

# The 4 in the Light-Bending Angle Is Not $4\pi$ but $2(1 + \gamma)$

— The Doubling Belongs to  $v = c$ , Not to Gravity: a Distinct-Root Census of the Deflection Angle, the Critical Surface Density and the Einstein Ring (the Yamagishi Census of Lensing) — [Paper 134]

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— *From thinking to syncing.* —

## ABSTRACT

The deflection of light by a point mass is  $\alpha = 4GM/(c^2b)$ . This paper writes what that 4 is made of. NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED. First, the 4 is not  $4\pi$ . It is  $2 \times 2$ . The first 2 is the value of an integral: integrating the transverse force along a straight line, this paper obtains  $\int_{-\infty}^{\infty} GMb/(b^2 + z^2)^{3/2} dz = 2GM/b$  symbolically. No  $\pi$  appears anywhere (Section 2). Second, one of the two cores. The second 2 is a measured quantity. In the parametrised post-Newtonian formalism  $\alpha = (1 + \gamma) 2GM/(c^2b)$ , and general relativity predicts  $\gamma = 1$ . But  $\gamma$  moves between theories: in Brans–Dicke  $\gamma = (1 + \omega)/(2 + \omega)$ , and the table here gives the “4” as 3.333 at  $\omega = 1$  and 3.980 at  $\omega = 100$ . The 4 is not a mathematical necessity. The first half follows from the equivalence principle; the second half is an empirical statement about the metric (Section 3). Third, the exact Schwarzschild deflection integral confirms it. Removing the endpoint singularity by substitution and quadrating, this paper obtains at  $b/m = 10^4$  a ratio of 1.00029464, agreeing to seven digits with the known second-order term  $1 + 15\pi m/(16b) = 1.00029452$  (Section 2). Fourth, the other core. The doubling belongs to  $v = c$ , not to gravity. The deflection of a particle of speed  $v$  is  $1 + v^2/c^2$  times the Newtonian value. The table here gives 2.000 at  $v/c = 1$ , 1.250 at 0.5, 1.010 at 0.1 and 1.0001 at 0.01. The picture that light falls twice as fast is wrong: the factor is 2 only because a photon has  $v = c$  (Section 4). Fifth, so where is the  $4\pi$ ? In the critical surface density.  $\Sigma_{\alpha} = c^2 D_s/(4\pi G D_l D_{ls})$ : that  $4\pi$  comes from Poisson's equation (the “fact” side of Paper 35) and lives in the step that reconstructs the mass, not in the bending angle. The same shape as Paper 130 (Section 5). Sixth, the content of the Einstein ring is that the mean convergence inside it is exactly 1, as verified symbolically here. The exponent 1/2 in  $\theta_E$  is the trace of solving a quadratic, not the geometry of a sphere (Section 6).

**Scope of this paper: NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED.**

Einstein (1915, 1936), Zwicky (1937), the PPN formalism (Will), Brans–Dicke (1961) and the Cassini measurement of  $\gamma$  (Bertotti et al. 2003) are **all standard**. What this paper adds is only **splitting the 4 into  $2 \times 2$  and making explicit that the second half is measured, tabulating the doubling as a property of  $v = c$ , placing the address of the  $4\pi$  at the critical surface density, and writing that the ring's content is a mean convergence of 1. No solid angle is read into the 4** (Papers 45, 101).

## 1. Notation and position

Write the mass as  $M$ , the impact parameter as  $b$ , and  $m \equiv GM/c^2$ . In a weak field the deflection of light is

$$\alpha = \frac{4GM}{c^2 b} = \frac{4m}{b}. \quad (1)$$

**Position:** existing results are laid side by side and taken apart. No new theorem is claimed. Paper 17 said that **calling things distinct-rooted is half the work; only writing the addresses makes the claim checkable. This paper does that for the 4 of (1).**

**And here is why it matters for the corpus: the corpus rests on  $4\pi$  (Papers 01, 02), so a 4 invites being read as a relative of  $4\pi$ . This paper stops that.**

## 2. The first 2 is the value of an integral

**Verification 2.1 (the Newtonian impulse integral):** Integrating the transverse gravitational force along a straight trajectory, symbolic computation here gives

$$\int_{-\infty}^{\infty} \frac{GMb}{(b^2 + z^2)^{3/2}} dz = \frac{2GM}{b},$$

and dividing by  $c^2$  gives  $\alpha_N = 2GM/(c^2 b)$ . (QED)

### Observation 2.2 (no $\pi$ appears)

**This 2 is the value of an integral, neither a solid angle nor a period. No  $\pi$  occurs anywhere in it.**

**Verification 2.3 (the exact deflection integral):** In the Schwarzschild metric the exact deflection is

$$\alpha = 2 \int_0^{u_{\max}} \frac{du}{\sqrt{b^{-2} - u^2 + 2mu^3}} - \pi$$

with  $u = 1/r$ . This paper removes the inverse-square-root endpoint singularity by the substitution  $u = u_{\text{max}} - t^2$ , divides out the simple root, and quadrates.

| $b/m$ | $\alpha$ (exact) | $4m/b$       | ratio             |
|-------|------------------|--------------|-------------------|
| 500   | 0.0080474680     | 0.0080000000 | 1.00593350        |
| 1000  | 0.0040118238     | 0.0040000000 | 1.00295595        |
| 10000 | 0.0004001179     | 0.0004000000 | <b>1.00029464</b> |

At  $b/m = 10^4$  the excess **1.00029464** agrees to **seven digits** with the known second-order term  $1 + 15\pi m/(16b) = 1.00029452$ . In a weak field the general-relativistic value is exactly twice the Newtonian one. (QED)

3. The second 2 is measured

**Established Fact 3.1 (the PPN formalism):** For metric theories generally, the deflection of light can be written

$$\alpha = (1 + \gamma) \frac{2GM}{c^2 b},$$

(4)

where  $\gamma$  measures **how much spatial curvature is produced per unit mass**, and **general relativity predicts  $\gamma = 1$** .

Observation 3.2 (the breakdown)

(4) splits the 4 in two.

| which half        | where it comes from                                      | standing            |
|-------------------|----------------------------------------------------------|---------------------|
| $2 \times 1$      | the warping of time (the equivalence principle suffices) | nearly forced       |
| $2 \times \gamma$ | spatial curvature                                        | a measured quantity |

**Verification 3.3 ( $\gamma$  moves between theories):** In Brans–Dicke theory  $\gamma = (1 + \omega)/(2 + \omega)$ , and the computation here gives

| $\omega$ | $\gamma$   | the value of the “4” |
|----------|------------|----------------------|
| 1        | 0.66666667 | 3.33333333           |
| 10       | 0.91666667 | 3.83333333           |
| 100      | 0.99019608 | 3.98039216           |
| 40000    | 0.99997500 | 3.99995000           |

The “4” moves. (QED)

**Observation 3.4 (so the 4 is a measurement)**

The Cassini radio-delay measurement puts  $\gamma - 1$  consistent with zero at the level of  $10^{-5}$  (Bertotti et al. 2003).  
The “4” is therefore a quantity measured to five digits. It is not a mathematical necessity.

This is an instance of Paper 95's sorting: the half coming from the equivalence principle sits inside the theory; the half coming from spatial curvature is held up by measurement. Within one formula, the two halves are of different standing.

#### 4. The doubling belongs to $v = c$

**Established Fact 4.1:** The weak-field deflection of a particle of speed  $v$  is standardly

$$\alpha(v) = \frac{2GM}{bv^2} \left( 1 + \frac{v^2}{c^2} \right),$$

so the ratio to the Newtonian value is  $1 + v^2/c^2$ .

#### Verification 4.2

| $v/c$     | ratio to Newtonian |                     |
|-----------|--------------------|---------------------|
| 1         | 2.00000000         | the famous doubling |
| 0.5       | 1.25000000         |                     |
| 0.1       | 1.01000000         |                     |
| 0.01      | 1.00010000         |                     |
| $10^{-4}$ | 1.00000001         |                     |

(QED)

**Observation 4.3 (correcting the picture)**

Saying that light falls twice as fast is not right. The factor is  $1 + v^2/c^2$ , and it is 2 only because a photon has  $v = c$ . A slow particle barely feels the second half.

**Observation 4.4 (why)**

The Newtonian half is carried by the **time component** and the other half by the **spatial components**. A **slow particle's worldline points almost entirely along time**, so it sweeps almost none of the spatial part. **Light sweeps time and space equally**. The **2** is the consequence of that.

And the separation from the corpus's other twos can be written (continuing the table of Paper 132).

| which 2                 | what fixes it                  | what moves it         |
|-------------------------|--------------------------------|-----------------------|
| the deflection's        | $v/c$ and $\gamma$             | speed, and the theory |
| Itô's (Paper 132)       | the Hurst exponent of the path | $H$                   |
| the $n = 2$ of Paper 01 | the spatial dimension          | $d$                   |
| the Hopf map's          | a fibration                    | nothing               |

## 5. So where is the $4\pi$ ?

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### Observation 5.1 (not in the deflection angle)

Neither (1) nor (4) contains a  $\pi$ . The 4 is  $2 \times 2$  and no relative of  $4\pi$ .

### Observation 5.2 (it is in the critical surface density)

Writing the lens equation produces the critical surface density

$$\Sigma_{\text{cr}} = \frac{c^2 D_s}{4\pi G D_l D_{ls}},$$

and that  $4\pi$  comes from Poisson's equation, the same one Paper 35 placed on the side of fact.

### Observation 5.3 (the address is the reconstruction step)

The  $4\pi$  appears where an observed distortion is turned back into a mass distribution. It is not in the bending angle. The same shape as Paper 130, where the  $4\pi$  lived not in Helmholtz's theorems but in Biot–Savart, that is, in the inverse problem. Lensing repeats it (Papers 89, 99).

To point at a  $4\pi$ , look not at the bending angle but at the formula that reads the mass back.

## 6. The ring's content is a mean convergence of 1

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**Verification 6.1:** The Einstein radius satisfies  $\theta_E^2 = 4GM D_{ls} / (c^2 D_l D_s)$ . Computing the mean convergence inside that radius, that is the mean surface density divided by the critical one, symbolic computation here gives **exactly 1**. (QED)

### Observation 6.2 (this is the geometric content)

The Einstein ring is the radius at which the projected mean density equals the critical density. That single sentence is the condition for the ring to exist.

### Observation 6.3 (where the exponent $1/2$ comes from)

The square root in  $\theta_E$  is a consequence of the lens equation being quadratic. It did not come from the geometry of a sphere. Paper 28 said that an exponent is not a dimension. Here too the exponent announces the degree of the equation that was solved.

## 7. Conclusion

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1. The 4 is not  $4\pi$  but  $2 \times 2$  (Ver. 2.1). The first 2 is the value of an integral, and no  $\pi$  appears.
2. The exact deflection integral confirms it (Ver. 2.3). At  $b/m = 10^4$  the ratio is 1.00029464, agreeing to seven digits with  $1 + 15\pi m/(16b) = 1.00029452$ .
3. The second 2 is measured (Ver. 3.3). With  $\alpha = (1 + \gamma)2GM/(c^2b)$ , Brans–Dicke at  $\omega = 1$  turns the “4” into 3.333. Cassini puts  $\gamma - 1$  at zero to  $10^{-5}$ . The 4 is a measurement, not a necessity.
4. The doubling belongs to  $v = c$  (Ver. 4.2). The ratio is  $1 + v^2/c^2$ , only 1.010 at  $v/c = 0.1$ . Light does not fall twice as fast.
5. The  $4\pi$  is in the critical surface density (Obs. 5.2), coming from Poisson's equation and living in the reconstruction step (Papers 35, 130).
6. The ring's content is a mean convergence of 1 (Ver. 6.1), and the exponent  $1/2$  is the trace of a quadratic (Paper 28).

**Closing:** Do not let a 4 call  $4\pi$  to mind. This one is made of a 2 from an integral and a 2 from a measured  $1 + \gamma$ . And the second is felt in full only because light travels at  $v = c$ . Until a number is broken into its parts, what it is saying is not known.

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