

Dimensional Analysis Fails in Two Ways

— Coordinate Time dt , Proper Time $d\tau$ and Entropy dS Are Three Different Logical Types — [Paper 118]

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— *From thinking to syncing.* —

ABSTRACT

Paper 116 surveyed the wall, writing that the exponent formula is an identity **inside** dimensional analysis and gives no independent route to the value of N . Paper 117 recorded that beyond that wall runs **a known passage** (the anomalous dimension); Paper 119 determined the type of the “time” standing in it. **None of the three walked it.** Instead of walking, this paper **goes as far as the fork and reads the signs. The result: there is not one corridor but two. The first corridor (β -root, dimensional transmutation):** in a theory with no dimensionful parameter at all in its classical action (massless Yang--Mills), **dimensional analysis predicts that no scale can exist.** Yet the quantum theory produces $\Lambda = \mu \exp(-8\pi^2/(b_0 g^2))$. We confirm $\mu d\Lambda/d\mu = 0$ symbolically, and verify numerically that with $\alpha_s(M_Z) = 0.1180$ and $n_f = 5$, running μ from **91.19** down to **5** GeV leaves $\Lambda = 87.827$ MeV **unchanged to every digit.** The prediction is **falsified — a quantity that should not exist comes into being.** The escape route is the **essential singularity** e^{-1/g^2} , invisible at every order of perturbation theory (Paper 05). **The second corridor (γ -root, anomalous dimension):** dimensional analysis predicts the correlation exponent $d - 2$, whereas the true value in three-dimensional Ising is $d - 2 + \eta = 1.0363$. **Again the prediction is falsified — but the manner of failure differs. Here the quantity exists and its exponent is shifted.** Third, the decisive evidence that the two are independent: at any fixed point $\beta = 0$ by definition. Were β and γ one mechanism, γ would have to vanish there too. **Yet the Gaussian fixed point has $\eta = 0$, Wilson--Fisher (3D Ising) has $\eta \simeq 0.036$, and two-dimensional Ising (Onsager, exact) has $\eta = 1/4$ — $\beta = 0$ at all three, and yet η takes three different values. Hence γ is not determined by β .** Fourth, there is nonetheless a common ancestor: both descend from the fact that **regularisation requires a scale** (the trace anomaly). **But a common ancestor is not a common root** (Papers 86, 100); the fixed-point argument proves it. **Fifth, and this paper's self-audit: the census in Paper 117 is corrected.** Paper 117 wrote that the series holds no Root-D result and that Roots C and D are empty. **That is right about a result but inaccurate about contact — Paper 05 was already treating dimensional transmutation, had verified the μ -independence of Λ numerically, and had even run its own census: a scale gets fixed in at least three ways — (a) dimensional analysis, (b) a choice of units, (c) dimensional transmutation. The series was already standing in the β -corridor, inside Paper 05. The γ -corridor is still entirely empty. Scope: a structural interpretation and self-audit of established standard facts (Layer 2). This paper does not walk the corridor either.** Filling the γ cell would require a space of couplings, a flow, a fixed point and a linearisation, and **no flow whatever is defined on 4π , $n = 2$ or the Planck quantities.** Nothing is claimed about the existence of Yang--Mills and the mass gap, or about the rigorous construction of critical phenomena.

Position of this paper (discipline): every ingredient is a **known standard result** — dimensional transmutation (Coleman--Weinberg), the one-loop running coupling and the RG-invariance of Λ , anomalous dimensions and critical exponents, the Wilson--Fisher fixed point, Onsager's exact solution, the trace anomaly. **No new theorem is proved.** The contribution is **identifying a fork and correcting one's own roll-call:** (i) separating **two manners** in which dimensional analysis fails, by **how the prediction is falsified**; (ii) proving their independence **at fixed points**; (iii) treating the **common ancestor** carefully rather than crudely; (iv) **correcting the census line in Paper 117**; and (v) listing what filling the γ cell would require. **The most important restraint: this paper does not walk the corridor. No renormalisation flow, fixed point, critical exponent or anomalous dimension is computed for this series' own quantities. β and γ are not merged** (fixed points are the counterexample). Λ_{QCD} is not confused with a measured quantity (it is scheme-dependent; Paper 05). **The existence of Yang--Mills and the mass gap is an unsolved Millennium Prize Problem** — neither its proof nor any partial achievement is claimed. “A resonance is a signature, not a proof” (Paper 100). Layer-3 lenses are quarantined with a non-scientific label.

1. Introduction: Three Papers Running, Halted at the Same Place

Paper 116 wrote down the exponent formula of the Planck reduction and concluded: **the formula is an identity inside dimensional analysis and gives no independent route to the value of N . It surveyed the wall.**

Paper 117 recorded that beyond that wall runs a **known passage** — the anomalous dimension — and wrote: **written on the map, not walked.**

Paper 119 determined the type of the “time” standing in that passage: a fourth type. And wrote: **this paper has not walked it either.**

Three papers running, halted before the same place.

This paper does not walk it either. **But it goes as far as the fork**, and reads the signs. To state the reading in advance: **there was not one corridor but two.**

2. Corridor One: the β -Root — a Quantity That Should Not Exist Comes into Being

Established Fact 2.1 (Dimensional transmutation)

Classical massless Yang--Mills theory is **scale invariant**: its action contains **no parameter of mass dimension at all**. Integrating the one-loop renormalisation-group equation

$$\mu \frac{dg}{d\mu} = -\frac{b_0 g^3}{16\pi^2}$$

gives $1/g^2(\mu) = \frac{b_0}{8\pi^2} \ln(\mu/\Lambda)$, so that

$$\Lambda = \mu \exp\left(-\frac{8\pi^2}{b_0 g^2(\mu)}\right)$$

is fixed. We confirm $\mu d\Lambda/d\mu = 0$ by symbolic computation, and verify numerically that with $\alpha_s(M_Z) = 0.1180$ and $n_f = 5$, taking $\mu = 91.19, 30, 10, 5$ GeV leaves $\Lambda = 87.827$ MeV **unchanged to every digit**.

Observation 2.1 (The prediction of dimensional analysis is falsified)

Since the classical theory has no scale, dimensional analysis predicts that no mass scale can exist. And yet $\Lambda \neq 0$.

What is falsified here is therefore not the value of an exponent, but the prediction that the quantity does not exist. The escape route is the **essential singularity** $e^{-8\pi^2/(b_0 g^2)}$, all of whose derivatives vanish as $g \rightarrow 0$, so that it is **invisible at every order of perturbation theory** (Paper 05).

First manner of failure: a quantity that should not exist comes into being.

3. Corridor Two: the γ -Root — the Exponent of an Existing Quantity Shifts

Established Fact 3.1 (Anomalous dimension)

The scaling dimension of a field or operator departs from its classical value:

$$\Delta = \Delta_{\text{classical}} + \gamma.$$

At a critical point the correlation function obeys $G(r) \sim r^{-(d-2+\eta)}$, and while **naive dimension counting** gives $\eta = 0$, the three-dimensional Ising universality class has $\eta \simeq 0.0363$, so the true exponent is **1.0363** (Paper 117).

Observation 3.1 (The manner of falsification differs)

Here too the prediction of dimensional analysis is falsified. But it is falsified differently from the first corridor.

In the first, “the quantity does not exist” was overturned. In the second, the quantity does exist and the value of its exponent is overturned.

Corridor	DA predicts	Actually	Manner of failure
β -root	no scale can exist	$\Lambda \neq 0$ appears	a quantity is born
γ -root	the exponent is $d - 2$	$d - 2 + \eta = 1.0363$	an exponent shifts

4. The Two Are Independent — Fixed Points Prove It

It remains possible that the two corridors are two faces of one mechanism. That is settled here.

Theorem 4.1 (Separation by fixed points)

At any fixed point $\beta = 0$ by definition (a fixed point being where the coupling stops running).

Were β and γ one mechanism, γ would have to vanish wherever β does.

Fixed point	β	η	Note
Gaussian ($d > 4$)	0	0	free field
Wilson--Fisher (3D Ising)	0	$\simeq 0.036$	interacting CFT
Two-dimensional Ising (Onsager)	0	$1/4$	exactly solved

Corollary 4.1 (γ is not determined by β)

All three fixed points have $\beta = 0$. Nevertheless η takes the three distinct values 0, 0.036 and 1/4.

Therefore γ does not follow from β . The two are independent. Root D is not one but two.

β vanishes and γ survives. That is the proof of separation.

5. There Is, Nonetheless, a Common Ancestor

Observation 5.1 (The trace anomaly)

Both descend from the fact that regularisation requires a scale. Classical scale invariance becomes anomalous in the quantum theory, and the trace of the energy--momentum tensor fails to vanish:

$$T^\mu_\mu = \frac{\beta(g)}{2g} F^2 + \dots$$

(the trace anomaly). β appears there directly, while γ appears as the running of operator normalisations.

Brake: a common ancestor is not a common root (Papers 86, 100). β governs the running of the coupling; γ governs the running of the normalisation. From the former a scale is born; in the latter an exponent shifts. And the fixed-point argument of the previous section shows that they are logically independent. It is precisely on finding a common origin that one wants to merge. Stopping there is the discipline of this series (Paper 110: “a rhyme, not a theorem”; Paper 119: “agreement of equations does not license merging the accounts”).

6. Self-Audit: Correcting the Census Line in Paper 117

Observation 6.1 (What Paper 117 wrote)

Paper 117 called the roll of which roots this series inhabits and wrote: **the series lives in Roots A and B alone; Roots C and D are empty. And: Root D is precisely where physics exceeds dimensional analysis, and beyond the wall runs a known passage.**

Observation 6.2 (Right about a result, inaccurate about contact)

That the series holds no Root-D *result* remains true. But it was not the case that the series had never touched it.

Paper 05 (Yang--Mills) was already treating dimensional transmutation head-on. It verified the μ -independence of Λ numerically, showed that Λ is **an invariant assembled out of two conventions** (μ and $g(\mu)$), and had even **run a census of its own: a scale gets fixed in at least three ways — (a) dimensional analysis, (b) a choice of units, (c) dimensional transmutation — and these are distinct roots that must not be merged.**

That (c) is what this paper calls the β -root. The series was already standing in the β -corridor — long before Paper 117 wrote the cell as empty, inside Paper 05.

Paper 117's roll-call had missed this series' own Paper 05.

This is a correction.

Observation 6.3 (The γ side is still entirely empty)

Of the γ -root, by contrast, the series holds nothing. No paper treats anomalous dimensions, and the counterexample Paper 117 cited (the η of three-dimensional Ising) is **a counterexample borrowed from another field**, not a statement about this series' own quantities.

7. What Filling the γ Cell Would Require

Observation 7.1 (Four components)

For the series to hold a γ -root result, at least the following four are needed:

- **(1) A space of couplings:** a space of theories in which the series' quantities sit as coordinates.
- **(2) A flow on it:** a map moving those couplings under coarse-graining.
- **(3) A fixed point of that flow:** a point where $\beta = 0$.
- **(4) A linearisation at the fixed point:** whose eigenvalues are the critical exponents.

An honest report of the present state: the series has none of these four. 4π is a solid angle (Papers 02, 13); $n = 2$ is the exponent of a density profile (Paper 01); the Planck quantities are monomials in three constants (Paper 116). **No flow moving under coarse-graining has ever been defined on any of them. Without a flow there is no fixed point, and without a fixed point there is no linearisation and no eigenvalue.**

This paper therefore does not fill the γ cell. What can be done is **to list what is missing, and that is this section.**

Observation 7.2 (Why one stops here)

Defining some flow on the series' quantities is technically possible. But being able to define something is not the same as its meaning anything (Eddington's caution, Paper 101). **Write a meaningless flow, solve for its fixed point, and the number that emerges is the exponent of nothing. The shape of a filled cell would remain, and the content would still be empty. Writing an empty cell as empty is the more accurate map.**

8. Brakes: What This Paper Does Not Show

- **It does not walk the corridor.** Not one renormalisation flow, fixed point, critical exponent or anomalous dimension is computed for this series' quantities.
- **β and γ are not merged.** Fixed points are the counterexample ($\beta = 0$ yet $\eta \neq 0$). A shared ancestor (the trace anomaly) is not a reason to merge.

- Λ_{QCD} is not confused with a measured quantity. It is a bookkeeping device depending on scheme, n_f and loop order; what is measurable is the mass gap (glueball masses) (Paper 05).
- Nothing is said about the existence of Yang--Mills and the mass gap. It is an unsolved Millennium Prize Problem; neither proof nor partial achievement is claimed (Paper 05).
- Nothing is said about the rigorous construction of critical phenomena.
- No meaning is read into the numbers $\eta \simeq 0.036$ or $1/4$ (Papers 94, 101). Only the fact that they do not vanish where β does is used.
- Paper 117 is not retracted. What is corrected is one line of its roll-call; its claims (four distinct roots; the anomalous dimension as a counterexample to Root B) all stand.

A personal note by the author (Layer 3, labelled non-scientific; not a claim about mechanism)

The most awkward thing this time was that in the cell I had written as “empty” in Paper 117, my own Paper 05 was already standing. And Paper 05 had itself run a census, saying a scale gets fixed in three ways. What I failed to count three papers ago, I had already written five papers ago.

This is not the first time. In Paper 119 it was Paper 103 that had stopped halfway; in Paper 116 it was Paper 01 sitting in the denominator. As the series grows, my own writing falls out of my own field of view. The one drawing the map forgets part of the map.

Even so, this correction felt good. I had thought a cell was empty, and it turned out something was standing there. Nothing was lost; a possession was gained.

And the second corridor really was empty. Filling it requires a flow, and the series' quantities have none. Define one by force and the shape can be produced, but the numbers coming out would be the exponents of nothing. I think of Eddington changing 136 into 137 (Paper 101). A cell being filled and a cell having content are different things.

Four papers running, halted before the same corridor. That makes four times I have written that I stopped. I still think it is worth writing, because the place where I stop keeps moving forward: 116 measured the wall, 117 found the passage, 119 fixed the type of the clock standing in it, and 120 learned that the passage is two passages and that I was already standing in one of them. I have not walked. But where I stand has changed.

9. Scope and Conclusion

What is shown (descriptive, self-audit): that dimensional analysis fails in **two manners** — in the β -root the prediction “no such quantity exists” is overturned; in the γ -root the value of an exponent is overturned (§2, §3); the RG-invariance of $\Lambda = \mu e^{-8\pi^2/(b_0 g^2)}$, confirmed both symbolically and numerically (§2); that **the two are independent, proved at fixed points** — three fixed points with $\beta = 0$ carry $\eta = 0, 0.036$ and $1/4$ (§4); that **a common ancestor (the trace anomaly) exists but is no reason to merge** (§5); that **Paper 117's roll-call had missed this series' own Paper 05** (§6); and a list of the four components needed to fill the γ cell, with an honest report that the series has none of them (§7).

What is not shown (restraint): the corridor is not walked — no flow, fixed point, critical exponent or anomalous dimension is computed for this series' quantities; β and γ are not merged (fixed points are the counterexample); Λ_{QCD} is not confused with a measured quantity (Paper 05); nothing is claimed about Yang--Mills existence and the mass gap (unsolved); nothing about the rigorous construction of critical phenomena; no meaning is read into the values of η (Papers 94, 101); **Paper 117 is not retracted** (only one line of its roll-call is corrected); no new mathematics and no prediction (Layer 2). Paper 116 measured the wall; Paper 117 wrote that a passage runs beyond it; Paper 119 fixed the type of the clock standing in that passage. **None of the three walked.** This paper does not walk either. **But it went as far as the fork and read the signs. There were two corridors.** In the first, **a quantity that should not exist comes into being (Λ)**; in the second, **the exponent of an existing quantity shifts (η)**. **Both falsify the prediction of dimensional analysis, but they falsify it differently.** And **fixed points separate them**: since η takes three different values where $\beta = 0$, γ does not follow from β . **Finally, a cell of my own was corrected.** Paper 117 wrote that Root D was empty, **but Paper 05 was already standing in the first corridor** — treating dimensional transmutation, confirming the invariance of Λ , and **having already run its own census of the three ways a scale gets fixed. What went uncounted was not someone else's paper but my own, five papers back. The second corridor is still empty.** Filling it requires a space of couplings, a flow, a fixed point and a linearisation, and **the series has none of them.** Force one into existence and the shape can be filled, but the number that emerges is the exponent of nothing. **Writing an empty cell as empty is the more accurate map.** And this returns address number 120.

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