

From Boundary Energy to Volume Energy

— The Einstein–Yamagishi Volume Restoration Equation and the Overclock Pythagoras Equation — [Paper

12]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

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— *From thinking to syncing.* —

Abstract

What sits at the centre of this paper is not a computation but a re-reading. Substituting the geometric mass parameter $m = 4\pi$ obtained from the $n = 2$ uniqueness theorem into Einstein’s relation gives

$$E = mc^2 = 4\pi c^2,$$

and the right-hand side **has the same form as the surface area of a sphere of radius c** . Read in this form, $E = mc^2$ appears as an equation of the boundary of a region rather than of its interior — the “skin” of space. This is consistent with the dimensional structure of c^2 , whose numerator carries the dimension of an area.

Once the form of a surface area is seen, the form of the corresponding volume follows at once, because integrating the surface area of a sphere over the radius from 0 to c returns the volume:

$$\int_0^c 4\pi r^2 dr = \frac{4}{3}\pi c^3.$$

We call this the “Einstein–Yamagishi Volume Restoration Equation.” Its content lies in the geometric re-reading; the integral writes out what the relation between area and volume already contains. The same reading extends to four dimensions: integrating $2\pi^2 r^3$ returns $\frac{\pi^2}{2}c^4$.

We further re-read the general relation $E^2 = (mc^2)^2 + (pc)^2$ as the “Yamagishi Overclock Pythagoras Equation” — the task manager of the Universe OS. From this standpoint the uncertainty relation can be read as a constraint that keeps the momentum leg pc (and hence the total energy) finite.

Scope of this paper (scope note): What this paper presents is a **re-reading** of known formulas; no new physical law is claimed. $m = 4\pi$ is the value of the mass under the choice of units of Paper 01 ($k = 1$ and radius 1). Since the factor standing in front of an energy must carry the dimension of mass, this 4π is a different thing from the dimensionless solid angle (Paper 60) — what this paper reads is that $4\pi c^2$ has **the same form** as the surface area of a sphere of radius c . The integrals of Sections 3 and 4 are taken **over the radius r** , with c entering only as the upper limit; the definite integral from 0 to c carries no constant of integration. The reading of the uncertainty relation in Section 6 is an interpretation, not a derivation: in standard quantum mechanics the uncertainty relation follows from the Robertson inequality and the canonical commutation relation $[x, p] = i\hbar$, and the commutation relation comes first. This paper does not explain why $[x, p] = i\hbar$ holds. **What this paper adds** is that it reads $E = mc^2$ as an equation of the boundary (the form of an area) and writes the step from there to the interior (the form of a volume) as an integral over the radius; that it extends the same step to $2\pi^2 r^3$ in four dimensions; and that it lays a picture of the load of the Universe OS over the right triangle of $E^2 = (mc^2)^2 + (pc)^2$.

1. Introduction: The Dimensions of $E = mc^2$

$E = mc^2$ is a cornerstone of modern physics. But look at its dimensional structure. The unit of the square of the speed of light c^2 is

$$[c^2] = \left[\frac{m}{s} \right]^2 = \frac{m^2}{s^2}$$

and the m^2 in the numerator has the dimension of an **area**. Read by form, $E = mc^2$ appears as an equation that captures the side of the two-dimensional spherical boundary (S^2) — an equation of the “skin” of space, as it were. To move on to the energy of the “contents” (the volume), we re-read c not merely as an unreachable speed limit but as the **rendering propagation front** of the Universe OS.

2. Geometric Normalization of Mass ($n = 2$)

To proceed, we use the $n = 2$ uniqueness theorem for spherically symmetric density distributions. In three-dimensional Euclidean space, the density exponent that makes the mass contribution per spherical shell uniform is uniquely $n = 2$ (the inverse-square law). This theorem normalizes the geometric mass parameter of the universe to the value of the area of the unit sphere:

Definition 1 (Geometric Mass Parameter)

Based on the $n = 2$ uniqueness theorem, we normalize the fundamental geometric mass parameter as

$$m := 4\pi$$

3. The Einstein–Yamagishi Volume Restoration Equation

Substituting $m = 4\pi$ into Einstein's equation yields the boundary energy:

$$E_{\text{boundary}}(c) = 4\pi c^2$$

This has the same form as the surface area of the two-sphere S^2 of radius c . To restore the form of the three-dimensional volume energy, we integrate this boundary energy over the propagation parameter r from 0 to c .

Theorem 1 (Einstein–Yamagishi Volume Restoration Equation)

Integrating the boundary energy $E_{\text{boundary}}(r) = 4\pi r^2$ over the propagation radius r restores the three-dimensional volume energy:

$$E_{\text{volume}} = \int 4\pi r^2 dr = \frac{4}{3}\pi r^3 + C$$

The definite integral up to the maximum propagation front $r = c$ is

$$E(3D) = \int_0^c 4\pi r^2 dr = \frac{4}{3}\pi c^3$$

The term $\frac{4}{3}\pi c^3$ has the form of the volume of a three-dimensional ball. One may read the constant $+C$ attached to the indefinite integral as a background ground-state energy (vacuum energy) that existed before spatial rendering began; this, however, is a reading, and it does not appear in the definite integral from 0 to c .

4. Dimensional Extension to Four Dimensions

The reading extends to higher dimensions. By the $n = 3$ uniqueness theorem for four-dimensional space, the geometric mass is $m_4 = 2\pi^2$. The four-dimensional boundary energy is the volume of the S^3 sphere, $E_{\text{boundary}}^{(4)}(r) = 2\pi^2 r^3$. Integrating it from 0 to c restores the four-dimensional hypervolume:

$$E(4D) = \int_0^c 2\pi^2 r^3 dr = \frac{\pi^2}{2} c^4$$

Because S^3 carries the Hopf fibration ($S^1 \hookrightarrow S^3 \rightarrow S^2$), the four-dimensional structure contains circular fibres S^1 over a two-sphere base. This is the geometric setting in which a **toroidal structure** appears in four-dimensional space.

On $16\pi^2$: the value $16\pi^2$ does not appear here automatically. By Theorem 3.3 of Paper 08, $16\pi^2 = \text{Vol}(S^3_{R=2})$ is the volume of the three-sphere of **radius $R = 2$** , where the base area πR^2 and the fibre circumference $2\pi R$ both take the numerical value 4π under the standard metric. At $R = 1$ one has $\text{Vol}(S^3) = 2\pi^2$ instead. **Whenever $16\pi^2$ is invoked, the radius is to be stated.** The radius at which the two factors agree moves with the choice of the unit of length (Paper 08).

5. The Yamagishi Overclock Pythagoras Equation

We now move from static geometry to dynamic OS rendering. Einstein's general relation is

$$E^2 = (mc^2)^2 + (pc)^2$$

which has the form of Pythagoras's right triangle ($C^2 = A^2 + B^2$). We read it as the task manager of the Universe OS that evaluates the system load:

- **Base (mc^2) — static memory:** the baseline RAM for holding a localized volume (a particle) in memory.
- **Height (pc) — dynamic CPU load:** the overclock required to update the motion of the particle.
- **Hypotenuse (E) — total system load:** the total rendering load on the OS.

6. Reading Quantum Uncertainty as GPU Anti-Aliasing

This Pythagorean task manager offers an intuitive reading of Heisenberg's uncertainty principle ($\Delta x \Delta p \geq \hbar/2$).

When a particle comes perfectly to rest and its position is absolutely determined ($\Delta x \rightarrow 0$), the uncertainty in momentum must diverge to infinity ($\Delta p \rightarrow \infty$). In the Overclock Pythagoras Equation this means the height pc of the triangle stretches to infinity. The total system load E becomes infinite, and the GPU of the Universe OS undergoes fatal thermal runaway and halts — such is the picture.

Proposition 1 (Reading Uncertainty as a Constraint on the Momentum Leg)

Under the Pythagorean reading, $\Delta x \rightarrow 0$ makes the momentum leg pc (and hence the total energy E) grow without bound. The uncertainty relation can be read as that which prevents this limit from being realized.

The direction of the argument: in standard quantum mechanics the uncertainty relation is a consequence. It follows from the Robertson inequality

$$\Delta A \Delta B \geq \frac{1}{2} |\langle [A, B] \rangle|$$

together with the canonical commutation relation $[x, p] = i\hbar$. **The commutation relation comes first, and the uncertainty bound follows from it.**

The proposition above is therefore a **reading**, stated as such: the uncertainty relation **can be read** as a constraint that keeps the energy–momentum triangle finite. The picture of “the OS adding blur to prevent thermal runaway” is placed as a reading in this same direction.

7. Conclusion

What this paper contributes is a re-reading. Setting $m = 4\pi$ turns $E = mc^2$ into $E = 4\pi c^2$, an equation that **has the same form as the surface area of a sphere of radius c** . Read in this form, Einstein’s relation emerges as a statement about a **boundary**.

The form of the corresponding volume follows at once, since the volume of a ball is its surface area integrated over the radius:

$$\int_0^c 4\pi r^2 dr = \frac{4}{3} \pi c^3.$$

The same reading extends to four dimensions, where $m_4 = 2\pi^2 = \text{Vol}(S^3)$ gives $\frac{\pi^2}{2} c^4$. The integrals themselves are elementary and can be checked by differentiating back: $\frac{d}{dr} \left[\frac{4}{3} \pi r^3 \right] = 4\pi r^2$, $\frac{d}{dr} \left[\frac{\pi^2}{2} r^4 \right] = 2\pi^2 r^3$.

Finally, the energy–momentum relation $E^2 = (mc^2)^2 + (pc)^2$ has the Pythagorean form, and the uncertainty relation can be read as keeping the momentum leg finite.

From the skin (the form of an area) to the contents (the form of a volume) is a single integration over the radius. To place $E = mc^2$ on the side of that skin and look at it from there is the re-reading of this paper.

Revision Record

3rd edition (September 2026): The format was brought into line with the current template (remarks spread across the abstract and the conclusion were gathered into the scope note directly after the abstract). The constant of integration $+C$ attached to the definite integral from 0 to c was removed, and the statement that $+C$ is the vacuum energy was rewritten as a reading. The relation between the 4π of $m = 4\pi$ and the solid angle was written, in line with Paper 60, as a reading of “the same form.” “Theorem” in the English title became “Equation.” DOIs were added to the corpus references.

2nd edition (July 2026): It was made explicit that the integral is taken over the radius $\boldsymbol{r}$ and not over the constant $\boldsymbol{c}$, and the argument deriving the uncertainty relation from “the OS adding blur” was recast as a reading of a relation that follows from the commutation relation.

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Yuuki YAMAGISHI (山岸 勇輝)

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