

How Much Do the Four Clocks Know About the Dimension They Are In

— Putting the Exponent Formula of the Yamagishi Patch to the Einstein–Yamagishi Dual-Clock Framework: the Tool Halts Between dt and $d\tau$ — [Paper 122]

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— *From thinking to syncing.* —

ABSTRACT

Paper 116 gave an exponent formula writing any quantity under the Yamagishi patch as $X = \hbar^p G_d^q c^N$, and showed that **the spatial dimension d sits only in the denominator $n = d - 1$** . Papers 118 and 119 showed that the single word “time” covers **four logical types**: the coordinate dt , the functional of a curve $d\tau$, the functional of a measure dS , and the parameter of a flow (Ricci and renormalisation-group, with $[t] = L^2$). This paper **puts the former tool to the latter census**. What it obtains is not a positive result but **the place where the tool halts**. **First, the four clocks have three different natural units** (Theorem 3.1, Corollary 3.2, verified symbolically). dt and $d\tau$ give $(\hbar G_d/c^3)^{1/n}/c$, the **Planck time**. The flow parameter gives $(\hbar G_d/c^3)^{2/n} = \ell_P^2$, the **Planck area** — not a second but an **area**. And dS **does not enter the formula at all**: $[S]$ contains temperature Θ , and Θ appears in no combination of $\hbar$, G_d and c . The natural unit of dS is **Boltzmann's constant k_B itself**, not a Planck quantity (Observation 3.3). **Second, and this is the core: the formula cannot distinguish dt from $d\tau$** . Their (p, q, N) are **identical** (Theorem 4.1). Moreover the formula itself is a **bijection** (Observation 2.2) and loses nothing. What is lost is lost **one step earlier** — where a coordinate and a functional of a curve, two objects of different logical type, are assigned **the same dimension**. Paper 118's recourse to a discriminant of logical type was therefore **forced, not a matter of taste**. **The dimensional tool provably halts there** (Corollary 4.2). **Third, this is neither of the two corridors counted in Paper 120**. That paper separated two ways in which dimensional analysis **breaks** — β (a quantity appears that was not in the input) and γ (the exponent itself shifts). Both return a **wrong** answer. Here the answer is **right and not unique**. It is not a breakdown but a **limit of resolution**, and **the only one of the three in which the tool is blameless** (Observation 4.3). **Fourth, the sensitivities to dimension are 1, 1, 0, 2** in units of $1/n$. **The flow parameter feels the dimension exactly twice as strongly as ordinary time**. Yet the dimension $[t] = L^2$ is itself the same in every d : **the dimension of a quantity and the route by which it reaches its Planck value are two different things** (Observation 5.2).

Scope of this paper: NO NEW PHYSICAL LAW AND NO NEW THEOREM. Both tools used are already published in this corpus (the exponent formula of Paper 116, the logical types of Papers 118 and 119). All this paper does is **put one to the other and write down how far it reaches and where it stops**. All external physics is standard (Planck units, the dimensions of the Ricci flow $\partial_t g_{ij} = -2R_{ij}$, Boltzmann's constant).

1. Introduction: putting a tool to one's own census

The corpus holds two independent pieces of work.

One is **a dimensional tool**. Paper 116 gave, for a quantity of dimension $[X] = L^\lambda M^\mu T^\tau$ in d spatial dimensions, the formula writing it as $X = \hbar^p G_d^q c^N$, and showed that **the “3” of c^3 is not the spatial dimension but a weight of time, while d enters only through the denominator $n = d - 1$** . It was not where it was being looked for; it was sitting inside the root.

The other is **a census of times**. Paper 118 showed that the one word “time” covers three logical types, and Paper 119 found within the corpus itself that the census was incomplete, identifying a fourth type.

This paper puts the former to the latter. The aim is not to obtain a positive result. It is **to measure how far the tool reaches, and to write down where it does not**.

2. The two tools

2.1 The exponent formula of the Yamagishi patch (Paper 116)

Theorem 2.1 (the exponent formula, Paper 116)

For a quantity of dimension $[X] = L^\lambda M^\mu T^\tau$ in d spatial dimensions written as $X = \hbar^p G_d^q c^N$, with $n := d - 1$,

$$q = \frac{\lambda + \tau - \mu}{n}, \quad p = \mu + q, \quad N = -(\mu + \tau) - 3q \quad (1)$$

and this solution is **unique** (the determinant of the system is $1 - d = -n$).

(Self-check) That this paper's implementation reproduces the formula is checked by feeding it the constants themselves (verified symbolically):

input	dimension	(p, q, N)
$\hbar$	ML^2T^{-1}	$(1, 0, 0)$
c	LT^{-1}	$(0, 0, 1)$
G_d	$L^dM^{-1}T^{-2}$	$(0, 1, 0)$

Each returns itself; the implementation is correct.

Observation 2.2 (the formula is a bijection — groundwork for Section 4)

Equation (1) is a map $(\lambda, \mu, \tau) \mapsto (p, q, N)$, and **it has an inverse** (verified symbolically):

$$\lambda = N + dq + 2p, \quad \mu = p - q, \quad \tau = -N - p - 2q$$

So **the formula itself discards nothing**. This matters in Section 4: the loss of a distinction found there **cannot be the formula's doing**.

2.2 The four logical types (Papers 118 and 119)

Fact 2.3: In the corpus's census, the single word “time” covers the following four.

Type	What it is	Source
dt	a coordinate — a label pinned to the manifold	Papers 108, 118
$d\tau$	a functional of a curve — proper time along a worldline	Papers 108, 118
dS	a functional of a measure — the monotone given by coarse-graining	Papers 77, 78, 118
flow parameter	the parameter of a path in $\mathbf{Met}(M)$, with $[t] = L^2$	Papers 103, 110, 119, 121

The fourth type is the one appearing in the Ricci flow $\partial_t g_{ij} = -2R_{ij}$ and in renormalisation-group flow, and **Paper 119 settled its identity by its dimension $[t] = L^2$** .

3. Putting the four clocks through the formula

Theorem 3.1 (the four routes)

Applying Theorem 2.1 to each type of Fact 2.3 gives, in d dimensions (verified symbolically):

Clock	dimension	p	q	N
dt	T	$1/n$	$1/n$	$-1 - 3/n$
$d\tau$	T	$1/n$	$1/n$	$-1 - 3/n$
dS	(contains Θ)	—	—	does not enter the formula (Obs. 3.3)
flow parameter	L^2	$2/n$	$2/n$	$-6/n$

Corollary 3.2 (natural units at $d = 3$)

Substituting $n = 2$ (verified numerically):

$$dt, d\tau \longrightarrow \hbar^{1/2} G^{1/2} c^{-5/2} = t_P = 5.3912 \times 10^{-44} \text{ s}$$

$$dS \longrightarrow k_B = 1.380649 \times 10^{-23} \text{ J/K}$$

$$(\text{flow parameter}) \longrightarrow \hbar G c^{-3} = \ell_P^2 = 2.6123 \times 10^{-70} \text{ m}^2$$

Four clocks, three different units.

Observation 3.3 (the third clock requires a fourth constant)

For dS one halts before applying the formula at all. Entropy has dimension $[S] = ML^2T^{-2}\Theta^{-1}$, which contains the temperature Θ ; and no combination whatever of $\hbar$, G_d and c produces Θ . Equation (1) therefore cannot be applied until either k_B is spent or $k_B = 1$ is declared (a convention; Paper 95).

Writing k_B explicitly as $X = \hbar^p G^q c^N k_B^s$ forces $s = 1$, and the remainder must then be dimensionless, giving $p = q = N = 0$. So the natural unit of dS is k_B itself — not a Planck quantity but a different currency.

(A recorded self-correction) In the early working of this paper it was written that “ dS has no scale”. **That was wrong.** It has a scale; the scale simply lies outside $\hbar, G, c$. “Having none” and “being elsewhere” are different.

Observation 3.4 (the fourth clock's unit is an area, not a time)

The route of the flow parameter is $q = 2/n$, that is

$$(\hbar G_d c^{-3})^{2/n} = \left[(\hbar G_d c^{-3})^{1/n} \right]^2 = \ell_P^2$$

the square of the Planck length in every dimension. At $d = 3$ this is $2.6123 \times 10^{-70} \text{ m}^2$.

To read it as a time one must divide by c^2 : $\ell_P^2/c^2 = t_P^2$ (verified numerically: both $2.9066 \times 10^{-87} \text{ s}^2$). **That step spends a convention** (Paper 95). **Calling the fourth clock a “time” at all has already passed through one convention, namely c .**

4. [The core] The tool halts between dt and $d\tau$

Theorem 4.1 (non-injectivity)

In Theorem 3.1, dt and $d\tau$ have identical (p, q, N) :

$$(p, q, N)_{dt} = (p, q, N)_{d\tau} = \left(\frac{1}{n}, \frac{1}{n}, -1 - \frac{3}{n} \right)$$

(verified symbolically: they agree with d left as a letter).

And by Observation 2.2 this is not the formula's doing. The formula is a bijection and carries (λ, μ, τ) across without loss. **The distinction is lost one step earlier** — where two objects of different logical type, a coordinate and a functional of a curve, are **assigned the same dimension T** .

Corollary 4.2 (Paper 118's logical types were forced, not chosen)

Paper 118 separated dt from $d\tau$ by **logical type** — a coordinate versus a functional of a curve. **That choice of discriminant was not a matter of style.** As Theorem 4.1 shows, **a tool that reads dimensions alone cannot separate these two in principle.** Paper 118 had no alternative.

This is not an endorsement of Paper 118 but a proof of its necessity. The corpus holds two tools, and **where one halts the other begins.**

Observation 4.3 (a third case beside Paper 120's two corridors)

Paper 120 separated **two ways in which dimensional analysis breaks** — the β corridor (a quantity absent from the input is born: dimensional transmutation) and the γ corridor (the exponent itself shifts: anomalous dimensions).

In both, dimensional analysis returns a wrong answer.

Case	What happens	The tool
β (Paper 120)	a quantity is born	errs
γ (Paper 120)	the exponent shifts	errs
this paper	right, but not unique	does not err; it merely does not reach

Here dimensional analysis returns a **correct** answer for dt and for $d\tau$ alike: the natural unit really is t_P in both cases. **The trouble is that the answers coincide, not that they are wrong.**

So this is not a breakdown but a limit of resolution, and the only one of the three in which the tool is blameless. The β and γ corridors mark where dimensional analysis must not be trusted. What this paper adds is **where it may be trusted and is still not enough.**

5. Sensitivity to the dimension

Observation 5.1 (the weights are 1, 1, 0, 2)

As Paper 116 showed, the spatial dimension enters only through the denominator $n = d - 1$, and its doorway is q . Taking $q \cdot n$ for the four clocks:

Clock	q	$q \cdot n$
dt	$1/n$	1
$d\tau$	$1/n$	1
dS	0	0
flow parameter	$2/n$	2

The flow parameter feels the dimension exactly twice as strongly as ordinary time. And the entropy clock **does not feel it at all** — a consequence of Observation 3.3: living outside the world of $\hbar, G, c$, it inherits none of that world's dependence on d .

Observation 5.2 (the dimension of a quantity and its route to a Planck value are different things)

The flow parameter has $[t] = L^2$. This follows from $\partial_t g_{ij} = -2R_{ij}$ with g dimensionless and R_{ij} of dimension L^{-2} , and it is the same in every spatial dimension: the dimension itself is blind to d .

Yet the route by which that quantity reaches a Planck value, $q = 2/n$, **does depend on d** . “The dimension of a quantity” and “how that quantity is built from the fundamental constants” are different matters, and one may be blind to d while the other is not.

This is **the same cut** Paper 116 made for c^3 , where the “3” was separated out as a weight of time rather than a spatial dimension. Here that cut is applied to the clocks.

6. A note: origin is not type

Observation 6.1 (a human-made clock is not a fifth type)

A calendar, a caesium clock, a Renko bar on a currency chart (Paper 82) — all are **clocks made by people**. But each has dimension T , and the map of Theorem 3.1 returns **the same image** as for dt .

So “made by people” is not a property this census can see. What the census sorts is **logical type**, not **origin**. The two are **orthogonal axes**.

The conclusion is that a human-made clock is not a fifth type but an instance of the first (the coordinate). The Renko bar of Paper 82 is a coordinate in which a person chose what counts as one bar; as a type it stands with dt . **The number of types remains four.**

The reason for placing this note is to avoid a collision of numbering. Paper 118 explicitly refused to call felt time “the fourth”, on the ground that “the number belongs to the types on the physics side”. This paper keeps the same discipline: “the fourth clock” means the flow parameter (Paper 119) and nothing else.

7. Scope

- **What this paper shows:** the routes (p, q, N) and natural units of the four logical types under Paper 116's formula (Theorem 3.1, Corollary 3.2); that dS lies outside the formula with k_B as its unit (Observation 3.3); that the fourth clock's unit is an **area** (Observation 3.4); **that the formula cannot separate dt from $d\tau$** (Theorem 4.1) and that Paper 118's logical types were therefore forced (Corollary 4.2); that this is a **third case** beside Paper 120's two corridors (Observation 4.3); and that the sensitivities are **1, 1, 0, 2** (Observation 5.1). **All of it is the application of existing tools, and none of it is new physics.**
- **What this paper does not show:** (i) any new physical law — **there is none**; (ii) that four logical types exhaust the list — **closure of the census is not demonstrated** (just as Paper 119 reopened Paper 118's census, a fifth is not excluded); (iii) that agreement of natural units implies physical identity — **dt and $d\tau$ share a unit and remain different objects**, which is the point of Theorem 4.1; (iv) the sorting “three belong to nature and one to us” — **origin is not a property this census can see** (Observation 6.1); (v) the legitimacy of calling the fourth clock a “time” — **that passes through the convention c^2** (Observation 3.4).
- **What remains: a tool has a reach.** Writing down where it does not reach strengthens the corpus exactly as much as writing down where it does.

8. Conclusion

1. Paper 116's exponent formula was applied to the four logical types of Papers 118 and 119, giving the routes (p, q, N) and the natural units at $d = 3$ (Theorem 3.1, Corollary 3.2, verified symbolically and numerically). **Four clocks carry three different units — t_P , k_B , ℓ_P^2 .**
2. **dS lies outside the formula:** $[S]$ contains temperature, and Θ appears in no combination of $\hbar$, G_d , c . Its natural unit is k_B itself, not a Planck quantity (Observation 3.3).
3. **The fourth clock's unit is the Planck area ℓ_P^2 , in every dimension.** Reading it as a time requires dividing by c^2 , and **that step spends a convention** (Observation 3.4).
4. **[The core] The formula cannot distinguish dt from $d\tau$** — their (p, q, N) coincide (Theorem 4.1). Since the formula is a bijection (Observation 2.2), **the distinction is lost one step earlier, where a dimension is assigned to an object.** Hence **Paper 118's discriminant of logical type was forced, not chosen** (Corollary 4.2).
5. **This is a third case beside Paper 120's two corridors:** in β and γ dimensional analysis **errs**, whereas here it is **right and not unique**. A limit of resolution rather than a breakdown, and **the only one of the three in which the tool is blameless** (Observation 4.3).
6. **The sensitivities to dimension are 1, 1, 0, 2** in units of $1/n$: the flow parameter twice that of ordinary time, the entropy clock zero (Observation 5.1). Yet $[t] = L^2$ is itself blind to d , so **a quantity's dimension and its route to a Planck value are different things** (Observation 5.2).

7. **A human-made clock is not a fifth type but an instance of the first.** Origin is an axis orthogonal to logical type and is not visible to this census (Observation 6.1). **“The fourth clock” means the flow parameter (Paper 119) and nothing else.**

Closing: Paper 116 asked where the dimension had been, and found it in a denominator. This paper turned the same tool on the clocks and watched it **halt between dt and $d\tau$** . It halted not through any fault of its own — **those two differ in a way that the word “dimension” cannot tell apart. Knowing the reach of a tool is part of having one.**

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