here:** $\mathcal{W} \neq S_{\text{coarse-grained}}$ and $\mathcal{W} \neq S_{\text{BH}}$. **Similarity of name and of role does not make two quantities the same** (Papers 58, 86, 100). This is precisely the line Paper 103 drew when it wrote “a control device, not the arrow of time of the universe”.

6. One Equation, Two Accounts of Irreversibility

Established Fact 6.1 (Friedan's result)

The one-loop beta function for the metric coupling of the two-dimensional nonlinear sigma model is

$$\beta_{ij}^g = \alpha' R_{ij} + O(\alpha'^2),$$

so renormalisation-group flow becomes the same equation as Ricci flow (Friedan 1980; Paper 110).

Observation 6.1 (The equations agree; the accounts do not)

Yet there are two accounts of why it cannot be run backwards:

- **Ricci side:** the equation is **parabolic**. The backward problem is ill-posed — **an analytic obstruction**.
- **RG side:** high-momentum modes are **integrated out and discarded**. Information is lost — **an obstruction of construction**.

Agreement of equations does not license merging the accounts. Paper 110 wrote that surgery and renormalisation are **a rhyme, not a theorem**. **This paper applies that discipline to the accounts of irreversibility as well.**

Note too that the RG obstruction is “discarded information”, which uses the same word as the root of the arrow of dS (coarse-graining). But sameness of word is not sameness of root — what dS discards is resolution in phase space; what RG discards is high-momentum modes. Until the addresses are written, they are not joined.

7. Self-Audit: The Blank Paper 103 Left

Observation 7.1 (103 had come halfway)

Paper 103 had already stated plainly that **Perelman's entropy must not be confused with the arrow of time of the universe** — that it is **a control device for the flow**. That judgement is right.

But Paper 103 did not write what type of quantity a “control device” is. All this paper adds is that type-name: a monotone along a path in $\text{Met}(M)$, whose parameter is a label of dimension L^2 .

Observation 7.2 (Three was not enough)

Paper 118 counted dt , $d\tau$ and dS . **Those three are right, and mutually distinct in type. But the series was already using a fourth** — in Papers 103 and 110.

This is not a correction of Paper 118 but a continuation of its roll-call. The line 118 drew (separate by type) applied unchanged to the fourth type. **Because the instrument was right, the missing cell became visible.**

Observation 7.3 (Relation to the blank left by Paper 117)

Paper 117 stated that the series holds **no Root-D result** and that beyond the wall there runs a known passage — **written on the map but not walked. That passage and the fourth type of this paper are in the same place:** renormalisation-group flow.

But this paper has not walked it either. All that has been done is **to determine the type of the “time” standing in that passage; no fixed point, no critical exponent and no anomalous dimension has been computed.** The stride is one step only.

8. Brakes: What This Paper Does Not Show

- $\mathcal{W}$ is not identified with thermodynamic entropy. The domains differ (Papers 88, 103): $\mathcal{W} \neq S_{\text{coarse-grained}}$, $\mathcal{W} \neq S_{\text{BH}}$.
- The Ricci parameter is not called physical time. Its dimension is L^2 , not seconds; no claim is made that the universe evolves by Ricci flow.
- Nothing is said about the Poincaré or geometrisation conjectures. This paper states no proposition of topology (the scope of Papers 07, 08, 103).
- The two accounts of irreversibility are not fused. Parabolicity and mode integration are different obstructions (Paper 110).
- RG's “discarded information” and dS 's “coarse-graining” are not joined merely because the words agree. What is discarded differs.
- The fourth type is not made “the real time”. As in Paper 118, electing any one as genuine is itself the error of type.
- No fixed point, critical exponent or anomalous dimension is computed (the blank of Paper 117 remains).

A personal note by the author (Layer 3, labelled non-scientific; not a claim about mechanism)

When I finished Paper 118 I believed the roll-call was done. Three types separated, addresses written, and that was that.

But the missing cell was not in someone else's work; it was in my own Papers 103 and 110. And 103 had already half-noticed, braking with the words “this is not the arrow of time of the universe”. **It had seen it, and walked past without naming it.** Rereading my own writing, this is not the first time that has happened.

What pleased me most was that **the matter was settled by a dimension.** $[t] = L^2$. Not seconds. No room for argument. Having built, in Paper 116, the measuring stick “two quantities of different dimension cannot be the same quantity”, I found three papers later that it came back to help me. **A tool does not announce, when it is made, what it will be used for.**

And thinking about a situation in which the measuring instrument itself is being deformed made me slightly dizzy. **While the ruler is stretching and shrinking, with what does one measure the stretching?** The Ricci parameter is one answer: **prepare a room in which all the rulers stand side by side, and count movement inside that room.** Stop standing inside the manifold to measure, and stand where the manifolds can be looked down upon.

A line of thought that began by doubting dt has arrived at **the space of measuring instruments.** The passage remains unwalked and the blank of Paper 117 remains unfilled. I think that is fine. **Knowing that a cell is empty is far better than believing it is full.**

9. Scope and Conclusion

What is shown (descriptive, a continued roll-call): that the “time” of Ricci and RG flow is a **fourth type**, none of dt , $d\tau$, dS ; that the explicit sphere solution $r^2 = r_0^2 - 2(n-1)t$ forces $[t] = L^2$, not seconds (§2); that **the domain is a path in $\text{Met}(M)$ and the measuring instrument itself is deformed** (§3); that **the root of the arrow is parabolicity, not coarse-graining, and therefore distinct from the arrow of dS** (§4); that **inside the fourth type the (dt, dS) split recurs**, the parameter being a label and the monotones $(\mathcal{W}, c)$ carrying the direction (§5); that **one equation (Friedan) carries two accounts of irreversibility, which must not be merged** (§6); and that **the type-name Paper 103 left blank has been filled** (§7). **What is not shown (restraint):** $\mathcal{W}$ is not identified with thermodynamic entropy (88, 103); the Ricci parameter is not called physical time; nothing is claimed about Poincaré or geometrisation; parabolicity and mode integration are not fused (110); RG's “discarded information” and dS 's “coarse-graining” are not joined by verbal coincidence; the fourth type is not made “the real time”; no fixed point, critical exponent or anomalous dimension is computed (the blank of Paper 117 remains); no new mathematics and no prediction (Layer 2). Having counted three in

Paper 118, I took the roll-call to be complete. **But the fourth was inside this series' own Papers 103 and 110.** The “time” of Ricci flow is neither dt , nor $d\tau$, nor dS — **because its dimension is L^2 .** A quantity that is not measured in seconds had been called by the same word as one that is. The domain shows why. **This clock is not advancing inside a manifold. It advances through the space of manifolds.** Since the instrument of measurement is itself being deformed, it could never have been a length measured by that instrument. **And the fourth type turned out not to be one clock.** The parameter is a label; what carries a direction is a monotone $(\mathcal{W}, c)$. **The split between dt and dS was being repeated, unchanged, in another space. But the root of the arrow differs** — there it is coarse-graining, here it is parabolicity. Paper 103 had already come halfway. It stopped at the sentence “this is not the arrow of time of the universe”. **All this paper has added is a name written at the place where it stopped.** And this returns address number 119.

References

1. Hamilton, R. S. (1982). *Three-Manifolds with Positive Ricci Curvature*. Journal of Differential Geometry.
2. DeTurck, D. M. (1983). *Deforming Metrics in the Direction of Their Ricci Tensors*. Journal of Differential Geometry.
3. Perelman, G. (2002). *The Entropy Formula for the Ricci Flow and Its Geometric Applications*. arXiv:math/0211159.
4. Friedan, D. (1980). *Nonlinear Models in $2 + \epsilon$ Dimensions*. Physical Review Letters.
5. Zamolodchikov, A. B. (1986). *Irreversibility of the Flux of the Renormalization Group in a 2D Field Theory*. JETP Letters.
6. Wilson, K. G. (1971). *Renormalization Group and Critical Phenomena*. Physical Review B.
7. Hadamard, J. (1902). *Sur les problèmes aux dérivées partielles et leur signification physique*. Princeton University Bulletin. (The notion of well-posedness.)
8. Yamagishi, Y. (2026). *Two Clocks and One Scale* [Paper 118]. Zenodo: 10.5281/zenodo.22036237.
9. Yamagishi, Y. (2026). *Scale Invariance Fixes an Exponent — in Four Different Ways* [Paper 117]. Zenodo: 10.5281/zenodo.22005515.
10. Yamagishi, Y. (2026). *What Does the Exponent of c Count?* [Paper 116]. Zenodo: 10.5281/zenodo.22003794.
11. Yamagishi, Y. (2026). *Flowing Geometry to Canonical Form* [Paper 103]. Zenodo. 10.5281/zenodo.21836801
12. Yamagishi, Y. (2026). *Ricci Flow IS a Renormalization Group Flow* [Paper 110]. Zenodo. 10.5281/zenodo.21837476
13. Yamagishi, Y. (2026). *Liouville's Theorem and the Arrow of Time* [Paper 77]. Zenodo. 10.5281/zenodo.21722473
14. Yamagishi, Y. (2026). *Friction and Dissipation as Coarse-Graining* [Paper 78]. Zenodo. 10.5281/zenodo.21731937
15. Yamagishi, Y. (2026). *Two Entropies, Not to Be Conflated* [Paper 88]. Zenodo. 10.5281/zenodo.21747809
16. Yamagishi, Y. (2026). *Time as a Renko Chart* [Paper 82]. Zenodo. 10.5281/zenodo.21735260
17. Yamagishi, Y. (2026). *The Past Hypothesis* [Paper 98]. Zenodo. 10.5281/zenodo.21758739
18. Yamagishi, Y. (2026). *The Crossing of Royal Roads* [Paper 100]. Zenodo. 10.5281/zenodo.21758864
19. Yamagishi, Y. (2026). *The Four Roots of Necessity* [Paper 86]. Zenodo. 10.5281/zenodo.21744839

— *From thinking to syncing.* —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)