

Of $k_B T \ln 2$, Only the T Is Physics

— The Three Parts of Landauer's Bound, and Why an Irreversible Gate Does Not Pay One Bit (the Yamagishi Census of Erasure) — [Paper 131]

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August 2026

— *From thinking to syncing.* —

ABSTRACT

The third edition of Paper 04 wrote that Landauer's bound constrains only **irreversible implementations** (Bennett 1973). **This paper takes the number itself apart. NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED. First, the 2 in $\ln 2$ is a count of states, and it moves.** The minimum work to erase into N states is $k_B T \ln N$. **The 2 appears only because the bit was chosen as the unit of information** — in nats the coefficient is exactly 1, in trits it is $\ln 3$. **It is not a constant of nature** (Section 2). **Second, k_B is a conversion factor.** Since the 2019 SI revision k_B is a **defined value**, and measuring temperature in energy units removes it from the formula altogether. **A quantity that disappears when the units change is not physics** (Papers 95, 117) (Section 3). **Third, and this is the core. What survives every choice of units is only the dimensionless content, namely the entropy lost in the erasure, that is $W_{\min}/(k_B T)$.** Two things remain as physics — that the bound is proportional to T , and that it is not zero. **Neither k_B nor $\ln 2$ is among them** (Section 4). **Fourth, an irreversible gate does not pay one bit.** For an AND gate on two uniform inputs the reduction of logical entropy is **1.1887 bits**, not 1 (computed here). And it **moves with the input distribution** — for Bernoulli inputs of parameter **0.3** it is **1.3261**, and of **0.9** it is **0.2365**. **What sets the cost is not the number of wires but the reduction of logical entropy** (Section 5). **Fifth, the same $k_B T \ln 2$ has historically been placed at two different steps.** Szilard (1929) and Brillouin (1951) put it at **measurement**; Landauer (1961) and Bennett (1982) put it at **erasure**. **The number is the same and the address is different. And the latter superseded the former:** measurement can in principle be made reversible, but a memory must eventually be reset (Section 6). **Sixth, the bound is a boundary that is never reached.** In the finite-time form $W(\tau) = k_B T \ln 2 + A/\tau$, the excess A/τ is **positive for every finite τ** . **The bound is a statement about work and says nothing whatever about time** (Paper 113) (Section 7).

Scope of this paper: NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED.

Landauer (1961), Bennett (1973, 1982), Szilard (1929), Brillouin (1951) and the experiment of Bérut et al. (2012) are **all standard**. What this paper adds is only **sorting $k_B T \ln 2$ into convention, convention and fact, computing the cost of an irreversible gate and showing it is not one bit, and setting the two addresses of one number side by side. It is not claimed that Landauer's bound is wrong** (it is not, and it has been verified experimentally).

1. Notation and position

Erasure is **the operation of collapsing several logical states into one**. Landauer's principle says that performing it on a system in contact with an environment at temperature T has a **lower bound on the work**,

$$W_{\min} = k_B T \ln 2 \quad (\text{for the erasure of one bit}). \quad (1)$$

What the third edition of Paper 04 laid down: that this bound constrains **irreversible implementations** rather than search itself (Bennett 1973), and that **only the logarithm does not move**.

What this paper lays down: a **decomposition of the right-hand side** of (1). Paper 95 separated **convention from fact**, and Paper 129 said that **mathematics supplies the shape and physics the scale. This paper turns the same blade on Landauer's bound**.

2. The 2 in $\ln 2$ is a count of states

Established Fact 2.1 (the general form): The minimum work to collapse N equiprobable logical states into one is

$$W_{\min} = k_B T \ln N.$$

Verification 2.2: This paper tabulates $\ln N$ for $N = 2, 3, 4, 8, 10$. **Only at $N = 2$ does it read $\ln 2$.** (QED)

Observation 2.3 (the 2 moves)

Take the unit of information to be the **nat** and the coefficient is **exactly 1**; take the **trit** and it is **$\ln 3$** . **The 2 is not a number nature chose but a unit we chose.**

Observation 2.4 (a distinct root from other twos)

The corpus is full of twos. **The separator is whether the number moves when something is moved. The 2 of $\ln 2$ moves when the number of states changes. The 2 of the Hopf map (Paper 68) does not — it is the structure of a fibration, not a choice of unit. The same character, a different root (Paper 17).**

3. k_B is a conversion factor

Observation 3.1 (it is a defined value)

Since the 2019 SI revision, k_B is **not measured but defined. Improving measurement does not change its value**; what improves is the realisation of the kelvin.

Observation 3.2 (it can be removed)

Measuring temperature in energy units, that is setting $\theta = k_B T$, turns (1) into

$$W_{\min} = \theta \ln 2,$$

in which k_B occurs nowhere. **A quantity that a change of units removes is not physics** (Papers 95, 117).

Observation 3.3 (the discipline of Paper 10)

Paper 10 said that **only dimensionless quantities can be tuned. $k_B T \ln 2$ carries a dimension (energy). Its value is therefore not a prediction.**

4. Only the dimensionless content survives

Observation 4.1 (the invariant part)

What survives every choice of units is

$$\frac{W_{\min}}{k_B T} = \Delta S/k_B = \text{the entropy lost in the erasure, in nats,}$$

and neither k_B nor $\ln 2$ occurs on the right.

Observation 4.2 (two things remain as physics)

part of $k_B T \ln 2$	standing
k_B	convention (a conversion factor; defined since 2019)
$\ln 2$	convention (the unit of information; $\ln N$ for N states)
proportionality to T	fact
the bound being nonzero	fact

Paper 129 said that $\hbar$ is not on the mathematical side of the inequality. Here the census runs the other way: in Landauer's bound it is the number that is mostly convention, and the physics resides only in two statements, a proportionality and a non-vanishing.

And those two are not light. That erasing information carries a thermodynamic price is itself a discovery, and it has been verified experimentally (Bérut et al. 2012). Strip the conventions and what remains, remains.

5. An irreversible gate does not pay one bit

Observation 5.1 (a common formulation)

It is sometimes said that an irreversible logic gate dissipates $k_B T \ln 2$ per operation. This is not correct.

Verification 5.2 (the AND gate): For two independent inputs, each Bernoulli with parameter p , this paper computes the logical entropies of input and output and obtains

p	$H(\text{input})$ bits	$H(\text{output})$ bits	reduction (bits)
0.50	2.0000	0.8113	1.1887
0.30	1.7626	0.4365	1.3261
0.10	0.9380	0.0808	0.8572
0.90	0.9380	0.7015	0.2365

Even for uniform inputs it is not 1 (it is 1.1887). And it moves with the input distribution, above 1 and below it. (QED)

Observation 5.3 (what sets the cost)

What sets the cost is **neither the number of wires nor the number of gates but the reduction of logical entropy**. The phrase “a one-bit gate” is naming two different quantities with one word — the width of the output and the information lost. They agree only in special cases.

Observation 5.4 (and erasure is coarse-graining)

Logical irreversibility is **two states collapsing into one**, and since the microscopic dynamics is reversible (Paper 77) that much **must be handed to the environment** (Paper 78). Landauer's bound is the lower bound on that hand-over.

6. One number, placed at two steps

Observation 6.1 (the historical census)

who	where it was placed	what is claimed
Szilard 1929, Brillouin 1951	measurement	measurement costs $k_B T \ln 2$
Landauer 1961, Bennett 1982	erasure	measurement is free, erasure costs

Observation 6.2 (same number, different address)

Both assign the same $k_B T \ln 2$ to different steps of the same cycle. And the second replaced the first: measurement into a blank memory can be built reversibly, but memory is finite, and closing the cycle requires resetting it. That is where the price falls due.

Observation 6.3 (the lesson for the corpus)

That a number explains something successfully does not establish where it was placed. Brillouin's assignment was numerically right. What was wrong was the address (Paper 17). An agreement of numbers is not a proof of address (Paper 100).

7. The bound is a boundary never reached

Verification 7.1: In the standard finite-time form, the work of erasure in time τ is

$$W(\tau) = k_B T \ln 2 + \frac{A}{\tau} \quad (A > 0),$$

and this paper confirms that the limit as $\tau \rightarrow \infty$ is $k_B T \ln 2$ while the excess A/τ is positive for every finite τ . (QED)

Observation 7.2 (what it does not say)

Landauer's bound is a statement about work and says nothing about time. The phrase “the minimum energy” is doubly inaccurate: it is work, not energy, and it is not attained in finite time.

Observation 7.3 (the frame of Paper 113)

This is a boundary approached and not reached. Paper 113 drew the distinction between a wall and a slope. Landauer's bound is the foot of a slope, not a wall.

8. Conclusion

1. **The 2 in $\ln 2$ is a count of states, and it moves** (Ver. 2.2). Erasure into N states costs $k_B T \ln N$; **in nats the coefficient is 1. It is a unit, not a constant of nature.**
2. **k_B is a conversion factor** (Obs. 3.2). Measure temperature in energy and it **leaves the formula. What can be removed is not physics** (Papers 95, 117).
3. **Only the dimensionless content survives** (Obs. 4.2). The physics is **proportionality to T and non-vanishing**, and **neither k_B nor $\ln 2$ is in it. Those two are not light** — the bound has been verified experimentally.
4. **An irreversible gate does not pay one bit** (Ver. 5.2). AND on uniform inputs loses **1.1887 bits**, and the figure ranges from **0.2365 to 1.3261** with the input distribution. **The cost is the reduction of logical entropy, not a count of wires.**
5. **One number has been placed at two steps** (Obs. 6.1): measurement by Szilard and Brillouin, erasure by Landauer and Bennett. **An agreement of numbers is not a proof of address** (Papers 17, 100).
6. **The bound is a boundary never reached** (Ver. 7.1). **It speaks of work and says nothing about time** (Paper 113).

Closing: Seeing $k_B T \ln 2$, do not read it as nature having chosen $\ln 2$. The $\ln 2$ is the trace of our having chosen the bit, and the k_B the trace of our having chosen the kelvin. **What nature chose is the single fact that erasure has a price. And that alone is large enough.**

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