

There Are Three Hopfs

— A Proper Name Also Needs an Address: Applying One Word, Distinct Roots to People (the Yamagishi Distinct-Root Census, Applied to Names) — [Paper 125]

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— *From thinking to syncing.* —

ABSTRACT

This corpus has built up a discipline for the case where the same number appears in different places: trace each occurrence back along its own path and call the roll (Papers 17, 34, 100, 124). This paper applies that discipline **not to a number but to a proper name. The occasion was a research roadmap produced by an external generative AI.** It instructed that the Cole--Hopf transformation (which linearises Burgers' equation into the heat equation) be connected with **“Hopf's geometric connection (Paper 67)”**. But **the Hopf of Cole--Hopf and the Hopf of Paper 67 are different people.** The same roadmap treated Heinz Hopf under one item and Eberhard Hopf under another, and placed them side by side as connectable. **First, calling the roll gives three.** The work began under the title “there are two Hopfs”. Counting properly gives **Heinz Hopf** (1894--1971, topology), **Eberhard Hopf** (1902--1983, analysis and dynamical systems), and **Ludwig Hopf** (1884--1939, physics; Einstein's first assistant) — **three** (Section 3). **Second, a criterion can be written.** Heinz's objects are **topological invariants** and survive continuous deformation. Eberhard's objects **depend on the equation** and fail if the equation is changed. Ludwig's objects are **physical models carrying dimensions. The criterion separates the two at exactly the point where they were conflated:** the Cole--Hopf transformation depends on the quadratic nonlinearity of Burgers' equation, while the Hopf fibration is not destroyed by any homeomorphism (Section 4). **Third, name collisions come in three degrees.** (i) one person, one name, two theorems (Noether's first and second, Paper 107); (ii) one person, one name, unrelated objects (Heinz's fibration and Hopf algebras); (iii) **several people, one name** (three Hopfs, two Cartans). **Paper 107 treated (i); this paper adds (iii)** (Section 5). **Fourth, the corpus audited itself.** Paper 92 writes “Élie Cartan” with the given name; Papers 80, 85 and 107 write “Emmy” for Noether; every mention of Hopf in the corpus is Heinz. **There is no misattribution.** One point, however: **the term “Hopf decomposition” used in Paper 45 denotes, in ergodic theory, a theorem of Eberhard Hopf — the object is right and the word collides** (Section 6). **Fifth, proper names fail more quietly than numbers do.** A reader who meets a number eventually asks where it came from. A proper name, by contrast, **appears to state its own provenance.** “Hopf's theorem” looks self-documenting; it is not (Section 7). **NO NEW MATHEMATICS IS CLAIMED.**

Scope of this paper: NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED. That the three Hopfs are different people is obvious to any mathematician, and every theorem cited here is standard. What this paper adds is only: **extending the corpus's “write the address” discipline to proper names, classifying the collisions, writing a criterion, and auditing the corpus's own vocabulary.**

1. Notation and position

This corpus has been checking “one word, distinct roots”. When the same symbol or the same number appears in different places, whether they share a root is settled by **tracing the path of the calculation — the address** (Papers 17, 34, 100, 124).

What this paper adds is an extension of the object under inspection. The roll has so far been called on **numbers: 4π , 8π , c^5** . Here the same roll is called on **proper names**.

Position: No new mathematics is claimed. The subject is **a discipline of attribution**, not mathematics.

2. The occasion: a roadmap connected two people

Observation 2.1 (what actually happened)

In August 2026 a research roadmap, produced for this corpus by an external generative AI, contained the following instruction.

Take the mechanism by which the Cole--Hopf transformation reduces a nonlinear shock wave to the linear heat equation, and census it as the meeting point of **Hopf's geometric connection (Paper 67)** and thermal diffusion.

The Hopf of the Cole--Hopf transformation is Eberhard Hopf. The Hopf of Paper 67 (Poincaré--Hopf, the Hopf map) is **Heinz Hopf**. They are different people, and there is no scholarly relation between them.

Observation 2.2 (the evidence is inside the roadmap)

The same roadmap treated the **Poincaré--Hopf index theorem** (Heinz) under one item and the **Cole--Hopf transformation** (Eberhard) under another. **It placed the two men in one document and treated them as connectable.**

This is not a reproach. Where only a surname is given, that reading is available. **It is, rather, a good illustration of how smoothly a proper name can mislead.**

3. Calling the roll: how many Hopfs are there

Observation 3.1 (it began at two)

This paper was first drafted under the title “There Are Two Hopfs”. **Calling the roll properly gave three.** That too illustrates the nature of the exercise: **do not fix the count before counting.**

3.1 Heinz Hopf (1894--1971)

Born in Germany, later at ETH Zurich. A man of **topology**.

- The Hopf fibration $S^3 \rightarrow S^2$ (1931)
- The Poincaré--Hopf index theorem (1926--27)
- The Hopf invariant, Hopf algebras, the Hopf--Rinow theorem, the Hopf link

Every Hopf this corpus has dealt with so far is this man (Papers 45, 67, 68, 104).

3.2 Eberhard Hopf (1902--1983)

Born in Austria; active in Germany and the United States. A man of **analysis and dynamical systems**.

- The Wiener--Hopf method (1931, with Norbert Wiener)
- The Hopf bifurcation (1942)
- The Cole--Hopf transformation (1950; Julian Cole 1951)
- Hopf's maximum principle, Hopf's lemma, the Hopf decomposition of ergodic theory

3.3 Ludwig Hopf (1884--1939)

A German **physicist**. Einstein's first assistant, and a student of Sommerfeld on turbulence and hydrodynamic stability. He left Germany under the Nazi regime and died in Dublin in 1939.

- The Einstein--Hopf paper (1910), on the Brownian motion of a resonator in a radiation field
- Work on hydrodynamic stability and turbulence; aerodynamics

Observation 3.2 (two of them inside fluid dynamics)

Ludwig worked on **hydrodynamic stability**; Eberhard produced the **Hopf bifurcation**, a concept central to the onset of turbulence. **Two different Hopfs inside one field**. Written by surname alone, they will certainly be mixed.

4. The criterion: topologically invariant, or equation-dependent

Proposition 4.1 (the three men's objects separate by character)

Who	Character of the object	Test
Heinz	a topological invariant	does it survive a homeomorphism?
Eberhard	depends on the equation	does changing the equation destroy it?
Ludwig	a physical model with dimensions	does a dimensional quantity enter?

Observation 4.2 (the criterion actually separates them at the point in question)

The **Cole--Hopf transformation** depends on the **quadratic nonlinearity** of Burgers' equation. Change the form of the nonlinear term and the transformation fails. **It is equation-dependent** — hence Eberhard.

The **Hopf fibration** is a **topological property** of the space S^3 . Continuous deformation does not destroy it, and no equation is required. **It is a topological invariant** — hence Heinz.

The two stand on opposite sides of the criterion. One vanishes if the equation changes; the other survives any deformation. **There is no room for a connection.**

5. Name collisions come in three degrees

Observation 5.1 (a three-level roll)

Degree	Shape	Example
(i)	one person, one name, two theorems	Noether's first and second theorems (Paper 107)
(ii)	one person, one name, unrelated objects	Heinz's fibration, and Hopf algebras
(iii)	several people , one name	Hopf (three), Cartan (two)

Paper 107 treated (i). It showed that within the work of one Emmy Noether, the first theorem (global symmetry to conserved quantity) and the second (local symmetry to identities) have **distinct roots**. **Two theorems parted inside one person.**

This paper adds (iii). The name is the same and **the person is different**.

(ii) is also real. Heinz Hopf's name attaches both to the Hopf fibration (topological) and to Hopf algebras (an algebraic structure), and **these two are not directly related objects**. One person, one name, two unrelated things.

There are also two Cartans. Élie Cartan (1869--1951; exterior calculus, connections) and his son Henri Cartan (1904--2008; sheaves, homological algebra). **The one treated in Paper 92 of this corpus is the father, Élie.**

6. Auditing this corpus

A discipline must first be applied to oneself. In preparing this paper, the existing corpus was inspected.

Observation 6.1 (no misattribution was found)

- **Cartan:** Paper 92 writes the given name, “**Élie Cartan**”.
- **Noether:** Papers 80, 85 and 107 write “**Emmy**”.
- **Hopf:** every mention in the corpus is **Heinz** (the fibration, Poincaré--Hopf, 1931).

The discipline was already inside the corpus. It had simply not been made systematic.

Observation 6.2 (one word, however, collides)

Paper 45 calls the splitting of S^3 into base and fibre a “**Hopf decomposition**”. The object it denotes is correct: **Heinz's fibration**.

But the term “**Hopf decomposition**” has an established and different meaning in ergodic theory — the splitting of a measure space, under a nonsingular transformation, into its **conservative and dissipative parts** — and that theorem is **Eberhard Hopf's**.

So the object is right while the word collides with a different man's theorem. The repair is trivial: writing “**the decomposition by the Hopf fibration**” removes the collision. Nothing in Paper 45's claim changes.

7. Why a proper name fails more quietly than a number

Observation 7.1 (numbers get questioned; names do not)

A reader who meets a number will eventually ask where it came from. What this corpus has built is the way of asking that question.

A proper name, by contrast, appears to state its own provenance. The phrase “Hopf's theorem” reads as though **the attribution were already written down**. Whose theorem it is looks like it is right there.

But what is written there is only a surname. A surname is not an address. An address is who the person was, when, in which paper, and what they showed.

Observation 7.2 (hence names fail more quietly)

A mistaken number is caught by checking dimensions (Paper 60), or by tracing the path of the calculation (Paper 52). **A mistaken person, however, never appears in the calculation at all.** The equations run correctly. The dimensions match. **Nothing warns.**

The rule fits on one line.

When citing a person, do not settle for the surname. Add the dates, or at least the given name.

The cost is almost nothing. Yet for want of it, an external roadmap connected two Hopfs.

8. Conclusion

1. **There are three Hopfs** (Section 3): Heinz (1894--1971, topology), Eberhard (1902--1983, analysis and dynamical systems), Ludwig (1884--1939, physics). **The work began at two and the roll returned three. Do not fix the count before counting.**
2. **A criterion can be written** (Proposition 4.1): topologically invariant, equation-dependent, or a dimensional physical model. **It separates the two at exactly the point where they were conflated** — Cole--Hopf depends on the equation, the Hopf fibration survives any homeomorphism (Observation 4.2).
3. **Name collisions come in three degrees** (Observation 5.1): one person and two theorems (Paper 107); one person and unrelated objects; **several people and one name. This paper adds the third.**
4. **No misattribution was found in this corpus** (Observation 6.1). Cartan is Élie, Noether is Emmy, and every Hopf is Heinz. **The discipline was already there; it had not been made systematic.**
5. **One word does collide** (Observation 6.2). Paper 45's "Hopf decomposition" **denotes the right object**, but in ergodic theory that term is **Eberhard's theorem**. Writing "the decomposition by the Hopf fibration" removes it.
6. **Proper names fail more quietly than numbers** (Section 7). A wrong number is caught by dimension or by path; **a wrong person never enters the calculation**. The equations run, the dimensions match, and nothing warns.

Closing: This corpus has built the discipline of asking, of a number, where it came from. **The same question must be put to a name.** The phrase "Hopf's theorem" looks as though the attribution were written into it. **Only the surname is written. A surname is not an address.**

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